CHARACTERISTIC PHASE E-SEQUENCES IN EFFICIENT PULSE-COMPRESSION METHODS USING DISCRETE WAVELET DECOMPOSITION

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ABSTRACT

We derive a family of sequences which obey a characteristic-phase constraint as it applies to finite sequences. We show these sequences are special cases of the well-known Welti sequences. We construct a transmit waveform which allows extremely efficient decomposition, whose outputs are simultaneously interpretable as adjacent correlation-filter outputs, and as lossless input signal representations. We discuss implementation of receive and transmit functions.

Index Terms— Synthetic aperture radar, pulse compression methods, wavelet transform

1. INTRODUCTION

Modern synthetic aperture radar (SAR) system performance demands, the opportunities of low-power, complex VLSI processing, and miniaturization pressures have converged to produce interesting system challenges with many tradeoffs between performance, complexity, power consumption, and size. Current research proposals include wafer-scale phased-array antenna systems wherein most of the supporting electronics, including the transmit/receive modules, waveform generation circuits, receive-waveform digitization, phase shifters (or time delays), control hardware, interconnect I/O, signal processing, and supervising microcontroller, are to be implemented a single silicon wafer. Such a system should use digital architectures where feasible to help achieve performance, cost, and yield objectives.

If such a system is also to achieve advances in signal processing capabilities, then there exists a critical need to achieve these gains with the simplest- and smallest-possible hardware, because a phased-array system on a wafer may service as many as a thousand antenna elements. In a SAR imaging system, for example, information from all antenna elements is gathered in a central processing location to develop an image. Circuit power and size constraints require that not all of the processing can be implemented at a central location in miniaturized systems.

In this work, we propose a new system architecture for a ranging system using long sequences suitable for pulse compression. We employ concepts related to wavelet processing to design a transmit waveform which can be digitally decomposed in using very simple, high-speed VLSI. The outputs of the proposed decomposition are simultaneously approximate a matched-filter output of a pulse-compression waveform, and *also* form a basis for the sampled signal such that (if needed) the central processing unit can do further processing on the waveform in a lossless manner.

2. CHARACTERISTIC PHASE E-SEQUENCES

In this section, we seek a real finite discrete sequence C which obeys two constraints. The first constraint is the orthogonality condition required of sequences constructing an orthogonal wavelet [1]:

$$\sum_k c_k c_{k+2m} = 0, \forall m \neq 0 \tag{1}$$

$$\sum_k c_k^2 = N \neq 0 \tag{2}$$

for some number N which we will take as the length L of the sequence. A sequence obeying these constraints is known as an *e-sequence*. In addition, we impose a *characteristic phase* constraint. Many infinite sequences possess the property that successive downsamplings of the sequence, in its characteristic phase, replicate the sequence. We modify this concept to admit finite sequences, and apply it as a constraint on c_k. We require that successive decimations (by 2) of the finite sequence be equal to the original sequence up to the end of the finite decimation. The proposed constraint is illustrated in Table 1.

The table shows there are five degrees of freedom, X_1^5, in choosing an arbitrary sequence of length 8, c_1^8, obeying the characteristic-phase constraint. Each degree of freedom specifies one or several values in the sequence: for example, X_2 is the value of c_2, c_3, and c_5.

In general, a characteristic-phase sequence of length L will admit $L/2 + 1$ degrees of freedom. The orthogonality conditions generate $L/2$ nonlinear equations. If we spend one degree of freedom to set a nonzero sequence amplitude (say,

X_1	c_1							
X_2		c_2	c_3		c_5			
X_3				c_4			c_7	
X_4						c_6		
X_5								c_8

Table 1. Illustration of *characteristic phase* for finite sequences up to length 8.

$$C_1 = \{1, 1\}$$
$$C_2 = \{1, 1, 1, -1\}$$
$$C_3 = \{1, 1, 1, -1, 1, 1, -1, 1\}$$
$$C_4 = \{1, 1, 1, -1, 1, 1, -1, 1, 1, 1, 1, -1, -1, -1, 1, -1\}$$
$$C_5 = \{1, 1, 1, -1, 1, 1, -1, 1, 1, 1, 1, -1, -1, -1, 1, ...$$
$$-1, 1, 1, 1, -1, 1, 1, -1, 1, -1, -1, -1, 1, 1, 1, -1, 1\}$$

Table 2. Some Type I sequences.

$c_0 = 1$) then we have a system of $L/2$ nonlinear equations in $L/2$ unknowns. In general, these may have no solutions or multiple solutions.

It is mathematically interesting to note that these equations (for $L \leq 16$) have no solutions for $L \neq 2^n$, as verified by exhaustive check. There are two solutions for each $L = 2^n$, and each forms a binary sequence with $c_k \in \{\pm 1\}$. Sequence solutions C' of length $L = 2^{n+1}$ may be generated by using a valid C sequence of length 2^n in the initial half, filling in a portion of the values in the second half determinable from the first half by the characteristic phase property, and backsolving for the remaining values using the orthogonal conditions. The resulting sequences are also binary. These recursive operations generate one sequence set of lengths $2^n, n > 1$ using the initial sequence $C = 1, 1$. We name these "Type I" sequences. Another sequence set is generated using the initial sequence $C = 1, -1$. We name these "Type II" sequences.

We list a sampling of such sequences in Table 2.

We note that these sequences are special cases of the Welti sequences [2]. These sequences, taken in pairs, are complimentary in the sense first defined by Golay in his classic paper [3]. In his paper, Welti shows that sequences obeying his constraints may be generated by one of two transformations from a shorter such sequence $C^{(i)} = \{A \vdots B\}$:

$$C^{(i+1)} = \{A \vdots B \vdots A \vdots \bar{B}\} \quad (3)$$

or

$$C^{(i+1)} = \{A \vdots B \vdots \bar{A} \vdots B\}. \quad (4)$$

where $\bar{A} = -1 * A$.

The set of Welti sequences are generated from successive transformations, which may be of either type at each iteration. Therefore there are $2 \log_2 L$ Welti sequences of length L. The Type I sequences of this work are the special case where the transformations are all of the first type (equation 3) with initial seed $C = \{1, 1\}$. Type II sequences are derived from transformations of the second kind (equation 4) with initial seed $C = \{1, -1\}$.

The interested reader is referred to the classic and prescient papers of the 1960s [3, 2, 4] for discussions on the fundamental relationships between (in modern terms) complementarity, quadrature matched sequences, and the orthogonality condition.

3. DECOMPOSITION AND THE RANGING PROBLEM

We now apply the sequences developed in the previous section to the problem of range-finding. Recall that the goal of this work is to send a transmit waveform which is simply related to a sequence C such that an efficient decomposition is a basis for the input waveform. Furthermore, the decomposition should provide outputs from which high-performance pulse compression is directly attained.

We now consider decomposing an input discrete-time signal using a wavelet decomposition and the sequence $C^\star$ (where the $\star$ represents time reversal) and its causally-adjusted quadrature matched sequence $D^\star$, where (by components)

$$d_k = (-1)^k c_{L-k-1}, 0 \leq k < L. \quad (5)$$

If we treat the vectors $C^\star$ and $D^\star$ as the taps of different FIR filters, standard discrete wavelet decomposition methods [1] allow a *lossless* representation of the input waveform by two half-rate data streams, each representing the alternate outputs of the $C^\star$ and $D^\star$ filters.

If the input signal coincides with the non-reversed sequence C, then we expect to see every-other output of the full-rate matched filter on one of the decomposition signals. Depending on the phase of the input waveform, we will see even autocorrelations (which are a perfect delta function) or the odd autocorrelations (which are nonzero but perhaps small). The situation is as depicted in Figure 1 for a decomposition based on the length-32 Type I sequence C_5 with an input signal equal to a zero-padded version of $C_5^\star$. With the reference delay, the decimated filter C output is a perfect delta function, and the filter D output is identically zero, as we have arranged. However, with a simple unit delay, the same input signal gives "noisy" signals on both filter C and D, which are difficult to interpret. With two unit delays, the decomposition signals return to their reference-delay structure. This behavior highlights an unfortunate property of the discrete wavelet transform for ranging applications: the frame coefficients are *not* shift-invariant. Unless we can arrange all of our targets

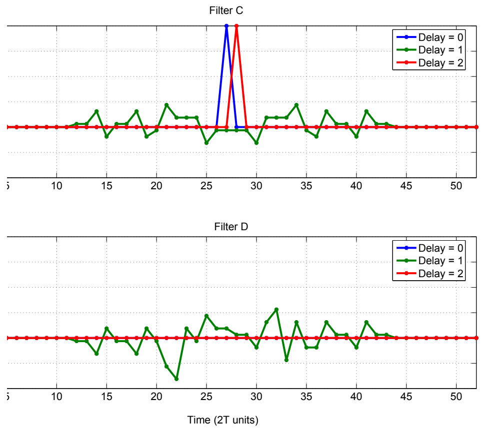

Fig. 1. Decomposition signals for a C_5 system using three different integral delays for the input waveform.

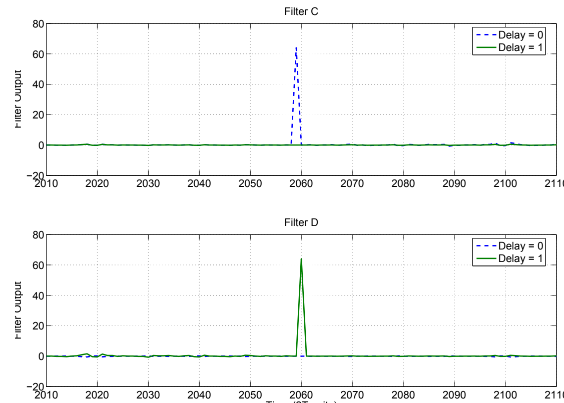

Fig. 2. Pulse compression performance for $L = 2^{12}$ for autocorrelation indices near the peaks.

to be at 2T intervals, this decomposition is not particularly useful as a pulse compression method.

Thus far the discussion of matched filtering has followed the lines of related work in the spread-spectrum wireless communications literature (see, for example, [5]). We now form a new input sequence $E^\star$ whose elements are formed from $C^\star$ and $D^\star$ in the following way:

$$e_k^\star = c_k^\star + d_{k-1}^\star, \tag{6}$$

that is, the input waveform is the sequence $C^\star$ added to a shifted version of the sequence $D^\star$. Note that the interior elements of $E^\star$ are now ternary with $e_k^\star \in \{0, \pm 2\}$. We may now expect filter C and filter D to alternately take the roles of "delta function" and "noise" as the phase of the input waveform takes its two important values (modulo 2).

In fact this is the case, as we see in Figure 2 for a larger value of $L = 2^{12}$. At the reference delay for the input signal, we see that filter C recognizes the peak, while filter D is small noise. With input delay of 1, now filter D recognizes the same peak, while filter C output is very small noise. We claim that by arranging for the approximate ping-pong of filters C and D by a unit delay (instead of the original 2-unit delay), we have greatly improved the continuity of the filter outputs for non-integral analog delays. Therefore the decimated filter outputs may be approximately taken as consecutive outputs of a full-rate pulse compression filter assuming that the transmitted waveform was C (not E!) with acceptable performance if L is large enough. This performance depends on the inherent near-orthogonality in our construction of C and D sequences at large L, in addition to each sequence's autocorrelation performance.

4. DECOMPOSITION IMPLEMENTATION

We now apply the concepts of multirate processing to provide highly optimized implementations of the proposed filter banks based on C.

We proceed by example. The decomposition matrix corresponding to a Type I C of length 8 is given by:

$$\mathbf{H}(z) = \begin{bmatrix} 1 + z^{-1} - z^{-2} + z^{-3} & -1 + z^{-1} + z^{-2} + z^{-3} \\ -1 - z^{-1} - z^{-2} + z^{-3} & 1 - z^{-1} + z^{-2} + z^{-3} \end{bmatrix} \tag{7}$$

Successive planar factorization by the angles $\pi/4$, 0, and $\pi/4$ yields the following factorization:

$$\mathbf{H}(z) = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{8}$$

$$\times \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \tag{9}$$

Therefore, $\mathbf{H}$ can be implemented using 3 delays, 6 adders, and no multiplications. The identity matrix in the factorization can be removed. This is an efficient and easily-implemented circuit.

We note that, by construction, the decomposition filter matrix $\mathbf{H}(\mathbf{z})$ is a 2 by 2 paraunitary matrix and is therefore amenable to factorization by the either the planar rotation method or the dyadic factorization method [6]. Both methods give factorizations which are fully characterized by a sequence of angles. Figure 3 shows the factorizations for sequences of length $L = 128$ by both methods. The dyadic factorizations lead to very regular but not particularly simple implementations. However, the planar factorizations show many factors with $\theta_k = 0$, which is implemented simply by a wire and a delay. In particular, the Type-II-related factorizations show only $\theta_k \in \{0, \pi/2\}$. In general the planar factorization

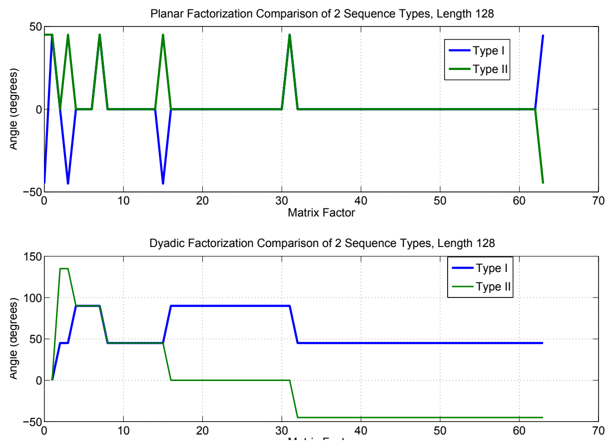

Fig. 3. Matrix factorization angle sequences for Type I and Type II systems of length L = 128

for Type II, length L sequence systems is:

$$\mathbf{H}(\mathbf{z}) = (\mathbf{RV^{L/2}}) \cdots (\mathbf{RV^4})(\mathbf{RV^2})(\mathbf{RV^1})\mathbf{R^T} \tag{10}$$

$$= [\prod_{k=L/2}^{0} \mathbf{RV^{2^k}}]\mathbf{R^T} \tag{11}$$

where

$$\mathbf{R} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \mathbf{V} = \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \tag{12}$$

These functions can be implemented using $\log_2 L$ additions, $L/2 - 1$ delays, and no multiplications. Type I factorizations have similar form, with alternating values of $\mathbf{R}$.

5. COMMENTS ON TRANSMIT WAVEFORM GENERATION

We have focused attention on the decomposition and simultaneous pulse compression of a discrete-time waveform. We now briefly comment on the generation of the transmit waveform. Evidently, by equation 6, generation of the C and D sequences will suffice.

We computed the linear complexity of these sequences using the Massey algorithm [7]. Our implementation of this algorithm assumes the elements of C and D are from the Galois field GF(2), and finds the length of the smallest linear feedback shift which reproduces the sequence. Many pn-style binary sequences (such as pseudorandom sequences) have linear complexity on the order of $\log_2 L$. However, our numerical study of the subject sequences showed the linear complexity to be $\approx L/2$ in all tested cases, which is quite large. Therefore, shift-register implementations may not be especially advantageous in this case.

We note that both C and D may be generated by an impulse function input to $\mathbf{H}(\mathbf{z})$, as in [8]. Because the inputs are one-bit numbers, the hardware precision required to implement the adders and the delay units are significantly reduced from the decomposition case (which presumably gets its data from a multi-bit ADC).

6. CONCLUSIONS

We have proposed a ranging system in which the transmit waveform is the sum of two binary sequences. The waveform is designed such that a very simple half-rate 2 by 2 decomposition filter provides excellent approximations to a full-rate pulse compression on each output, when L is large enough. In addition, the two outputs exactly comprise a basis, so that further downstream processing may reconstruct the input waveform exactly, and simply, if required.

We constructed a sequence using a wavelet orthogonality condition and a characteristic-phase constraint. We showed that these constraint lead to binary waveforms which are a special case of the Welti complementary sequences. We hope to show, in future work, the pulse-compression performance in our proposed construction compared to that using other types of sequences, including the many other Welti constructions.

7. REFERENCES

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