# A Subspace Method for MIMO Radar Space-Time Adaptive Processing

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***Abstract*— In the traditional transmitting beamforming radar system, the transmitting antennas send coherent waveforms which form a highly focused beam. In the MIMO radar system, the transmitter sends noncoherent (possibly orthogonal) broad (possibly omnidirectional) waveforms. These waveforms can be extracted by a matched filterbank. The extracted signals can be used to obtain more diversity or improve the clutter resolution. In this paper, we focus on space-time adaptive processing (STAP) for MIMO radar systems which improves the clutter resolution.**

**With a slight modification, STAP methods for the SIMO radar case can also be used in MIMO radar. However, in the MIMO radar, the rank of the jammer-and-clutter subspace becomes very large, especially the jammer subspace. It affects both the complexity and the convergence of the STAP. In this paper, a new subspace method is proposed. It computes the clutter subspace using the geometry of the problem rather than data and utilizes the block diagonal property of the jammer covariance matrix. Because of fully utilizing the geometry and the structure of the covariance matrix, the method is very effective for STAP in MIMO radar.** [1]

***Index Terms*— Beamforming, Space-Time Adaptive Processing (STAP), MIMO Radar, Prolate Spheroidal Wave Function, Clutter Subspaces, Brennan's rule.**

## I. INTRODUCTION

According to the literature [1]- [8], a MIMO radar is defined as a radar system which transmits orthogonal waveforms [1]- [5] or noncoherent [6]- [8] waveforms instead of transmitting coherent waveforms which form a focused beam in the traditional transmit beamforming. In the MIMO radar receiver, a matched filterbank is used to extract the orthogonal waveform components. There are two major advantages of the system. First, these orthogonal components are transmitted from different antennas. If these antennas are far enough, the target radar cross sections (RCS) for different transmitting paths will become independent random variables. Thus each orthogonal waveform carries independent information about the target. This spatial diversity can be utilized to perform better detection [3]. Second, the phase differences caused by different transmitting antennas along with the phase differences caused by different receiving antennas can form a new *virtual array* steering vector. With judiciously designed antenna positions, one can create a very long and critically sampled array steering vector with a small number of antennas. Thus the clutter resolution can be dramatically increased [1], [2] with a small cost. In this paper, we focus on the second advantage.

There have been many subspace methods proposed in [17]-[21] and the references therein for improving the complexity and convergence of the STAP in the traditional SIMO radar. With a slight modification, these methods can also be applied to the MIMO radar case. However, in the MIMO radar, the space-time adaptive processing (STAP) becomes even more challenging because of the extra dimension created by the orthogonal waveforms. On the one hand, the extra dimension increases the rank of the jammer and clutter subspace, especially the jammer subspace. This makes the STAP more complex. On the other hand, the extra degree-of-freedom created by the MIMO radar allows us to filter out more clutter subspace without affecting the SINR much. In this paper, we propose a STAP method which computes the clutter subspace using the geometry of the problem rather than data and utilizes the block-diagonal structure of the jammer covariance matrix. Because of fully utilizing the geometry and the structure of the covariance matrix, our method is very effective for the STAP in MIMO radar.

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## II. SIGNAL MODEL

Fig. 1 shows the geometry of the MIMO radar, where $d_T$ is the spacing of the transmitting antennas, $d_R$ is the spacing of the receiver antennas, $M$ is the number of transmitting antennas, $N$ is the number of the receiving antennas, $T$ is the radar pulse period, $l$ indicates the index of radar pulse (slow time), $\tau$ represents the time within the pulse (fast time), $v_t$ is the target moving speed toward the radar station, and $v$ is the speed of the radar station. The radar station movement is assumed to be parallel to the antenna array. This assumption has been made in most of the ground moving target indicator (GMTI) systems. The transmitted signals of the $m$th antenna can be expressed as

$$x_m(lT+\tau) = \sqrt{E}\phi_m(\tau)e^{j2\pi f(lT+\tau)},$$

for $m = 1, 2, \cdots, M-1$, where $\phi_m(\tau)$ is the unmodulated waveform, $f$ is the carrier frequency, and $E$ is the transmitted energy for the pulse. The demodulated received signal of the $n$th antenna can be expressed as

$$\begin{aligned} & y_n(lT+\tau-\frac{2r}{c}) \approx \\ & \sum_{m=0}^{M-1}\rho_t\phi_m(lT+\tau)e^{j\frac{2\pi}{\lambda}(\sin\theta_t(2vTl+d_Rn+d_Tm)+2v_tTl)} \\ & +\sum_{i=0}^{N_c-1}\sum_{m=0}^{M-1}\rho_i\phi_m(lT+\tau)e^{j\frac{2\pi}{\lambda}(\sin\theta_i(2vTl+d_Rn+d_Tm))} \\ & +v_n(lT+\tau)+w_n(lT+\tau), \end{aligned} \quad (1)$$

where $r$ is the distance of the range bin of interest, $c$ is the speed of light, $\rho_t$ is the amplitude of the signal reflected by the target, $\rho_i$ is the amplitude of the signal reflected by the $i$th clutter, $\theta_t$ is the looking direction of the target, $\theta_i$ is the looking direction of the $i$th clutter, $N_c$ is the number of clutter signals, $v_n$ is the jammer signal received by the $n$th antenna, and $w_n$

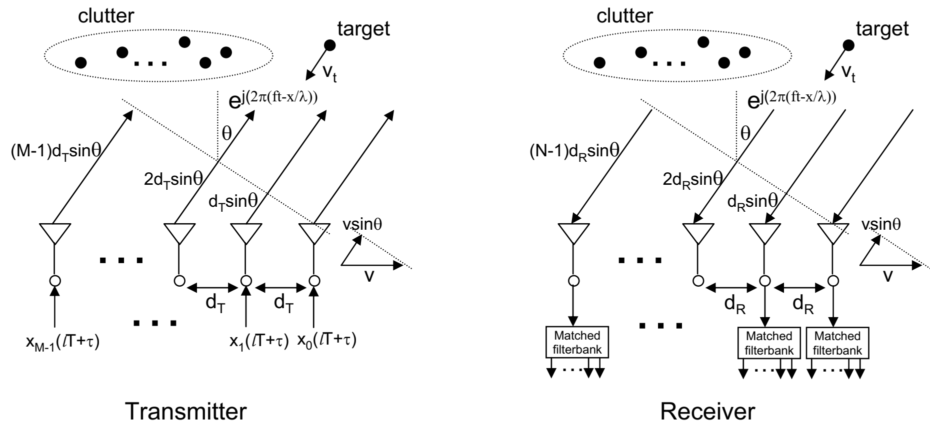


Fig. 1. MIMO radar scheme.

is the white noise in the $n$th antenna. The first term in Eq. (1) represents the signal reflected by the target. The second term is the signal reflected by the clutter. The last line represents the jammer signal and white noise. We assume there is no internal clutter motion (ICM) or antenna array misalignment [20]. The phase differences in the reflected signals are caused by the Doppler shift, the differences of the receiving antenna locations, and the differences of the transmitting antenna locations. In the MIMO radar, the transmitting waveforms $\phi_m(\tau)$ satisfy $\int \phi_m(\tau)\phi_k^*(\tau)d\tau = \delta_{mk}$. Therefore, one can extract the sufficient statistics by a matched filterbank as shown in Fig. 1. The extracted signals can be expressed as

$$\begin{aligned} & y_{n,m,l} \triangleq \int y_n(lT+\tau-\Delta)\phi_m^*(\tau)d\tau = \\ & \rho_t e^{j\frac{2\pi}{\lambda}(\sin\theta_t(2vTl+d_Rn+d_Tm)+2v_tTl)} + \\ & \sum_{i=0}^{N_c-1} \rho_i e^{j\frac{2\pi}{\lambda}(\sin\theta_i(2vTl+d_Rn+d_Tm))} + v_{n,m,l} + w_{n,m,l}, \end{aligned} \tag{2}$$

for $n = 0, 1, \cdots, N-1$, $m = 0, 1, \cdots, M-1$, and $l = 0, 1, \cdots, L-1$, where $v_{n,m,l}$ is the corresponding jammer signal, $w_{n,m,l}$ is the corresponding white noise, and $L$ is the number of the pulses in a coherent processing interval (CPI). To simplify the above equation, we define the following normalized spatial and Doppler frequencies:

$$\begin{aligned} f_s &\triangleq \frac{d_R}{\lambda}\sin\theta_t,\ f_{s,i} \triangleq \frac{d_R}{\lambda}\sin\theta_i \\ f_D &\triangleq \frac{2vT}{\lambda}\sin\theta_t + \frac{2v_tT}{\lambda}. \end{aligned} \tag{3}$$

One can observe that the normalized Doppler frequency of the target is a function of both target looking direction and speed. Usually $d_R = \lambda/2$ is chosen to avoid aliasing in spatial frequency. Using the above definition we can rewrite the extracted signal in Eq. (2) as

$$\begin{aligned} y_{n,m,l} &= \rho_t e^{j2\pi f_s(n+\gamma m)} e^{j2\pi f_D l} + \\ & \sum_{i=0}^{Nc-1} \rho_i e^{j2\pi f_{s,i}(n+\gamma m+\beta l)} + v_{n,m,l} + w_{n,m,l}, \end{aligned} \tag{4}$$

for $n = 0, 1, \cdots, N-1$, $m = 0, 1, \cdots, M-1$, and $l = 0, 1, \cdots, L-1$, where $\gamma \triangleq d_T/d_R$ and $\beta \triangleq 2vT/d_R$.

The goal of space-time adaptive processing (STAP) is to find a linear combination of the extracted signals so that the SINR can be maximized. Thus the target signal can be extracted from the interferences, clutter, and noise to perform the detection. Stacking the MIMO-STAP signals in Eq. (4), we obtain the $NML \times 1$ vector

$$\mathbf{y} = \begin{pmatrix} y_{0,0,0} & y_{1,0,0} & \cdots & y_{N-1,M-1,L-1} \end{pmatrix}^T. \tag{5}$$

Then the linear combination can be expressed as $\mathbf{w}^\dagger\mathbf{y}$, where $\mathbf{w}$ is the weighting for the linear combination. The SINR maximization can be obtained by minimizing the total variance under the constraint that the target response is unity. It can be expressed as the following optimization problem:

$$\begin{aligned} & \min_{\mathbf{w}} \mathbf{w}^\dagger\mathbf{R}\mathbf{w} \\ & \text{subject to } \mathbf{w}^\dagger\mathbf{s}(f_s, f_D) = 1, \end{aligned} \tag{6}$$

where $\mathbf{R} \triangleq E[\mathbf{y}\mathbf{y}^\dagger]$, and $\mathbf{s}(f_s, f_D)$ is an $NML \times 1$ vector which consists of the elements

$$e^{j2\pi f_s(n+\gamma m)} e^{j2\pi f_D}, \tag{7}$$

for $n = 0, 1, \cdots, N-1$, $m = 0, 1, \cdots, M-1$, and $l = 0, 1, \cdots, L-1$. This $\mathbf{w}$ is called minimum variance distortionless response (MVDR) beamformer. The covariance matrix $\mathbf{R}$ can be estimated by using the neighbor range bin cells. In practice, in order to prevent self-nulling, a target-free covariance matrix can be estimated by using guard cells [20]. The well-known solution to the above problem is [14]

$$\mathbf{w} = \mathbf{R}^{-1}\mathbf{s}(f_s, f_D)/(\mathbf{s}(f_s, f_D)^\dagger\mathbf{R}^{-1}\mathbf{s}(f_s, f_D)). \tag{8}$$

However, the covariance matrix $\mathbf{R}$ is $NML \times NML$. It is much larger than in the SIMO case because of the extra dimension. The complexity of the inversion of such a large matrix is very high. In the sample matrix inversion (SMI) method [20], the covariance matrix $\mathbf{R}$ is directly estimated as

$$\hat{\mathbf{R}} = \frac{1}{K}\sum_{k=0}^{K-1} \mathbf{y}_k\mathbf{y}_k^\dagger, \tag{9}$$

where $\{\mathbf{y}_k\}$ are the MIMO-STAP vectors of the neighbor range bin cells of the range bin of interest. Then the estimated covariance matrix is directly substituted into Eq. (8)

for computing the MVDR beamformer. The estimation of such a large covariance matrix converges slowly [16]. In the next section, we develop a faster method with significantly improved performance.

## III. New STAP Method for MIMO Radar

In this section, we introduce a new STAP method for MIMO radar. In this method, the clutter subspace is computed using parameters $\gamma$ and $\beta$ defined in Eq. (4) and the jammer-plus-noise covariance matrix is estimated from clutter-free signals.

### A. Computation of the clutter space

For convenience, we study only the clutter term in Eq. (4) which is expressed as

$$y_{c,n,m,l} = \sum_{i=0}^{Nc-1} \rho_i e^{j2\pi f_{s,i}(n+\gamma m+\beta l)},$$

for $n = 0, 1, \cdots, N-1$, $m = 0, 1, \cdots, M-1$, and $l = 0, 1, \cdots, L-1$. Note that $-0.5 < f_{s,i} < 0.5$ because $d_R = \lambda/2$. Define $c_{i,n,m,l} = e^{j2\pi f_{s,i}(n+\gamma m+\beta l)}$ and

$$\mathbf{c}_i = \begin{pmatrix} c_{i,0,0,0}, c_{i,1,0,0}, \cdots, c_{i,N-1,M-1,L-1} \end{pmatrix}^T. \quad (10)$$

By stacking the signal $\{y_{c,n,m,l}\}$ into vector, one can obtain

$$\mathbf{y}_c = \sum_{i=0}^{N_c-1} \rho_i \mathbf{c}_i.$$

Assume that $\rho_i$ are zero-mean independent random variables with variance $\sigma_{c,i}^2$. The the clutter covariance matrix can be expressed as

$$\mathbf{R}_c = E\mathbf{y}_c\mathbf{y}_c^\dagger = \sum_{i=0}^{N_c-1} \sigma_{c,i}^2 \mathbf{c}_i\mathbf{c}_i^\dagger.$$

Therefore, span($\mathbf{R}_c$) = span($\mathbf{C}$), where

$$\mathbf{C} \triangleq \begin{pmatrix} \mathbf{c}_0, \mathbf{c}_1, \cdots, \mathbf{c}_{N_c-1} \end{pmatrix}.$$

The vector $\mathbf{c}_i$ can be viewed as a *nonuniformly* sampled version of the time-and-bandlimited signal $e^{j2\pi f_{s,i}x}$, $x \in [0, N-1+\gamma(M-1)+\beta(L-1)]$ and $f_{s,i} \in [-0.5, 0.5]$. Such signals can be well approximated by linear combinations of $2WX+1$ prolate spheroidal wave functions (PSWF) [11], where $W$ is the bandwidth and $X$ is the duration of the time-limited functions. PSWF were first used in STAP context by [12]. In this case, we have $W = 0.5$ and $2WX+1 = N+\gamma(M-1)+\beta(L-1)$. This result can be viewed as a generalization of the Brennan's rule [10] for the MIMO radar case. Therefore, the nonuniformly sampled version of the time-and-band-limited function, namely $\mathbf{c}_i$ can also be approximated as

$$\mathbf{c}_i \approx \sum_{k=0}^{r_c-1} \alpha_{i,k}\mathbf{u}_k,$$

for some $\{\alpha_{i,k}\}$, where $r_c \triangleq N+\gamma(M-1)+\beta(L-1)$ and $\mathbf{u}_k$ is the corresponding nonuniformly sampled version of the PSWF [9]. Therefore, we obtain

$$\text{span}(\mathbf{R}_c) = \text{span}(\mathbf{C}) \approx \text{span}(\mathbf{U}_c), \quad (11)$$

where $\mathbf{U}_c \triangleq \begin{pmatrix} \mathbf{u}_0 & \mathbf{u}_1 & \cdots & \mathbf{u}_{r_c-1} \end{pmatrix}$. In practice, the PSWF can be computed off-line. When the parameters change, one can obtain the vectors $\mathbf{u}_k$ by resampling the PSWF. The details of this method can be found in [9].

### B. New STAP method

The target-free covariance matrix can be expressed as $\mathbf{R} = \mathbf{R}_J + \mathbf{R}_c + \sigma^2\mathbf{I}$, where $\mathbf{R}_J$ is the covariance matrix of the jammer signals, $\mathbf{R}_c$ is the covariance matrix of the clutter signals, and $\sigma^2$ is the variance of the white noise. By Eq. (11), there exists a $r_c \times r_c$ matrix $\mathbf{A}_c$ so that $\mathbf{R}_c \approx \mathbf{U}_c\mathbf{A}_c\mathbf{U}_c^\dagger$. Thus the covariance matrix can be approximated by

$$\mathbf{R} \approx \underbrace{\mathbf{R}_J + \sigma^2\mathbf{I}}_{\text{call this } \mathbf{R}_v} + \mathbf{U}_c\mathbf{A}_c\mathbf{U}_c^\dagger. \quad (12)$$

We assume the jammer signals $v_{n,m,l}$ in Eq. (4) are statistically independent in different pulses and different orthogonal waveforms and they satisfy [20]

$$E[v_{n,m,l} \cdot v_{n',m',l'}^\dagger] = \begin{cases} r_{J,n,n'}, & m = m', l = l' \\ 0, & \text{otherwise}, \end{cases}$$

for $n, n' = 0, 1, \cdots, N$, $m, m' = 0, 1, \cdots, M$, and $l, l' = 0, 1, \cdots, L$. Using this fact, the jammer-plus-noise covariance matrix $\mathbf{R}_v$ defined in Eq. (12) can be expressed as

$$\mathbf{R}_v = \text{diag}(\mathbf{R}_{vs}, \mathbf{R}_{vs}, \cdots, \mathbf{R}_{vs}),$$

where $\mathbf{R}_{vs}$ is an $N \times N$ matrix with elements $[\mathbf{R}_{vs}]_{n,n'} = r_{J,n,n'} + \sigma^2$ for $n, n' = 0, 1, \cdots, N$. Therefore the covariance matrix $\mathbf{R}$ in Eq. (12) consists of a *low-rank* clutter covariance matrix and a *block-diagonal* jammer-pulse-noise covariance matrix. By using the matrix inversion lemma [22], one can obtain

$$\mathbf{R}^{-1} \approx \mathbf{R}_v^{-1} - \mathbf{R}_v^{-1}\mathbf{U}_c(\mathbf{A}_c^{-1} + \mathbf{U}_c^\dagger\mathbf{R}_v^{-1}\mathbf{U}_c)^{-1}\mathbf{U}_c^\dagger\mathbf{R}_v^{-1}. \quad (13)$$

Assume that the clutter to noise ratio is very large and therefore all of the eigenvalues of $\mathbf{A}_c$ approach infinity. Then we have $\mathbf{A}_c^{-1} \approx \mathbf{0}$. Substituting $\mathbf{A}_c^{-1} = \mathbf{0}$ and Eq. (13) into Eq. (8), one can obtain the MIMO-STAP beamformer for spatial frequency $f_s$ and Doppler frequency $f_D$ as

$$\mathbf{w} \propto (\mathbf{R}_v^{-1} - \mathbf{R}_v^{-1}\mathbf{U}_c(\mathbf{U}_c^\dagger\mathbf{R}_v^{-1}\mathbf{U}_c)^{-1}\mathbf{U}_c^\dagger\mathbf{R}_v^{-1})\mathbf{s}(f_s, f_D) \quad (14)$$

The inversion of the block-diagonal matrix $\mathbf{R}_v^{-1}$ can be obtained by computing diag($\mathbf{R}_{vs}^{-1}, \mathbf{R}_{vs}^{-1}, \cdots, \mathbf{R}_{vs}^{-1}$) and the multiplication of the block-diagonal matrix is also simpler. The complexity of directly inverting the $NML \times NML$ covariance matrix $\mathbf{R}$ is $O(N^3M^3L^3)$. Taking advantage of the block-diagonal matrix and the low rank matrix, in Eq. (13), the complexity of computing $\mathbf{R}_v^{-1}$ is only $O(N^3)$ and the complexity of computing the inversion of $(\mathbf{U}_c^\dagger\mathbf{R}_v^{-1}\mathbf{U}_c)^{-1}$ is only $O(r_c^3)$. The overall complexity of computing Eq. (13) is reduced to $O(r_cN^2M^2L^2)$. The bottleneck now becomes to compute the multiplication of an $NML \times r_c$ matrix and a $r_c \times NML$ matrix.

In Eq. (14), the matrix $\mathbf{U}_c$ can be obtained by the nonuniform sampling of the PSWF as described in the last section. The jammer-pulse-noise covariance matrix $\mathbf{R}_v$ requires further estimation from the received signals. Because of the block-diagonal structure, one can estimate the covariance matrix $\mathbf{R}_v$ by estimating its submatrix $\mathbf{R}_{vs}$. The matrix $\mathbf{R}_{vs}$ can be estimated when there is no clutter and target signals. For this, the radar transmitter *operates in passive mode* so that the receiver can collect the signals with only jammer signals and white noise [21]. The submatrix $\mathbf{R}_{vs}$ can be estimated as

$$\hat{\mathbf{R}}_{vs} = \frac{1}{K_v}\sum_{k=0}^{K_v-1} \mathbf{r}_k\mathbf{r}_k^\dagger, \quad (15)$$

where $\mathbf{r}_k$ is an $N \times 1$ vector which represents the target-free and clutter-free signals received in $N$ receiving antennas.

## IV. Numerical Examples

In this section, we compare the SINR performance for different STAP methods. The SINR is defined as

$$\text{SINR} \triangleq \frac{|\mathbf{w}^\dagger \mathbf{s}(f_s, f_D)|^2}{\mathbf{w}^\dagger \mathbf{R} \mathbf{w}},$$

where $\mathbf{R}$ is the target-free covariance matrix. Fig. 2 shows the comparison of the SINR for $f_s = 0$ and different Doppler frequencies. In the example, the parameters are $M = 5$, $N = 10$, $L = 16$, $\beta = 1.5$ and $\gamma = 10$. The latitude is 9000m and the range of interest is 12728m. For this latitude and range, the clutter is generated by using the model in [13]. The clutter to noise ratio (CNR) is 40dB. There are two jammers at $20^\circ$ and $-30^\circ$ degree. The jammer to noise ratio (JNR) of both jammers equals 50dB. The SINR is normalized so that the maximum SINR equals 0dB. The SINR performance for MVDR with perfect covariance matrix is shown in the figure as an upper bound of the SINR performance of the MIMO-STAP methods. The corresponding upper bound for the SIMO system with $\gamma = 1$ is also compared. Note that we have normalized the total transmitting energy so that each antenna uses the same energy to illuminate all directions. One can see that the MIMO system has better idea SINR performance than the SIMO system. This is because the MIMO system has a better spatial resolution. Four MIMO-STAP methods are compared. Our method is described in Eq. (14) and the clutter-free covariance matrix is estimated using Eq. (15). The SMI method [20] directly estimates the covariance matrix by Eq. (9) and substitutes it into Eq. (8). The loaded sample matrix inversion (LSMI) [15] applies an extra diagonal loading before computing Eq. (8). In this example, the diagonal loading factor is chosen as ten times the white noise level. The principal component (PC) method is described in [20]. Lacking use of the covariance matrix structure, the SMI method has the slowest convergence. The PC method and LSMI method utilize the fact that the jammer-plus-clutter covariance matrix has low rank. The performance of these are about the same. Our method not only utilizes the low rank property but also the geometry of the problem. It converges to a satisfactory SINR with very few clutter-free samples.

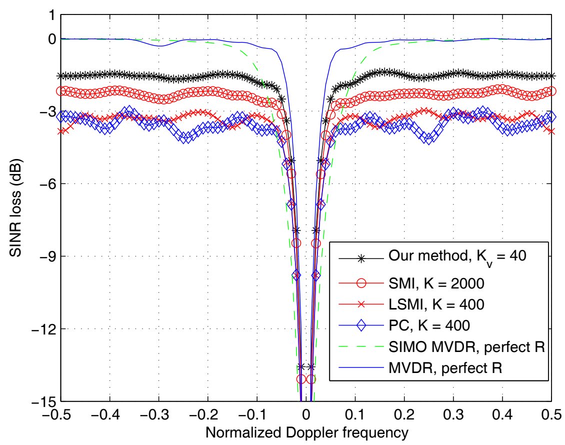


Fig. 2. SINR at looking direction zero.

## V. Conclusions

The method described in this paper utilizes the knowledge of the parameters $\gamma$ and $\beta$, the structure of the clutter space, and the block diagonal structure of the jammer covariance matrix. We first estimate the clutter subspace by using the parameters $\gamma$ and $\beta$ in Eq. (11). Using the fact that the jammer matrix is block diagonal and the clutter matrix has low rank with known subspace, we can break the inversion of a large matrix $\mathbf{R}$ into the inversions of some smaller matrices with matrix inversion lemma. Therefore our method has much smaller computational complexity. Moreover, by using the structure, fewer parameters need to be estimated. In our method, only the $N \times N$ matrix $\mathbf{R}_{vs}$ need to be estimated instead of the the $NML \times NML$ matrix $\mathbf{R}$ in the SMI method. Therefore, for a given number of data samples, our methods has better performance.

In this paper, we only consider the ideal case. In fact, the clutter subspace might change because of effects such as the internal clutter motion (ICM) or velocity misalignment [20]. In this case, a better way might be estimating the clutter subspace by using both the geometry and the received signals. This idea will be explored in the future.

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