# NON-LINEAR PREDICTION OF INVERSE COVARIANCE MATRIX FOR STAP

Chin-Heng Lim †, Chong-Meng Samson See \* †and Bernard Mulgrew ‡

†Temasek Laboratories@NTU, 50 Nanyang Drive, Singapore 637553
\* DSO National Laboratories, 20 Science Park Drive, Singapore 118230
‡Institute for Digital Communications, University of Edinburgh, United Kingdom

## ABSTRACT

For bistatic ground moving target indication radar, the clutter Doppler frequency depends on range for all array geometries. This range dependency leads to problems in clutter suppression through STAP techniques. In this paper, we propose a new approach of applying non-linear prediction theory to address the range dependency problem in bistatic airborne radar systems. This technique uses a non-linear function to obtain an estimate of the range-dependent inverse covariance matrix. Simulation results suggest a non-linear fit for the model (nonlinear relationship between the inverse covariance matrices) and show an improvement in processor performance as compared to conventional STAP methods.

*Index Terms*— Space-time adaptive processing, ground moving target indication, non-linear prediction, inverse co-variance matrix, clutter suppression

#### 1. INTRODUCTION

Space-time adaptive processing (STAP) is a well-established technique for detection of moving targets by an airborne radar. Interest in bistatic STAP, where the transmitter and receiver are separated, has picked up in recent years. Bistatic radar offers several advantages over its monostatic counterpart, such as the higher possibility of detecting stealth targets.

Training and updating of the clutter covariance matrix is a key step in the implementation and effectiveness of any STAP system. In a bistatic or multi-static environment, the usual impediment and possible clutter in-homogeneity experienced in the linear monostatic side-looking case is further complicated by the range-dependent nature of the clutter ridge in the angle-Doppler plane induced by the physical geometry of the two (or more) aircraft [1]. Thus the bistatic range-dependent clutter spectrum complicates the clutter suppression problem and degrades the performance. A mismatch in the clutter statistics between the training range cells and the test range cell will result in the widening of the STAP filter clutter notch. This will cause target returns from relatively slow-velocity or low-flying targets to be suppressed or even go undetected.

E-mail: ChinHeng@ntu.edu.sg (corresponding author), schongme@dso.org.sg, B.Mulgrew@ed.ac.uk

A number of compensation approaches [2, 3] exist to mitigate the impact of this range dependency. However, these techniques effectively only manage to map the mainlobe clutter peak and take little account of sidelobe clutter. Other techniques [4–6] have been proposed that attempt to map the sidelobe clutter. The major draw-back with these techniques is that they require knowledge of the radar system navigational data. Although the data can be estimated, like in [5] and [6], knowledge of the theoretical direction-Doppler curves or an initial estimate is required respectively.

A new technique was proposed in [7] to obtain the rangedependent inverse covariance matrix using linear prediction theory. In this paper, we extend the prediction to a non-linear function fit, which highlights the non-linear relationship between the inverse covariance matrices. This technique retains the same advantages in that no navigational data or parameters estimation has to be performed as only the clutter data is required. Moreover, the technique is not restricted to uniform linear array (ULA) applications. The paper is organized as follows: The data model of the STAP processor is formulated in Section 2. Section 3 deals with the analysis of the new approach and simulation results are presented in Section 4. A brief conclusion can be found in Section 5.

## 2. PROBLEM STATEMENT

Consider a radar system utilizing an *N*-element array with inter-element spacing *d*, which transmits an *M*-pulse waveform in its coherent processing interval (CPI). The received data for each range gate can be organized into an  $(NM \times 1)$  space-time snapshot **x** by stacking the spatial snapshots from each pulse. The space-time interference (clutter + noise) covariance matrix is defined as **Q**, where  $\mathbf{Q} = \mathbf{E}[\mathbf{x}\mathbf{x}^H]$  and  $\mathbf{E}[.]$  is the expectation operator. Under the assumption of Gaussian interference, the optimum processor (Wiener filter) is [8]:

$$\mathbf{w}_{opt} = \mathbf{Q}^{-1} \mathbf{s},\tag{1}$$

where  $\mathbf{s} = \mathbf{s}_t \otimes \mathbf{s}_s$  is the  $(NM \times 1)$  target signal steering vector and  $\otimes$  is the Kronecker product. The temporal and spatial dimensions of the target steering vector are respectively:

$$\mathbf{s}_t = \left[1 e^{j2\pi f_D} \dots e^{j2\pi (M-1)f_D}\right]^T \tag{2}$$

$$\mathbf{s}_s = \left[1 \ e^{j2\pi f_s} \ \dots \ e^{j2\pi (N-1)f_s}\right]^T. \tag{3}$$

For a bistatic forward-looking radar configuration, the normalized target Doppler and spatial frequencies are:

$$f_D = \frac{v_{rad,rx} + v_{rad,tx}}{\lambda \text{PRF}} \tag{4}$$

$$f_s = \frac{d\sin\varphi_r\cos\theta_r}{\lambda}.$$
 (5)

 $v_{rad,rx}$  and  $v_{rad,tx}$  are the relative velocities of the target to the receiver and transmitter respectively,  $\lambda$  is the wavelength of the radar signal, PRF is the pulse repetition frequency,  $\varphi_r$  and  $\theta_r$  are the azimuth and elevation angle of the receiver to the point of interest on the ground respectively.

For signal processing applications in a practical radar system, it is highly unlikely that an infinite sequence of snapshots can be obtained in each range gate to get the exact clutter covariance matrix  $\mathbf{Q}$ . Thus some performance loss will be incurred from estimating the covariance matrix. In addition, the clutter statistics of the test range gate may be unknown. The data from the adjacent range gates, conventionally referred to as the training data, is then used for the estimation of the clutter sample covariance matrix. This gives the sample matrix inversion (SMI) algorithm [8]. When the above two restrictions are present, the estimated covariance matrix  $\widehat{\mathbf{Q}}$  obtained from the maximum likelihood (ML) estimator [8] is:

$$\widehat{\mathbf{Q}} = \frac{1}{Z} \sum_{i}^{I} (\mathbf{z}_i) (\mathbf{z}_i)^H, \qquad (6)$$

where Z is the total number of "snapshots" used (sample support) and z is the training data.

The performance metric used to evaluate the performance of the processors in this paper is the improvement factor loss  $(IF_{loss})$  [1], widely-used within the radar community:

$$IF_{loss} = \frac{\mathbf{s}^H \mathbf{Q}^{-1} \mathbf{s} \mathbf{s}^H \widehat{\mathbf{Q}}^{-1} \mathbf{Q} \widehat{\mathbf{Q}}^{-1} \mathbf{s}}{\mathbf{s}^H \widehat{\mathbf{Q}}^{-1} \mathbf{s} \mathbf{s}^H \widehat{\mathbf{Q}}^{-1} \mathbf{s}}.$$
 (7)

The maximum attainable value of  $IF_{loss}$  is unity, indicating that the processor performance is not degraded by clutter. In practice, the processor performance is degraded by estimation losses and the clutter range-dependency problem.

#### 3. NON-LINEAR PREDICTION OF INVERSE COVARIANCE MATRIX (NL-PICM)

In [7], a technique (PICM) was proposed to use linear prediction theory to address the bistatic clutter Doppler range dependency problem. For this paper, we propose a non-linear function (but linear in parameter) fit to predict the inverse interference covariance matrix for bistatic STAP. It must be noted that the non-linear prediction technique (NL-PICM) is applied to the inverse covariance matrix sequences, as shown



Fig. 1. Non-linear prediction of inverse covariance matrix.

in Fig. 1, and *not* the uncorrelated data snapshots (obtained by using a sliding window in the space and/or time domain).

This non-linear (polynomial/Volterra series) function, with linear parameters, allows removing of target 'spikes' and provides good clutter suppression performance in the presence of aliasing (range and/or Doppler ambiguities) effects. In this paper, we use the Volterra series (up to the 2nd order) to model the non-linear fit (other non-linear functions or a L1-norm can be used). Denote the test range gate as the  $r^{th}$ range gate and each index  $\lambda$  of the stacked inverse covariance matrix of the  $k^{th}$  training range gate as  $\bar{\mathbf{Q}}_k^{-1}(\lambda)$ , where  $\lambda = 1, 2, \dots, (NM)^2$ , the non-linear prediction is:

$$\widehat{\mathbf{Q}}_{k}^{-1}(\lambda) = \alpha_{+1}(\lambda)\,\overline{\mathbf{Q}}_{k+1}^{-1}(\lambda) + \alpha_{-1}(\lambda)\,\overline{\mathbf{Q}}_{k-1}^{-1}(\lambda) \\
+ \alpha_{+2}(\lambda)\,\overline{\mathbf{Q}}_{k+1}^{-2}(\lambda) + \alpha_{-2}(\lambda)\,\overline{\mathbf{Q}}_{k-1}^{-2}(\lambda) \\
+ \alpha_{12}(\lambda)\,\overline{\mathbf{Q}}_{k+1}^{-1}(\lambda)\,\overline{\mathbf{Q}}_{k-1}^{-1}(\lambda),$$
(8)

for 
$$k = r - \frac{K}{2} - 1, \dots, r - 2, r + 2, \dots, r + \frac{K}{2} + 1,$$

where  $\widehat{\mathbf{Q}}_{k}^{-1}$ ,  $\overline{\mathbf{Q}}_{k+1}^{-1}$  and  $\overline{\mathbf{Q}}_{k-1}^{-1}$  are the estimated stacked inverse covariance matrix for the  $k^{th}$  range gate and the stacked inverse covariance matrices for the  $(k + 1)^{th}$  and  $(k - 1)^{th}$  range gate respectively;  $\alpha_{+1}$ ,  $\alpha_{-1}$ ,  $\alpha_{+2}$ ,  $\alpha_{-2}$  and  $\alpha_{12}$  are the NL prediction weights and K is the total number of training range gates required for the prediction sequence. The  $(r-1)^{th}$  and  $(r+1)^{th}$  range gates (guard-gates) are excluded for computation of the NL prediction weights.

The inverse covariance matrices and prediction weights are a function of  $\lambda$ . The solution of the NL prediction weights

are given by the minimum mean squared error (MMSE) of the true and estimated stacked inverse covariance matrix

$$\arg\min_{\lambda} \sum_{k} \left| \bar{\mathbf{Q}}_{k}^{-1}(\lambda) - \widehat{\bar{\mathbf{Q}}}_{k}^{-1}(\lambda) \right|^{2}.$$
(9)

The series of resulting linear simultaneous equations from equation (8) can be re-arranged into a matrix form and a solution can be obtained by solving the system of linear simultaneous equations. Rewriting equation (8):

$$\widehat{\bar{\mathbf{Q}}}_{k}^{-1}(\lambda) = \boldsymbol{\Theta} \, \boldsymbol{\Gamma},\tag{10}$$

$$\boldsymbol{\Theta} = \begin{bmatrix} \bar{\mathbf{Q}}_{k+1}^{-1}(\lambda) & \bar{\mathbf{Q}}_{k-1}^{-1}(\lambda) & \bar{\mathbf{Q}}_{k+1}^{-2}(\lambda) & \dots \\ \bar{\mathbf{Q}}_{k-1}^{-2}(\lambda) & \bar{\mathbf{Q}}_{k+1}^{-1}(\lambda)\bar{\mathbf{Q}}_{k-1}^{-1}(\lambda) \end{bmatrix}$$
(11)

$$\boldsymbol{\Gamma} = \begin{bmatrix} \alpha_{+1}(\lambda) & \alpha_{-1}(\lambda) & \alpha_{+2}(\lambda) & \alpha_{-2}(\lambda) & \alpha_{12}(\lambda) \end{bmatrix}^T \quad (12)$$

The linear prediction is done on each  $\lambda$  for the inverse covariance matrix. The prediction weights  $\widehat{\Gamma}$  for each  $\lambda$  of the inverse covariance matrix is obtained from the surrounding training data (regressors). From equation (10),

$$\left(\boldsymbol{\Theta}^{H}\boldsymbol{\Theta}\right)^{-1}\boldsymbol{\Theta}^{H}\widehat{\bar{\mathbf{Q}}}_{k}^{-1}(\lambda) = \widehat{\boldsymbol{\Gamma}}$$
(13)

Equation (8) is simply a standard least squares (LS) problem with a known solution. Technically, this is smoothing [9] rather than forward or backward prediction. It should also be noted that the NL fit can be extended to multi-dimensional prediction (using more prediction coefficients within the same range gate). However, increasing the number of prediction weights results in a corresponding increase in the number of training range gates required. Hence, there is a trade-off between more accurate estimates and computational complexity. The number of prediction weights that can be used is limited since the benefits of using more prediction coefficients will be negated by the bistatic clutter Doppler range dependency problem.

By exploiting the Hermitian property of the inverse covariance matrix, the linear prediction only needs to be carried out for the upper triangular portion. The linear prediction weights for the lower triangular portion of the inverse covariance matrix are simply the complex conjugate of its upper triangular portion's counterparts. By implementing PICM, there is no need to exploit the Toeplitz-block-Toeplitz structure of the theoretical covariance matrix, thus eliminating the requirement for a uniform linear array (ULA), as required in [5, 6]. Without any such restrictions, PICM can be applied to arrays of arbitrary configuration.

#### 4. SIMULATION RESULTS

For the simulation analysis, the clutter model is shown in Chapter 2 of [1]. The bistatic forward-looking (direction of travel normal to linear antenna array with half wavelength spacing) radar parameters are shown in Table 1. The transmitter's flight path is 90° from that of the receiver and the backlobe of the clutter scatterers' response was ignored. The baseline separation (along flight direction) is 2000m and the receiver maximum sensor pattern direction is  $\varphi_0 = 45^\circ$ .

Table 1. Radar Parameters		
number of antenna elements	N	8
number of pulses delay	M	24
pulse repetition frequency	PRF	20 kHz
operating frequency	$f_o$	10 GHz
receiver & transmitter height	$H_R, H_T$	1000 m
receiver & transmitter velocity	$v_R, v_T$	90 m/s
receiver look angle	$\varphi_r$	45°
clutter-to-noise ratio (CNR)		30dB

For all the algorithms shown in this section, a total of K = 20 training range gates (10 on either side of the range gate under test) are used for the prediction sequence and the simulation results are obtained from a Monte Carlo simulation comprising of 500 runs.

Fig. 2 shows the  $IF_{loss}$  plot of the NL-PICM technique (red solid line) and the PICM technique with four-taps linear prediction (blue dashed line). The solid line with pluses shows the sample matrix inversion (SMI) algorithm [8], which is obtained by straight averaging the covariance matrices  $\mathbf{Q}_k$ from the training sequence. Lastly, the solid line with squares shows the SMI algorithm obtained by straight averaging of the inverse covariance matrices  $\mathbf{Q}_k^{-1}$ . This algorithm is included as a reference since the proposed non-linear prediction is carried out on the inverse covariance matrices.

At the critical clutter notch frequency of 3000Hz, it can be observed that the NL-PICM technique gives the best performance. There is an improvement of 4dB over the PICM technique. Another important point to note is the narrower clutter notch provided by NL-PICM over the two SMI processors, hence enhancing the capability of detecting relatively slowmoving, low-flying targets. The importance of this region is indicated by the fact that such target signals tend to fall within this region and will be attenuated.

For the next set of simulation results in Fig. 3, we show the performance of the various algorithms in the presence of range ambiguities. A pulse Doppler radar can be ambiguous in either range or Doppler frequency [1]. For conventional pulse Doppler radar, range ambiguities occur because of the transmission of repetitive pulses and the clutter echoes of a certain range gate includes the clutter contributions from other range gates. The different mainlobe in ambiguous range gates move in different directions and do not coincide, thus resulting in additional clutter notches.

The figure is zoomed into the region of interest and the same plots are used to denote the various algorithms, as in Fig.



**Fig. 2**. *I F*<sub>loss</sub> plot for a bistatic forward-looking radar: PICM (4-taps), NL-PICM, SMI and SMI-(INV Q)

2. In addition, the performance of the angle-Doppler compensation (ADC) algorithm [3] is shown by the dash-dotted line with circles. This algorithm mitigates for the bistatic clutter Doppler range dependency problem by compensating with the peak angle and Doppler frequencies. Thus it performs well within the mainlobe. However, aliasing results in additional clutter notches due to the overlapping of clutter spectrums and thus the algorithm does not perform as well as NL-PICM. The NL-PICM technique has a narrower clutter notch than the other algorithms, which highlights the benefits of using NL-PICM in the presence of aliasing effects.

## 5. CONCLUSION

In this paper, we provided the analysis of using non-linear prediction theory to obtain an estimate of the range-dependent inverse covariance matrix. Simulation results indicate a nonlinear fit for the model and show the improvement in clutter suppression performance over conventional STAP techniques. The proposed technique is also able to mitigate against the additional clutter notches in the presence of aliasing effects and can be applied to arrays of arbitrary configuration. No navigational data or parameter estimation is necessary as only the clutter data is required.

#### 6. REFERENCES

- R. Klemm, *Principles of space-time adaptive processing*, 2nd edition. IEE Radar, Sonar, Navigation and Avionics Series, 2002.
- [2] C. H. Lim, E. Aboutanios, and B. Mulgrew, "Modi-



**Fig. 3**.  $IF_{loss}$  plot for a bistatic forward-looking radar with range ambiguities: PICM (4-taps), NL-PICM, SMI and ADC

fied JDL with Doppler compensation for airborne bistatic radar," *IEEE Radar Conference*, pp. 854–858, May 2005.

- [3] B. Himed, Y. Zhang, and A. Haijari, "STAP with angle-Doppler compensation for bistatic airborne radars," *Proceedings of IEEE Radar Conference*, pp. 311–317, 2002.
- [4] C. H. Lim and B. Mulgrew, "Filter banks based JDL with angle and separate Doppler compensation for airborne bistatic radar," *International Radar Symposium*, pp. 583– 587, September 2005.
- [5] F. D. Lapierre, M. V. Droogenbroeck, and J. Verley, "New solutions to the problem of range dependence in bistatic STAP radars," *Proceedings of IEEE Radar Conference*, pp. 452–459, May 2003.
- [6] A. G. Jaffer, B. Himed, and P. T. Ho, "Estimation of range-dependent clutter covariance by configuration system parameter estimation," *Proceedings of IEEE Radar Conference*, pp. 596–601, May 2005.
- [7] C. H. Lim and B. Mulgrew, "Prediction of inverse covariance matrix (PICM) sequences for STAP," *IEEE Signal Processing Letters*, vol. 13, no. 4, pp. 236–239, April 2006.
- [8] L. E. Brennan and I. S. Reed, "Theory of adaptive radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. AES-9, no. 2, pp. 237–252, March 1973.
- [9] S. Haykin, *Adaptive Filter Theory, 4th edition*. Prentice Hall, 2002.