MAXIMUM LIKELIHOOD RANGE DEPENDENCE COMPENSATION FOR STAP

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ABSTRACT

We present a new method to estimate the clutter-plus-noise covariance matrix used to compute an adaptive filter in space-time adaptive processing (STAP). The method computes a ML estimate of the clutter scattering coefficients using a Bayesian framework and knowledge on the structure of the covariance matrix. A priori information on the clutter statistics is used to regularize the estimation method.

Other estimation methods based on the computation of the power spectrum using for instance the periodogram are compared to our method. The result in terms of SINR loss shows that the proposed method outperforms the other ones.

Index Terms— STAP, Bayes, range-dependence, structured covariance matrix

1. INTRODUCTION

In downlooking airborne radars, the received echoes are typically contaminated with clutter returns that compete with slow-moving targets. Echoes from slow-moving targets can be separated from clutter returns by using space-time adaptive processing (STAP) [1]. The computation of the optimum filter for STAP requires an accurate estimate of the covariance matrix (CM) of the clutter-plus-noise signal at the range of interest. This estimate is usually obtained by averaging single-realization sample CM obtained at different ranges around the range of interest. This averaging only provides an accurate estimate if the clutter echoes are independent and identically distributed (IID) at all ranges. Clutter echoes are typically IID when obtained for a monostatic sidelooking configuration. This is however not the case for non sidelooking monostatic configurations and for most bistatic [2] and multistatic configurations [3]. The geometry-induced non-stationarities of the clutter statistics in range strongly affect the accuracy of the CM estimate.

We present a model-based method to compensate for the geometry-induced non-stationarities. The method is relatively close to that presented in [4, 5] which is based on the registration of the clutter notch at different ranges prior to averaging. Another similar method is presented in [6]. These methods are all based on the estimation of the clutter scattering coefficients at the range of interest, making use of the data at all available ranges, prior to a synthesis of the covariance matrix. These methods will be briefly discussed and, when possible, a comparison will be presented.

Section 2 describes the signal model we consider. Section 3 presents the Bayesian framework used to estimate the clutter map from the measurements. In this section, we also compare the performance of the proposed estimation method to other estimation methods in the case of a simulated clutter. Section 4 describes how the clutter scattering coefficients can be computed from the clutter map obtained in the previous section. It shows how to construct the clutter-plus-noise CM from these clutter scattering coefficients. Section 5 presents the results of the CM estimation methods in terms of end-to-end signal to interference plus noise ratio (SINR) loss. Section 6 concludes the paper.

2. SIGNAL MODEL

Let us consider the signal received by a pulse-Doppler radar. No assumption is made regarding the relative position of the transmitter and the receiver. The receiver is assumed to be equipped with an array. Again, no assumption is made regarding the location of the receiving elements included in the array. The receiver might even be made of different collaborating platforms as described in [3], as long as the system is coherent. We further assume to have a perfect knowledge of the system geometry.

Each coherent processing interval consists of M pulses. For each pulse, one data sample at the range of interest is taken at each of the N receiving elements. The lexically-ordered $N \times M$ samples of the received signal corresponding to the range of interest are denoted by $\mathbf{y}$, which is modeled by [7]

$$\mathbf{y} = \sum_{i=1}^{L} a_{c_i} \mathbf{v}_i + \mathbf{n} \tag{1}$$

where L is the number of clutter patches contributing to the signal, a_{c_i} is the (complex) amplitude of the signal contributed by clutter patch i, $\mathbf{v}_i$ is the $N \times M$ steering vector associated with clutter patch i, and $\mathbf{n}$ is the white Gaussian thermal noise with CM $R_n = \sigma^2 I$. Equation (1) can be rewritten in matrix notation as

$$\mathbf{y} = V\mathbf{a}_c + \mathbf{n}, \tag{2}$$

where $V = \{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_L\}$ and $\mathbf{a}_c = \{a_{c_1}, ..., a_{c_L}\}^T$. Given the assumptions above, the only unknowns in this equation are $\mathbf{a}_c$ and $\mathbf{n}$.

The complex amplitude coefficients a_{c_i} can be decomposed in a known factor c_i that groups the geometric terms of the radar equation (range attenuation, antenna radiation pattern, ...) and a complex unknown factor a_i that represents the random fluctuations of the returns due to speckle effects. One can thus write $a_{c_i} = a_i c_i$ where c_i are known. Taking this into account, (2) can be rewritten as

$$\mathbf{y} = V_c\mathbf{a} + \mathbf{n} \tag{3}$$

where $V_c = \{c_1\mathbf{v}_1, c_2\mathbf{v}_2, ..., c_L\mathbf{v}_L\}$, and $\mathbf{a} = \{a_1, ..., a_L\}^T$.

The clutter-plus-noise CM $R = E\{\mathbf{y}\mathbf{y}^\dagger\}$ can be synthesized using the model (3)

$$R = V_c \tilde{A} V_c^\dagger + R_n, \quad (4)$$

where $\tilde{A} = \text{diag}\{\tilde{a}_1^2, \tilde{a}_2^2, ..., \tilde{a}_L^2\}$ and $\tilde{a}_i^2 = E\{|a_i|^2\}$.

3. BAYESIAN ESTIMATION METHOD

Our ultimate goal consists in estimating the scattering coefficient of clutter patch i, i.e. the mean value of $|a_i|^2$. However, in order to be able to compare the proposed method to existing methods, we will first estimate $\mathbf{a}_c$ using a single realization of (1).

An optimal estimate of $\mathbf{a}_c$ is obtained by maximizing $p(\mathbf{a}_c|\mathbf{y})$ with respect to $\mathbf{a}_c$, where $p(x|y)$ denotes the conditional probability of x knowing y. This can be done by using the Bayes identity

$$p(\mathbf{a}_c|\mathbf{y})p(\mathbf{y}) = p(\mathbf{y}|\mathbf{a}_c)p(\mathbf{a}_c). \quad (5)$$

As $p(\mathbf{y})$ does not depend on $\mathbf{a}_c$, it does not need to be computed since it will not affect the maximum. From (2), one has $\mathbf{n} = \mathbf{y} - V\mathbf{a}_c$. As we assume a white Gaussian noise,

$$p(\mathbf{n}) \propto e^{-\mathbf{n}^\dagger R_n^{-1}\mathbf{n}}. \quad (6)$$

Hence, if $\mathbf{a}_c$ is known, (2) and (6) yield

$$p(\mathbf{y}|\mathbf{a}_c) \propto e^{-(\mathbf{y}-V\mathbf{a}_c)^\dagger R_n^{-1}(\mathbf{y}-V\mathbf{a}_c)}. \quad (7)$$

The prior probability $p(\mathbf{a}_c)$ expresses the a priori knowledge on $\mathbf{a}_c$. As a_{c_i} represents a single realization of the complex amplitude of the signal scattered by the clutter, we will assume it is independent and complex-Gaussian distributed [8], and

$$p(\mathbf{a}_c) \propto e^{-\mathbf{a}_c^\dagger R_{a_c}^{-1}\mathbf{a}_c}, \quad (8)$$

where R_{a_c} is taken proportional to a diagonal matrix with $\mathbf{c} = \{c_1, ..., c_L\}$ on its diagonal.

Finally,

$$p(\mathbf{a}_c|\mathbf{y}) \propto e^{-(\mathbf{y}-V\mathbf{a}_c)^\dagger R_n^{-1}(\mathbf{y}-V\mathbf{a}_c)-\mathbf{a}_c^\dagger R_{a_c}^{-1}\mathbf{a}_c} \quad (9)$$

and as the matrices R_n and R_{a_c} are positive definite, the maximum is reached for

$$\mathbf{a}_c = (V^\dagger R_n^{-1} V + R_{a_c}^{-1})^{-1} V^\dagger R_n^{-1}\mathbf{y}. \quad (10)$$

Due to the low-rank nature of the problem, $V^\dagger R_n^{-1} V$ is typically rank-deficient and $R_{a_c}^{-1}$ — the prior knowledge about $\mathbf{a}_c$ — acts as a regularizing term.

Let us compare this estimator to other estimators proposed in the literature. A periodogram is used in [5] to provide an estimate of $|\mathbf{a}_{c_i}|^2$ arguing that estimates of $|\mathbf{a}_{c_i}|^2$ can be obtained from a spectral analysis of the signal $\mathbf{y}$. Using the notation of this paper, the corresponding estimator can be expressed as

$$\mathbf{a}_c = V^\dagger \mathbf{y} \quad (11)$$

Another estimator that can be used is

$$a_{c_i} = \frac{\mathbf{v}_i^\dagger R_z^{-1}\mathbf{y}}{\mathbf{v}_i^\dagger R_z^{-1}\mathbf{v}_i}, \quad (12)$$

with $R_z = E\{\mathbf{z}\mathbf{z}^\dagger\}$ and $\mathbf{z} = \sum_{k=1,k\neq i}^{L} a_{c_k}\mathbf{v}_k + \mathbf{n}$. This estimator is optimum [9] if $\mathbf{v}_i$ is not correlated with $\mathbf{z}$, which is obviously not the case here. Under this assumption, (12) can be interpreted as an adapted matched filter (AMF), which optimally rejects the other components present in $\mathbf{y}$ [9]. An estimate of the CM R_z could be obtained iteratively in a similar way as in [10]. In the comparison below, the clairvoyant CM R_z is used.

In [6], the least-squares (LS) estimator is considered

$$\mathbf{a}_c = (V^\dagger V)^{-1} V^\dagger \mathbf{y}. \quad (13)$$

This expression is very similar to (10) without regularization term $R_{a_c}^{-1}$. The expression thus only exists if $V^\dagger V$ is full rank. This means that the number L of clutter patches considered in the model (1) needs to be taken smaller or equal to the rank of $V^\dagger V$. This approach requires an ad-hoc method to estimate the number of clutter patches L. In our approach, by using a regularization term, the only condition on L is that it should be large enough so that (1) accurately approximate the underlying continuous clutter integral [7].

Figures 1 and 2 present results of the estimation in the case of a bistatic scenario involving two aircrafts flying on perpendicular trajectories and the receiving aircraft is equipped with an uniform linear array. A constant value $a_i = 1$ was considered. In this case, the steering vectors $\mathbf{v}_i$ are characterized by a spatial frequency ν_s and a Doppler frequency ν_d (the frequencies are normalized). The steering vector $\mathbf{v}_i$ of each clutter patch along an isorange corresponds to a different frequency coordinate (ν_{s_i}, ν_{d_i}). The values of the amplitude of the estimated coefficients $|a_{c_i}|$ are plotted along the curve described by the frequency coordinate of the corresponding steering vectors. The antenna diagram is assumed omnidirectional in Fig. 1

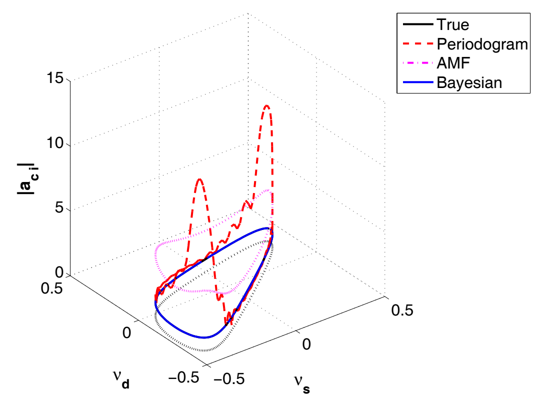

Fig. 1. Comparison of estimates of $|a_{c_i}|$ in the case of a scenario involving omnidirectional antennas.

and sinc-shaped in Fig. 2.

As can be seen, the Bayesian method accurately estimates the coefficients (the corresponding curve actually hides the curve corresponding to the true values). Moreover, as expected, the periodogram and the AMF fail to correctly estimate the coefficients. In Fig. 2, the periodogram actually fails to correctly null out the coefficients in the backlobe of the antenna. This is due to the high sidelobes of the periodogram.

A development similar to the one leading to (10) can be achieved from (3) to obtain the following expression for the optimum value of $\mathbf{a}$

$$\mathbf{a} = (V_c^\dagger R_n^{-1} V_c + R_a^{-1})^{-1} V_c^\dagger R_n^{-1}\mathbf{y}. \quad (14)$$

The prior knowledge about $\mathbf{a}$ is supplied by R_a which again acts as a regularizing term. Information available about the actual ground

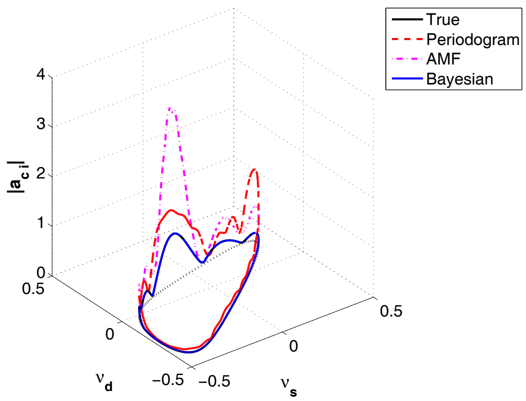

Fig. 2. Comparison of estimates of $|a_{c_i}|$ in the case of a scenario involving a directional receive antenna.

cover along the considered isorange can be introduced in the diagonal elements of R_a.

Figure 3 presents the results of a simulation in the same conditions as above, involving a directional receive antenna. According

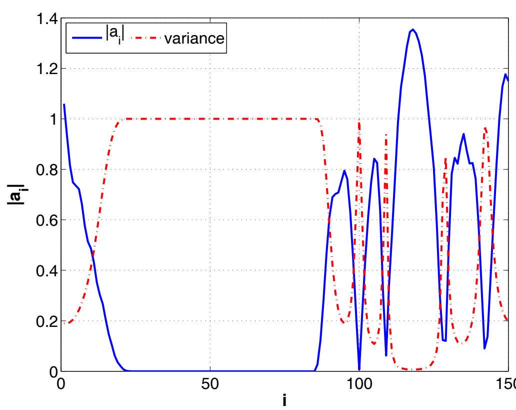

Fig. 3. Estimates of $|a_i|$ and corresponding variance in the case of a scenario involving a directional antenna.

to the simulation scenario, the true a_i's are constant and equal to one. The estimated value exhibits zeroes at the location of the zeroes of the antenna diagram. It should be stressed that the zeroes in the clutter map are not directly due to the zeroes in the antenna diagram, but are rather due to the fact that $\mathbf{y}$ contains no information about the scattering coefficients at those locations and the filter attempts to minimize the noise influence. This is illustrated by the variance of a_i (see Fig. 3), obtained from the diagonal terms of $(V_c^\dagger R_n^{-1} V_c + R_a^{-1})^{-1}$.

4. CLUTTER SCATTERING COEFFICIENTS ESTIMATION

By repeating the estimation process of $\mathbf{a}_c$ or $\mathbf{a}$ at the different available ranges, an estimation of the clutter map can be obtained. Due to the speckle [8], the estimated values will exhibit random variations. This is illustrated in Figs. 4 and 5 where the values of the $|a_{c_i}|$ resp. $|a_i|$ are displayed as a function of the actual location on the ground.

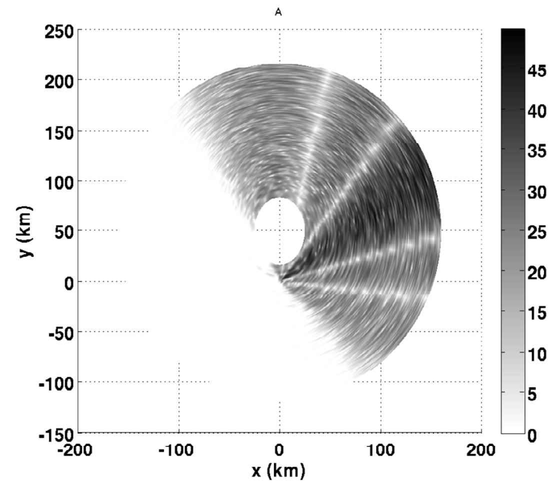

Fig. 4. Map of the estimated $|a_{c_i}|$ (in dB) at different range in the case of a scenario involving a directional antenna, plotted in geographic coordinates.

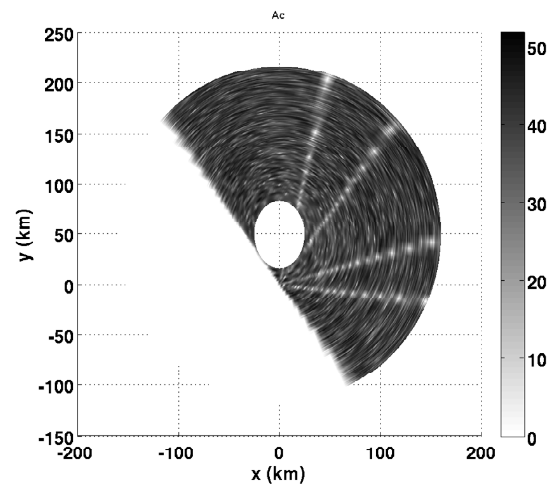

Fig. 5. Map of the estimated $|a_i|$ (in dB) at different range in the case of a scenario involving a directional antenna, plotted in geographic coordinates.

The scenario is the same as above, but with realistic values for $\mathbf{a}$, i.e. drawn from a circular Gaussian distribution. These clutter maps exhibit the usual speckle effect. The influence of the sinc-shaped antenna diagram is clearly visible in Fig. 4, where the amplitude of a_{c_i} is depicted. Figure 5 shows the amplitude of a_i and the white stripes are due to the zeroes in the antenna diagram, where the value of a_i cannot be estimated.

In order to synthesize the clutter-plus-noise CM, the mean squared magnitude $\tilde{a}_i^2 = E\{|a_i|^2\}$ of a_i is required. Estimation of the mean $\tilde{a}_i^2$ from these random values is obtained by spatially averaging the clutter map which is also known as multilooking. The spatial averaging only makes sense if the averaged quantities are identically distributed. This means that the averaging may only be performed over areas having similar ground cover. Moreover, data from ranges containing targets and more generally data from ranges where the model (1) is not valid should be excluded from the spatial averaging.

In the present application, the average was computed along straight radial lines from the center of the considered isorange in order to be able to compare the results with those obtained with other methods. It should be noted that this implies that the ground cover is homogeneous along straight lines, which is probably not very realistic. A more realistic assumption is that the ground cover is homogeneous over large patches (fields, forest, ...) over which averaging could be performed. The resulting estimate in the case of averaging along straight radial lines is illustrated in Fig. 6. As can be seen, the

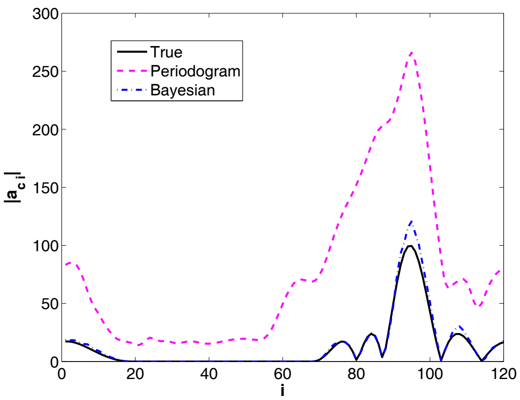

Fig. 6. Comparison of estimates of $|a_{c_i}|$ after spatial averaging.

averages obtained using the Bayesian method is in close agreement with the true values. However, spatially averaging the periodogram output fails to exhibit the zeroes of the antenna diagram.

5. END-TO-END PERFORMANCE

The quality of the estimated clutter-plus-noise CM can be measured by the SINR loss. Figure 7 shows the SINR loss obtained

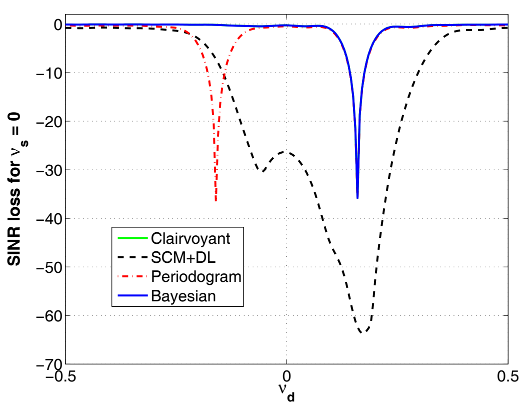

Fig. 7. Comparison of the SINR loss for the proposed method.

using different estimates for the CM. The curve corresponding to the optimum processor, that uses the clairvoyant CM, is actually hidden by the curve corresponding to the proposed Bayesian method. The effect of the range dependency on the SINR loss is clearly visible (overnulling) when the sample CM with diagonal loading (SCM+DL) is used. The SINR loss curve corresponding to the periodogram-based estimator exhibits a secondary notch due to an erroneous estimate of the scattering coefficients in the backlobe of the antenna.

6. SUMMARY AND CONCLUSIONS

The proposed method can be subdivided in three steps: (1) an analysis step, where a clutter map is computed from the data; (2) an averaging step where clutter scattering coefficients are obtained by spatially averaging the clutter map, and (3) a synthesis step, where the clutter scattering coefficients are used to reconstruct a CM.

The method relies on a model of the signal. This implies a perfect knowledge of the geometric configuration of the scenario considered and, of course, an accurate model. The estimation of the clutter map makes an optimum use of the available knowledge in a rigorous framework. The estimation is shown to outperform other methods in the literature.

A procedure where a clutter map is first estimated and then spatially averaged to obtain an estimate of the clutter scattering coefficients allows to consider arbitrary spatial averaging windows.

The proposed clutter-plus-noise CM estimation method provides a CM estimate that performs nearly as good as the optimum processor.

Finally, the proposed method can be used with any monostatic or bistatic radar configuration, and also with some multistatic configurations.

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