LDPC CODE DESIGN FOR HALF-DUPLEX RELAY NETWORKS

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ABSTRACT

In this study, we consider the design of LDPC codes for cooperative relay systems in half-duplex mode (namely, "cheap" relay) that are of practical interest. We transform the code design problem into the design of rate-compatible LDPC codes where the SNRs in different parts of one codeword are different. Due to the SNR variation, the conventional density evolution (DE) or extrinsic-mutual-information-transfer (EXIT) is not capable of accurately predicting the code performance. We develop a more refined definition of code ensembles and present a modified DE based algorithm related to the new relay code structure. Our results show that the proposed algorithm is more accurate than the conventional DE or EXIT in this case. We further employ the code optimization based on differential evolution. The optimized "cheap" relay code significantly outperforms existing codes.

Keywords: Cooperative relay network, half-duplex mode, LDPC

1. INTRODUCTION

Cooperative relay networks can provide not only "cooperative diversity" [1] but also higher transmission rates [2, 3], and thus, coding strategies that can improve the transmission rate of relay networks are of interest. The practical constraint for the wireless systems prohibits receiving and transmitting signals simultaneously in the same frequency. Hence, the current commercial relays have to work in half-duplex mode, namely, "cheap" relay [3].

LDPC code is a good candidate for capacity approaching in "cheap" relay channels because of its excellent performance [4]. LDPC code design typically refers to the design of code ensembles which characterizes a set of codewords sharing the same edge statistics of the parity-check matrices, via DE or EXIT [5, 6]. The design of rate-compatible LDPC codes have been studied in [7]. The existing rate-compatible LDPC code design employs the conventional DE algorithm with the extra edge constraints imposed due to the rate-compatible structure. Although the structure of "cheap" relay codes is in parts similar to the rate-compatible code structure, current design methods are not directly applicable because the SNR varies in different sub-blocks of one codeword, and thus, the code design problem becomes much more challenging.

In this study, we treat the design of rate-compatible LDPC code ensembles for "cheap" relay systems. We first present a new definition of the LDPC code ensemble for the codes which experience different SNRs within one codeword. This corresponds to the "cheap" relay scenario, as well as the ARQ scenario. Note that the new code ensemble definition characterizes the parity-check matrix in a more refined way. Based on this ensemble definition, we propose a modified DE algorithm to predict the "cheap" relay code performance. With this tool, the optimum ensemble of "cheap" relay codes can be searched via a modified differential evolution [5], where the rate-compatible constraints for "cheap" relay codes are treated. Our results show that the proposed algorithm is more accurate than the existing methods, and the optimized code achieves significant gain over the existing codes in "cheap" relay systems.

The remainder of the report is organized as follows. Section 2 describes the "cheap" relay systems, and formulates the problem of "cheap" relay code design. In Section 3, the entire framework for the "cheap" relay code design is developed, including the new ensemble definition for "cheap" relay codes, the modified DE algorithm, and the nonlinear optimization algorithm. Simulation results are shown in Section 4. Section 5 draws the conclusions.

2. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a multiple-relay channel consist of a source node (S) a destination node (D) and K relay nodes ($R^{(i)}$). Let x_S and $x_R^{(i)}$ be the transmit signals from the source and the relays, and h_{SD}, $h_{SR}^{(i)}$ and $h_{RD}^{(i)}$ be the path gains between the source-destination, source-relay, relay-destination, respectively. We consider half-duplex relay operation where relays receive in the odd and transmit in the even time slots. The received signals at $R^{(i)}$ and D are

$$y_R^{(i)}(2t-1) = h_{SR}^{(i)} x_S(2t-1) + n_R^{(i)}(2t-1), \quad (1)$$
$$y_D(2t-1) = h_{SD} x_S(2t-1) + n_D(2t-1), \quad (2)$$
$$y_D(2t) = h_{SD} x_S(2t) + h_{RD} x_R(2t) + n_D(2t), \quad (3)$$

where $n_R^{(i)}, n_D \sim \mathcal{N}(0, N_0)$ denote the AWGNs.

During the odd time slot, S transmits a LDPC codeword $\underline{w}_1$, i.e., $x_S(2t-1) = \sqrt{P_{S,1}}\underline{w}_1$, where $E\{\|\underline{w}_1\|^2\} = 1$. In the case that D cannot successfully decode $\underline{w}_1$, $R^{(i)}$ attempts to decode $\underline{w}_1$ using (1), and then, S and $R^{(i)}$ cooperatively transmit signals related to $\underline{w}_1$ during the even time slot. Assuming perfect decoding at $R^{(i)}$, both of S and $R^{(i)}$ transmit the same bits $\underline{w}_e$ during the even time slot, i.e., $x_S(2t) = \sqrt{P_{S,2}}\underline{w}_e$ and $x_R^{(i)}(2t) = \sqrt{P_R^{(i)}}\underline{w}_e$, $1 \le i \le K$, where $E\{\|\underline{w}_e\|^2\} = 1$. Then the received signals at D during the even time slot, (3), can be rewritten as $y_D(2t) = (h_{SD}\sqrt{P_{S,2}} + \sum_i h_{RD}^{(i)}\sqrt{P_R^{(i)}})\underline{w}_e + n_D(2t) = \tilde{h}_{SD}\underline{w}_e + n_D(2t)$.

The proposed "cheap" relay code in the form of $\underline{w}_2 = [\underline{w}_1, \underline{w}_e]$ can be interpreted as an extended LDPC code of the original codeword $\underline{w}_1$ with extended bits $\underline{w}_e$. Such a code structure allows for decoding of the code either as $\underline{w}_1$ or $\underline{w}_1$ with two different rates depending on the channel condition which defines a rate-compatible code. Therefore, the problem is formulated as designing an optimal code ensemble $\underline{w}_1$ (that has been extensively studied in the current literature) and the optimal extension of the code to the code ensemble $\underline{w}_2 = [\underline{w}_1; \underline{w}_e]$ by adding the extended parity bits $\underline{w}_e$. The rate-compatible LDPC code structure imposes new constraints on the parity-check matrix of the code [7]. The parity-check matrix $\boldsymbol{H}_2 = \begin{bmatrix} \boldsymbol{H}_1 & \boldsymbol{O} \\ \boldsymbol{A} & \boldsymbol{B} \end{bmatrix}$ for the entire codeword $\underline{w}_2$ contains the parity-check matrix for $\underline{w}_1$ ($\boldsymbol{H}_1$), non-zero sub-matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, and zero sub-matrix $\boldsymbol{O}$.

However, the main challenge in the "cheap" relay channel code design is the fact that the average received power corresponding to the two parts of the codeword $\underline{w}_1$ and $\underline{w}_e$ are in general not equal. Specifically, the SNRs for the decoding of $\underline{w}_1$ and $\underline{w}_e$ can be written, respectively, as $\gamma_1 = |h_{SD}|^2 P_{S,1}/N_0$ and $\gamma_2 = |\tilde{h}_{SD}|^2/N_0$, where $P_{S,1} + P_{S,2} + \sum_{i=1}^{K} P_R^{(i)} \leq P_T$ with P_T being the maximum overall transmit power. As a result of SNR variation in one codeword, the conventional DE or EXIT method cannot accurately predict the code behavior.

3. CODE DESIGN OF "CHEAP" RELAY LDPC CODES

3.1. Code Ensemble for "Cheap" Relay LDPC Codes

As is well known, a parity-check matrix fully describes the corresponding LDPC codeword. The code ensemble (i.e., edge distribution or node perspective) can be used to statistically characterize a set of codewords which share the same edge distribution, and thus, approximately have the same performance [5, 4]. Note that in the existing analysis, the SNR within one codeword is assumed the same, which applies in the block-fading or the fast-fading channels. However, for a "cheap" relay codeword, $\gamma_1 \neq \gamma_2$ for $\underline{w}_1$ and $\underline{w}_e$. As a result, the existing edge distribution is too rough for the existing ensemble analysis methods to evaluate "cheap" relay codes. Therefore, we propose a more refined definition as follows.

Let $D_V^{(k)}$ and $D_C^{(k)}$ be the maximum degrees of the variable nodes and the check nodes, respectively, for the code $\underline{w}_k$, $k = 1$ and 2. Denote $\{\lambda_{i,k}\}_{i=1}^{D_V^{(k)}}$ and $\{\rho_{i,k}\}_{i=1}^{D_C^{(k)}}$ as the variable edge distribution and the check edge distribution, respectively; denote $\{a_{i,k}\}_{i=1}^{D_V^{(k)}}$ and $\{b_{i,k}\}_{i=1}^{D_C^{(k)}}$ as the variable node perspective and the check node perspective, respectively. Suppose that one "cheap" relay codeword experiences two different SNRs, i.e., $\gamma_1 \neq \gamma_2$. For the code $\underline{w}_2$, we then have the following new ensemble definitions. Define $\pi_1(i)$ as the probability of a variable node which has a degree of i and falls in the region of $\underline{w}_1$ (i.e., $\boldsymbol{H}_1$ and $\boldsymbol{A}$), namely, region-1. Consequently, $\pi_2(i) = 1 - \pi_1(i)$ denotes the probability of the node falling in region-2 (i.e., $\boldsymbol{B}$ and $\boldsymbol{O}$ for $\underline{w}_e$). Clearly, the probability set $\{\pi_1(i), \pi_2(i)\}_{i=2}^{D_V^{(2)}}$ should satisfy the following constraint: $\sum_{i=1}^{D_V^{(2)}} \pi_1(i) a_{i,2} = \frac{R_2}{R_1}$ or $\sum_{i=1}^{D_V^{(2)}} \pi_2(i) a_{i,2} = 1 - \frac{R_2}{R_1}$, where R_l denotes the rate of $\underline{w}_l$, $l = 1, 2$. Then define the edge distribution within region-l, $l = 1$ and 2, as $\{\lambda_{i,2}^{(l)},\ l = 1, 2\}_{i=1}^{D_V^{(2)}}$ where $\lambda_{i,2}^{(l)} = \frac{\pi_l(i)\lambda_{i,2}}{\sum_{j=1}^{D_V^{(2)}} \pi_l(j)\lambda_{j,2}}$, $l = 1, 2$. Obviously, the new code ensemble of $\underline{w}_2$, $\{\{\lambda_{i,2}^{(1)}, \lambda_{i,2}^{(2)}\}_{i=2}^{D_V^{(2)}}, \{\rho_{j,2}\}_{j=2}^{D_C^{(2)}}\}$, depends on both the overall code ensemble $\{\{\lambda_{i,2}\}_i, \{\rho_{j,2}\}_j\}$ and the node probability set $\{\pi_1(i), \pi_2(i)\}_{i=2}^{D_V^{(2)}}$. Then for each overall code ensemble $\{\{\lambda_{i,2}\}_i, \{\rho_{j,2}\}_j\}$, there are a number of possible new code ensembles $\left\{\{\lambda_{i,2}^{(1)}, \lambda_{i,2}^{(2)}\}_i, \{\rho_{i,2}\}_j\right\}$, each of which can be fully determined by one node probability set $\{\pi_1(i), \pi_2(i)\}_i$. Note that the above new code ensemble describes the parity-check matrix of $\underline{w}_2$ in a more refined manner in the sense that the edge distributions within region-1 and region-2 can be described separately. Also note that the "cheap" relay codewords with the same overall code ensemble $\{\{\lambda_{i,2}\}_i, \{\rho_{j,2}\}_j\}$ but different probability set $\{\pi_1(i), \pi_2(i)\}_i$ typically have different performances.

3.2. Ensemble Performance Analysis of "Cheap" Relay Codes

As we mentioned in Section 3.1 and will demonstrate in Section 4, the fact $\gamma_1 \neq \gamma_2$ makes the existing methods based on the overall edge distribution $\{\{\lambda_{i,2}\}_i, \{\rho_{j,2}\}_j\}$, DE [5, 4] or EXIT chart [8, 6], not very accurate in performance prediction. We next develop an iterative algorithm to evaluate the "cheap" relay code based on the new code ensemble $\{\{\lambda_{i,2}^{(1)}, \lambda_{i,2}^{(2)}\}_i, \{\rho_{j,2}\}_j\}$. The main difference between this approach and the conventional DE approaches is that instead of assuming equal density function for the entire codeword, we treat different density functions separately for the two different region-l, $l = 1, 2$. For notational convenience, hereafter in Section 3.2, we drop the subscript "2" in those quantities for the codeword $\underline{w}_2$. Fig. 1 shows the general structure for iterative decoding of LDPC codes [6]. The variable node decoder (VND) denotes a repetition code decoder; the check node decoder (CND) denotes a single parity-check code decoder.

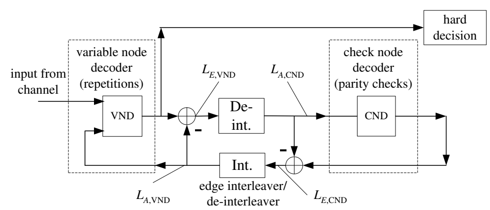

Fig. 1. Diagram of the iterative LDPC decoding structure.

VND process: The VND output LLR for a node with degree i is, in general, given by $L_{E,V}(i) = L_0 + \sum_{j \neq i} L_{A,V}(j)$, where $L_0 \sim \mathcal{N}(m_0, \sigma_0^2)$ denotes the VND input LLR from channel, and the mean $m_0 = 2\gamma_0$ depends on the channel SNR γ_0. Note that $L_{E,V}(i)$ in region-1 and region-2 are different since $\gamma_1 \neq \gamma_2$. Denote the quantities with superscript (l) as those for region-l, $l = 1, 2$. In particular, the VND output LLRs of a node with degree i for region-1 and region-2 are given, respectively, $L_{E,V}^{(1)}(i) = L_0^{(1)} + \sum_{j \neq i} L_{A,V}^{(1)}(j)$ and $L_{E,V}^{(2)}(i) = L_0^{(2)} + \sum_{j \neq i} L_{A,V}(j)$, where $L_0^{(1)} \sim \mathcal{N}(m_0^{(1)}, \sigma_0^{(1)})$ and $L_0^{(2)} \sim \mathcal{N}(m_0^{(2)}, \sigma_0^{(2)})$ correspond to the detector outputs; $m_0^{(1)} = 2\gamma_1$ and $m_0^{(2)} = 2\gamma_2$; $L_{A,V}^{(l)}(j)$ are i.i.d. symmetric Gaussian random variables, i.e., $L_{A,V}^{(l)}(j) \sim \mathcal{N}(m_C^{(l)}, 2m_C^{(l)})$, $l = 1, 2$, and will detailed later in this section. Then $L_{E,V}^{(l)}(i)$, $l = 1, 2$, can be well approximated by a symmetric Gaussian random variable [5, 4],

$$\mathcal{N}(\underbrace{m_0^{(l)} + (i-1)m_C^{(l)}}_{m_V^{(l)}(i)}, 2(m_0^{(l)} + (i-1)m_C^{(l)})). \tag{4}$$

Thus, the averaged VND output LLR per edge (for $l = 1, 2$) has the Gaussian mixture density function:

$$f_V^{(l)} = \sum_i \lambda_i^{(l)} \mathcal{N}(m_V^{(l)}(i), 2m_V^{(l)}(i)) = \mathcal{N}(m_V^{(l)}, 2m_V^{(l)}), \quad (5)$$

where $\{\lambda_i^{(l)}\}_i$ denotes the variable edge distribution in region-l.

CND process: The CND output LLR of a node with degree i is given by $L_{E,C}(i) = 2\arg\tanh(\prod_{j\neq i}\tanh(\frac{L_{A,C}(j)}{2}))$, where $L_{A,C}(j)$ denotes the CND input LLRs coming from the VND output edges. Taking expectation over each side, we have $E\{\tanh(\frac{L_{E,C}(i)}{2})\} = E\{\prod_{j\neq i}\tanh(\frac{L_{A,C}(j)}{2})\} = \prod_{j\neq i} E\{\tanh(\frac{L_{A,C}(j)}{2})\}$, where the second equality follows the fact that $L_{A,C}(j)$ is independently distributed. Note that $L_{E,C}(j)$ in region-1 and region-2 also have different density functions because $f_V^{(1)} \neq f_V^{(2)}$ as revealed in (4) and (5). Then the CND output LLRs in region-1 satisfy: $\psi(m_C^{(1)}(i)) = \left[\psi(m_V^{(1)})\right]^{d_{i,1}-1}\left[\psi(m_V^{(2)})\right]^{d_{i,2}}$, where $\psi(x) \triangleq E\{\tanh(y/2)\}$ with $y \sim \mathcal{N}(x, 2x)$; the second equality comes from the fact that $L_{A,C}^{(l)}(j)$ are i.i.d. within region-l; $d_{i,l}$, $l = 1, 2$, denotes the number of CND input edges coming from region-l, which is defined as $d_{i,l} = iP_l$ with $P_l \triangleq \frac{ia_i\pi_l(i)}{\sum_{j=2}^{D_V} ja_j}$ denoting the proportion of the number of edges falling within region-l. Furthermore, using the mixture Gaussian density function in (5), the mean of CND output LLRs in region-1, $m_C^{(1)}(i)$, is given by

$$\psi^{-1}([\psi(\sum_j \lambda_j^{(1)} m_V^{(1)}(i))]^{d_{i,1}-1}[\psi(\sum_j \lambda_j^{(2)} m_V^{(2)}(i))]^{d_{i,2}}). \quad (6)$$

Similarly, then mean of the CND output LLRs within region-2, $m_C^{(2)}(i)$, can then be written as

$$\psi^{-1}([\psi(\sum_j \lambda_j^{(1)} m_V^{(1)}(i))]^{d_{i,1}}[\psi(\sum_j \lambda_j^{(2)} m_V^{(2)}(i))]^{d_{i,2}-1}). \quad (7)$$

Using (6) and (7), the mixture Gaussian density functions of CND outputs are given by

$$f_C^{(l)} = \sum_i \rho_i \mathcal{N}(m_C^{(l)}(i), 2m_C^{(l)}(i)) = \mathcal{N}(m_C^{(l)}, 2m_C^{(l)}), \quad (8)$$

where $l = 1, 2$, and $\{\rho_i\}_i$ denotes the overall check distribution.

Our proposed new performance prediction algorithm can be summarized as follows. Calculate $m_V^{(l)}(i)$ using (4); calculate $m_C^{(l)}(i)$ using (6) and (7); then calculate $m_C^{(l)}$ using (8); repeat the above procedures until a certain iteration number is reached.

3.3. Optimization of "Cheap" Relay Code Ensemble

Note that the code ensemble optimization based on the differential evolution [5] can also be employed to find the optimum code ensemble $\{\{\lambda_{i,2}^{(1)}, \lambda_{i,2}^{(2)}\}_i, \{\rho_{j,2}\}_j\}$ for the "cheap" relay code $\underline{w}_2$ which is an extended LDPC code of the original one $\underline{w}_1$. Then the ensemble optimization procedures can be summarized as follows.

[Algorithm 1] Ensemble optimization

(a) Initialization:
given the original LDPC code with a rate of R_1 and an ensemble of $\{\{\lambda_{i,1}\}_{i=1}^{D_V^{(1)}}, \{\rho_{j,1}\}_j^{D_C^{(1)}}\}$, generate N codes each of which has a rate of R_2 and an ensemble of $C(n) = \{\{\lambda_{i,2}^{(1)}(n), \lambda_{i,2}^{(2)}(n)\}_{i=1}^{D_V^{(2)}}, \{\rho_{j,2}(n)\}_{j=1}^{D_C^{(2)}}\}$, $n = 1, 2, \cdots, N$.

(b) Code ensemble evaluation:
evaluate each code ensemble $C(n)$; and find the optimum code ensemble $C^* = \arg\max_{1\leq i\leq N} m_{\text{dec}}(n)$, where $C^* = \{\{\lambda_{i,2}^*, \lambda_{i,2}^*\}_{i=1}^{D_V^{(2)}}, \{\rho_{j,2}^*\}_{j=1}^{D_C^{(2)}}\}$, and $m_{\text{dec}}(n)$ is the mean of the decoder output for $C(n)$.

(c) Code ensemble updating:
randomly choose $\tilde{N}$ ensembles out of $\{C(n)\}_{n=1}^N$, denoted by $\tilde{C}(k) = \{\{\tilde{\lambda}_{i,2}(k), \tilde{\lambda}_{i,2}(k)\}_{i=1}^{D_V^{(2)}}, \{\tilde{\rho}_{j,2}(k)\}_{j=1}^{D_C^{(2)}}\}$, $k = 1, \cdots, \tilde{N}$; and then, update the n-th ensemble by $C(n) = C^* + \alpha\sum_{k=1}^{\tilde{N}}(-1)^{k-1}\tilde{C}(k)$, where $\alpha < 1$ is the step size.

(d) Repeat step (c) to update all code ensembles $C(n)$.

(e) If the iteration number exceeds a predefined maximum iteration number, then stop; otherwise, go back to step (b).

In Step (a), to generate new code ensembles using the original ensemble, some additional rate-compatible constraints have to be treated as follows. The overall edge distribution and node perspective of $\underline{w}_2$, $\{\lambda_{i,2}, a_{i,2}\}_{i=1}^{D_V^{(2)}}$ and $\{\rho_{j,2}, b_{j,2}\}_{j=1}^{D_C^{(2)}}$, satisfy [5]:

$$\begin{cases} \lambda_{2,2} = 1 - \sum_{j=3}^{D_V^{(2)}} \lambda_{j,2}, \text{ and } \rho_{2,2} = 1 - \sum_{j=3}^{D_C^{(2)}} \rho_{j,2}, \\ \lambda_{D_V^{(2)},2} = \frac{\frac{R_2}{2}\sum_{j=3}^{D_C^{(2)}}(\frac{1}{j}-\frac{1}{2}) - (1-R_2)\sum_{j=3}^{D_V^{(2)}-1}(\frac{1}{j}-\frac{1}{2})}{(1-R_2)(\frac{1}{D_V^{(2)}}-\frac{1}{2})}, \\ a_{i,2} = \frac{\frac{\lambda_{i,2}}{i}}{\sum_{j=2}^{D_V^{(2)}}\frac{\lambda_{j,2}}{j}}, \text{ and } b_{i,2} = \frac{\frac{\rho_{i,2}}{i}}{\sum_{j=2}^{D_C^{(2)}}\frac{\rho_{j,2}}{j}}. \end{cases} \quad (9)$$

Further consider the rate-compatible structure of the parity-check matrix $\boldsymbol{H}_2$. Since $\boldsymbol{O}$ is a zero sub-matrix, the overall check node perspective $\{b_{i,2}\}_{i=1}^{D_C^{(2)}}$ of $\boldsymbol{H}_2$ and the overall check node perspective $\{b_{i,1}\}_{i=1}^{D_C^{(1)}}$ of $\boldsymbol{H}_1$ satisfy the constraints [7]: $b_{i,2}M_2 \geq b_{i,1}M_1$, i.e., $b_{i,2} \geq \frac{M_1}{M_2}b_{i,1} = \frac{(1-R_1)N_1}{(1-R_2)N_2}b_{i,1} = \frac{(1-R_1)R_2}{(1-R_2)R_1}b_{i,1} = \Gamma b_{i,1}$, $i = 2, 3, \cdots, D_C^{(1)}$ where the first equality follows the fact $1 - R_i = \frac{M_i}{N_i}$ and the second equality comes from the inherent rate-compatible property, $R_1N_1 = N_1 - M_1 = N_2 - M_2 = R_2N_2$. Then using (9), the node perspective constraints can be equivalently transformed into the corresponding edge distribution constraints $\frac{\rho_{i,2}/i}{\sum_{j=2}^{D_C^{(2)}}\rho_{j,2}/j} \geq \Gamma b_{i,1}$, i.e., $\frac{\rho_{i,2}}{i} \geq \Gamma b_{i,1}[\frac{1}{2} + \sum_{j=3}^{D_C^{(2)}}\rho_{j,2}(\frac{1}{j}-\frac{1}{2})]$, $i = 2, 3, \cdots, D_C^{(1)}$. For the case of $i = 2$, using $\rho_{2,2} = 1 - \sum_{j=3}^{D_C^{(2)}}\rho_{j,2}$ in (9), we further have $\frac{1-\Gamma b_{2,1}}{2} \geq \sum_{j=3}^{D_C^{(2)}}\frac{\rho_{j,2}}{2} + \Gamma b_{2,1}[\sum_{j=3}^{D_C^{(2)}}\rho_{j,2}(\frac{1}{j}-\frac{1}{2})]$. On the other hand, the overall variable node perspective $\{a_{i,2}\}_{i=1}^{D_V^{(2)}}$ of $\boldsymbol{H}_2$ and $\{a_{i,1}\}_{i=1}^{D_V^{(1)}}$ of $\boldsymbol{H}_1$ satisfy the constraints [7]: $\sum_{k=l}^{D_V^{(2)}} a_{k,2} \geq \frac{N_1}{N_2}\sum_{k=l}^{D_V^{(1)}} a_{k,1} = \frac{R_2}{R_1}\sum_{k=l}^{D_V^{(1)}} a_{k,1} = \Upsilon\sum_{k=l}^{D_V^{(1)}} a_{k,1}$, $l = 2, 3, \cdots, D_V^{(1)}$. Then using (9), the node constraints can be equivalently transformed into the edge constraints: $\sum_{k=l}^{D_V^{(2)}}\frac{\lambda_{k,2}}{k} \geq (\Upsilon\sum_{k=l}^{D_V^{(1)}} a_{k,1})(\frac{\frac{1}{2}+\sum_{k=3}^{D_C^{(2)}}\rho_{k,2}(\frac{1}{k}-\frac{1}{2})}{1-R_2})$, $l = 2, 3, \cdots, D_V^{(1)}$.

code-1	structure-1	structure-2	structure-3
Algorithm 1	0.64dB	0.82dB	0.92dB
Simulation	0.87dB	0.99dB	1.32dB
DE	1.24dB	2.52dB	3.32dB
EXIT	1.07dB	2.61dB	3.01dB

Table 1. E_bN_0 threshold comparison among Algorithm 1 and the existing DE and EXIT chart approaches: $1/2$-rate codes; $\gamma_2 = 2\gamma_1$; $N_2 = 2N_1$.

	optimized code	repetition with code-1
Prediction	-2.90dB	-2.34dB
Simulation	-2.33dB	-1.80dB

Table 2. E_bN_0 threshold of the optimized "cheap" relay code.

In Step (b), the performance of each code ensemble $C(n) = \{\{\lambda_{i,2}^{(1)}(n), \lambda_{i,2}^{(2)}(n)\}_{i=2}^{D_V^{(2)}}, \{\rho_{i,2}(n)\}_{i=2}^{D_C^{(2)}}\}$ need to be evaluated. Hence, we need to obtain $\{\lambda_{i,2}^{(1)}(n), \lambda_{i,2}^{(2)}(n)\}_{i=2}^{D_V^{(2)}}$ for the given overall code ensemble $\{\{\lambda_{i,2}\}_i, \{\rho_{j,2}\}_j\}$. As we discussed in Section 3.1, for a given overall edge distribution $\{\lambda_{i,2}, \rho_{j,2}\}_{i,j}$, the edge distribution $\{\lambda_{i,2}^{(1)}(n), \lambda_{i,2}^{(2)}(n)\}_{i=2}^{D_V^{(2)}}$ fully depends on the node probability set $\{\pi_1(i), \pi_2(i)\}_i$, which is not unique and corresponds to a specific structure of the parity-check matrix.

4. SIMULATION RESULTS

Evaluation of "cheap" relay codes: We now show some comparisons between the new and existing algorithms. The averaged SNR within $\underline{w}_1$ is 3dB less than that within the other half codeword, i.e., $\gamma_2 = 2\gamma_1$ and $N_2 = 2N_1$. We consider a rate $1/2$ code (code-1) with $\lambda(x) = 0.2124x^2 + 0.1985x^3 + 0.0084x^5 + 0.0560x^6 + 0.0142x^7 + 0.1665x^8 + 0.0091x^9 + 0.0200x^{10} + 0.0025x^{20}$, and $\rho = 1$ and the node probability sets $\pi_1(i) = 0.5; 1 \leq i \leq D_V^{(2)}$, $\pi_1(2) = 1; \pi_1(3) = 0.0745$, and $\pi_1(3) = 0.9255, \pi(5) = \cdots = \pi(30) = 1$ for three structures 1, 2, and 3, respectively. Code length is $N_2 = 40000$. The results in Table 1 show that our algorithm more accurately predict the code performance than DE or EXIT. It is seen that the more the complexity of the edge distribution, the more the gap between the predicted threshold by conventional DE and EXIT versus the actual performance. The good prediction achieved by the proposed algorithm is very important, for the optimization of the cheap relay channel typically results in complicated edge distributions.

Optimized ensemble of the "cheap" relay code: Next we show the performance of Algorithm 1. The normalized distance between S and D is $d = 0.25$; $P_{S,1} = P_T/2$ and $P_{S_2} = P_R = P_T/4$; $R_1 = 1/2$ and $R_2 = 1/4$; since $D_V^{(1)} = 30$, we set $D_V^{(2)} = 33$. Code-1 above is used for $\underline{w}_1$, which is actually the optimum LDPC code for AWGN channels. The performance of the optimized relay code is shown in Table 2. As a comparison, the repetition scheme using code-1 is adopted here, i.e., during S transmits $\underline{w}_1$ (code-1), and then, S and R transmit the same code $\underline{w}_e = \underline{w}_1$. Fig. 2 shows the BER performance of the optimized code and the above repetition scheme with different code lengths. It is seen that the optimized code can achieve a significant gain (0.5dB) over the repetition scheme with the optimum AWGN code.

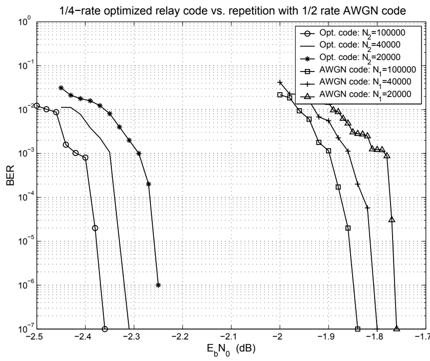

Fig. 2. BER performance comparison.

5. CONCLUSIONS

In this study, we treated the LDPC code design for "cheap" relay systems. We have presented the new definition of the LDPC code ensemble, based on which we have developed the DE based algorithm to predict the "cheap" relay code performance. Further considering the specific rate-compatible edge constraints, we have employed the optimization framework based on differential evolution to find the optimum code ensemble. We have shown that the proposed algorithm is more accurate than the existing methods, and the optimized code outperforms the existing AWGN codes.

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