# ERROR EXPONENTS FOR TARGET-CLASS DETECTION WITH NUISANCE PARAMETERS

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**ABSTRACT**

We study the target class detection performance of a sensor network having a structured topology. The target is in the far-field of the network, located at a distance $\gamma$ and angle $\theta$, and produces a random signal field that is sampled by sensors. It is assumed that samples have a correlation structure and power level that depend on $\gamma$, $\theta$ and the target's class $i$, $i \in \{0, 1\}$.

We study the Neyman-Pearson miss probability error exponent for this scenario using large deviations theory. When ($\gamma$, $\theta$) is known, we characterize the properties of the error exponent as a function of signal and design parameters. When ($\gamma$, $\theta$) has at least one unknown component, we use the theory of adaptive tests [2] to prove that there exists a test that achieves the same error exponent as if ($\gamma$, $\theta$) were known in some scenarios, but that there exists no such test in others.

***Index Terms—*** Error exponent, Gauss-Markov model, Neyman-Pearson detection, Adaptive test

## 1. INTRODUCTION

We study the identification of a target by a sensor network. We assume that the target belongs to one of two classes (e.g., friendly and enemy) and produces a stochastic "signal field" that evolves concentrically, and that is sampled with spacing $d$ by a set of $N$ sensors located in the far-field of the target at a distance $\gamma$ and angle $\theta$; see Figures 1 and 2. It is assumed that samples have a correlation structure and power level that depend on $\gamma$, $\theta$ and the target's class $i$, $i \in \{0, 1\}$. (For example, if the target is a tank it may produce an acoustic wavefront which can be sampled by sensors, creating a signal field. The variance and correlation structure of this field would depend on the class of tank, as well as the location of the sensors relative to the tank.) These samples are sent to a fusion center which fuses the data and makes a single global decision as to the target's class (this scenario is formalized in Section 2). We assume that the fusion center uses binary hypothesis testing to make a decision, where hypothesis $\mathcal{H}_i$ denotes the event that the target is of class $i \in \{0, 1\}$, and, in the absence of a prior probability on the class of target, uses the Neyman-Pearson (NP) testing framework [5].

We study the detection performance once the $N$ samples arrive at the fusion center.[1] Let $P_F$ denote the probability of false alarm and $P_M$ the probability of miss. To characterize detection performance we study the NP error exponent defined for a constraint $P_F \leq \alpha \ \in (0, 1)$ as the exponential rate of decay in $P_M$ as the number of signal samples approaches infinity, i.e.,

$$K \triangleq \lim_{N\to\infty} -\frac{1}{N} \log P_M. \tag{1}$$

[1]The communication protocols used to initiate the detection process and to deliver the samples to the fusion center are not considered in this work.

The error exponent is a useful metric. It provides an estimate on the number of observations needed to attain a given level of detection performance, is often parameterized by physical and design parameters (e.g., the signal to noise ratio (SNR) and sensor spacing) that allow us to quantify and optimize the impact of these parameters on detection performance. However, equation (1) is currently in an implicit form that is not amenable to analysis.

Suppose first that $(\gamma, \theta)$ is known perfectly at the fusion center. Let $p_i(s_1^N|\gamma, \theta)$ denote the probability density of the sequence of $N$ samples, $s_1, \ldots, s_N$, obtained when $\mathcal{H}_i$ is true. If the likelihood ratio test (LRT) is used at the fusion center, a generalization of Stein's lemma [8] yields

$$K = \lim_{N\to\infty} \frac{1}{N} \log \frac{p_0(s_1^N \mid \gamma, \theta)}{p_1(s_1^N \mid \gamma, \theta)} \qquad \text{(a.s. in } \mathcal{H}_0\text{)}, \tag{2}$$

provided that the limit exists, where the notation (a.s. in $\mathcal{H}_0$) means that the limit is to be taken in the almost sure sense under $\mathcal{H}_0$. Note that (2) is independent of $\alpha$.

Now suppose that at least one component of $(\gamma, \theta)$ is unknown. In this case, the fusion center cannot implement the (NP optimal) LRT, and (2) serves only as an upper bound on detection performance. We ask, can this upper bound be achieved by any implementable test? Recall that our criteria (1) is defined as the data size $N$ grows large. As an intuitive example of how such a test could exist, consider a test employing the LRT but with unbiased estimates of the unknown parameters used in place of the true values. Such a test would be able to achieve (2) if the estimators converge sufficiently fast in $N$. More generally, we are interested in the existence of *any* test which achieves the bound (2). We use the theory of adaptive tests [2] to prove that when ($\gamma$, $\theta$) has at least one unknown component, the existence of such a test is strongly dependent on the assumptions made on the knowledge of ($\gamma$, $\theta$). See Section 6 for a list of main results.

### 1.1. Related Work

This work addresses the distributed detection of a binary hypothesis using the criterion (1). Related works consider the evaluation of (1) for certain signal models. When the probability density of the data is known for each hypothesis, [7] considers evaluation of (1) when $s_1^N$ is i.i.d. receiver noise under $\mathcal{H}_0$ and a noisy ergodic Gauss-Markov signal under hypothesis $\mathcal{H}_1$. In [1, pp.138-139] and [2], error exponents are provided for the case of Gauss-Markov signals under either hypothesis where the correlation parameter or variance is hypothesis dependent. In [3], we characterized properties of the error exponent for equi-powered noiseless Gauss-Markov signals under both hypotheses using a physical model which linked the correlation parameter to network design parameters.

When knowledge of the probability density is incomplete, one approach is to model unknown parameter(s) ($\gamma$ and/or $\theta$ in this work) as nuisance parameters. Reference [2] provides a definition of and theorem on the existence of asymptotically optimal tests (known as adaptive tests), and evaluates it for a wide class of Markov processes. In [3], we showed that for a related target detection problem, the nuisance parameter is embedded within the correlation parameter. We proved that no adaptive test exists for such a scenario. This work is a generalization of [3] to signals of unequal power and with multiple nuisance parameters, and for several different assumptions on which parameters are known at the fusion center.

## 2. SYSTEM MODEL

Consider the system described in the introduction and in Figures 1 and 2. We model the signal field as a Gaussian random field that evolves with a Gauss-Markov correlation structure along any straight line originating from the target. In the far-field, the closest straight line intersects the collection line at an angle $\theta$. We assume $\theta \in \Theta \triangleq [\theta_{\min}, \theta_{\max}]$, where $0 \leq \theta_{\min} < \theta_{\max} \leq \pi/2$.[2]

Consider the observations $s_1^N$ taken by the sensors. We assume that $s_1^N$ are noiseless, with statistics under $\mathcal{H}_i$ described by

$$\mathcal{H}_i : s_k = a_i\ s_{k-1} + z_{i,k},\ \ i \in \{0,1\}, \tag{3}$$

where $a_i \in (0,1)$ describes the correlation strength, $s_k \sim \mathcal{N}(0, \gamma\ \sigma_i^2)$ is the $k^{\text{th}}$ signal sample, and $z_{i,k} \overset{i.i.d.}{\sim} \mathcal{N}(0, \gamma\ \sigma_i^2(1-a_i^2))$ is innovations noise.[3] The signal power is given by $\gamma\sigma_i^2$ where $\sigma_i^2 > 0$ denotes the power of the signal observed at the sensors if the network is located at some reference distance from the target, and where $\gamma$ is a scale factor that reflects the actual distance of the target. We take $\gamma \in \Gamma \triangleq [\gamma_{\min}, 1]$ with $\gamma_{\min} \in (0,1)$.[4] We assume that the coefficient $a_i$ decays exponentially in the *projected* sensor distance $d\cos(\theta)$ at a rate proportional to a specified constant $A_i$, i.e.,

$$a_i = \exp\{-A_i d\cos(\theta)\}, \tag{4}$$

where $A_i \in (0, \infty)$ and $A_0 \neq A_1$. All quantities are assumed known at the fusion center except for possibly $\gamma$ and $\theta$, discussed below.

Topography and logistics can limit the possible region over which the target can be located (e.g., due to roadways, mountains, or rules and regulations). If $\gamma$ and/or $\theta$ are unknown at the fusion center, the values of $\theta_{\min}, \theta_{\max}$, and/or $\gamma_{\min}$ can be used to quantify this a priori knowledge. It will be seen that knowledge of $(\gamma, \theta)$ plays a major role in the ability of the network to determine the target's class.

## 3. ERROR EXPONENT FOR KNOWN LOCATION

We start with the case where ($\gamma$, $\theta$) is known perfectly at the fusion center. We evaluate and analyze (2).

### 3.1. Calculation of the Error Exponent via the SLLN

Since $s_1^N$ is a Markov sequence, it follows that $\log P_i(s_1^N|\gamma,\theta) = \log P_i(s_1|\gamma,\theta) + \sum_{k=2}^{N} \log P_i(s_k|s_{k-1},\gamma,\theta)$. Each term above is Gaussian distributed and is easily evaluated [4, p.184]. Substituting the result into (2) and repeatedly applying the strong law of large numbers (SLLN) for weakly stationary processes [6, p.206], we get

$$K = \frac{1}{2}\log\left(R\frac{1-a_1^2}{1-a_0^2}\right) + \frac{1}{2}\left(\frac{1}{R}\frac{1-2a_0a_1+a_1^2}{1-a_1^2} - 1\right), \tag{5}$$

where $R \triangleq \sigma_1^2/\sigma_0^2$ is the ratio of signal powers. Next, we analyze (5) w.r.t. $R$ and the network parameters given by the relation (4).

### 3.2. Analytic Properties of the Error Exponent

In properties P1-P3 below, we fix $\{a_j\}$ and study the behavior of $K$ w.r.t. $R$. We can prove:

**P1 *(General behavior w.r.t.* $R$*).*** $K$ is monotone decreasing and convex for $R \in (0, R^*]$, monotone increasing and convex for $R \in [R^*, 2R^*]$, and monotone increasing and concave for $R \in [2R^*, \infty)$, where $R^*$ is given below.

**P2 *(Minimum w.r.t.* $R$*).*** The minimum $R^*$ is given by

$$R^* = \frac{1-2a_0a_1+a_1^2}{1-a_1^2}. \tag{6}$$

**P3 *(Maximum w.r.t.* $R$*).*** In the limits in $R$, we have

$$\lim_{R\to 0} K = \lim_{R\to\infty} K = \infty.$$

**Proofs.** The proofs of P1-P3 are straightforward and omitted.

From property P2, it follows that $K \geq 0$ with equality if and only if (i.f.f.) $R = 1$ and $a_0 = a_1$. Note that $R^* > 1$ if $a_1 > a_0$, and $R^* < 1$ if $a_1 < a_0$. This is because, in the model (3), the sample-by-sample innovation is decreasing in the correlation parameter $a_i$ and increasing in the signal power $\sigma_i^2$. If, e.g., $a_1 > a_0$, the sample-wise innovation due to correlation is greater under $\mathcal{H}_0$ than $\mathcal{H}_1$. Starting from the case that $\sigma_0^2 = \sigma_1^2$ and holding $\sigma_1^2$ fixed, the detection problem becomes more difficult as $\sigma_0^2$ decreases, since this would equalize the overall sample-wise innovation under the two hypotheses. However, for some value of $\sigma_0^2$, further decreasing $\sigma_0^2$ makes the detection problem easier, as the overall innovations under the two hypothesis begins to diverge. It is expected therefore that $1 < R^* < \infty$ when $a_1 > a_0$, which is true.

Next, we fix $R$ and study the variation of $K$ w.r.t. correlation parameters ($d$, $A_0$, $A_1$, $\theta$). Substituting (4) into (5), we can prove:

**P4 *(Vary* $A_i$*).*** Let $i, j \in \{0,1\}$ with $i \neq j$. Fix $A_j$. Then $K$ is monotone decreasing for $A_i \in (0, f_i(R)]$ and monotone increasing for $A_i \in [f_i(R), \infty)$, where $f_0(R)$ and $f_1(R)$ are given by the valid solution to a quadratic and cubic equation, respectively. From these, it can be verified that $f_i(1) = A_j$, and $f_0(R)$ is monotone increasing in $R$ while $f_1(R)$ is monotone decreasing.

**P5 *(Vary* $d$*).*** For $R \gg 1\ \ (R \ll 1)$[5],

$$K(d)\ \begin{cases} \uparrow\ \ (\downarrow)\ \ \text{in } d, \text{if } A_1 < A_0\ , \\ \downarrow\ \ (\uparrow)\ \ \text{in } d, \text{if } A_1 > A_0\ . \end{cases}$$

**P6 *(Vary* $d$*).*** In the limits

$$\lim_{d\to\chi} K = \begin{cases} \frac{1}{2}\left[\log\left(R\ \frac{A_1}{A_0}\right) + \frac{1}{R}\frac{A_0}{A_1} - 1\right], & \text{for } \chi = 0, \\ \frac{1}{2}\left[\log R + \frac{1}{R} - 1\right], & \text{for } \chi = \infty. \end{cases}$$

**Proofs.** The proofs of P4-P6 are omitted due to lack of space.

Property P4 is illustrated in Figure 3, where $A_1$ is fixed and we plot $K(A_0, R)$, i.e., $K$ as a function of $A_0$ and $R$. If $R = 1$, $K(A_0, R)$ is decreasing for $A_0 \in (0, A_1)$ and increasing for $A_0 \in (A_1, \infty)$. If $R > 1$, this critical point shifts to the right, and if $R < 1$, it shifts to the left. Note that the general shape of $K$ versus $A_0$ varies for different values of $R$, as shown in the figure. Property P5 has the following design implication: if $R \gg 1$ or $R \ll 1$, the optimal sensor spacing $d$ is chosen to make the signal samples either independent or *maximally* correlated, depending on if $A_0/A_1 > 1$. Finally, we note that in P5 and P6, the results hold if $d$ is replaced by $d\cos(\theta)$. Thus we can regard $d$ as fixed, and vary $\cos(\theta)$ instead.

[2] Choosing the right limit as $\frac{\pi}{2}$ will be seen to entail no loss of generality.

[3] $\mathcal{N}(0, \sigma^2)$ denotes a zero mean Gaussian random variable with var. $\sigma^2$.

[4] If $\gamma$ is decreasing in the target-to-network distance, $\gamma = 1$ represents the furthest, and $\gamma = \gamma_{\min}$ the closest, possible distance to the sensor network.

[5] The notation "$\uparrow$" ( "$\downarrow$" ) indicates monotone increasing (decreasing).

### 3.3. Numerical Results

Define the finite-$N$ exponential error rate by $K(N) \triangleq -\frac{1}{N}\log P_M$, and note that $\lim_{N\to\infty} K(N) = K$. In Figure 4, we plot $K(N), N \in \{20, 40\}$ (determined numerically), versus $R$ for $A_0 = 0.5$, $A_1 = 0.2$, $d = 1$, $\theta = 0$, $\alpha = 0.01$, $\sigma_1^2 = R$, and $\sigma_0^2 = 1$. We also plot $K$, determined from (5). Note that properties observed in P1-P3 hold for finite $N$ as well: $K(N)$ obeys the convexity structure of P1, has a minimum when $R > 1$ as in P2, and achieves maxima as $R \to \{0, \infty\}$, as in P3. Related simulations are seen to concur with the properties predicted by P4-P6 (results omitted for brevity).

## 4. ERROR EXPONENT FOR UNKNOWN LOCATION

When at least one component of $(\gamma, \theta)$ is unknown, (2) cannot be evaluated. We treat unknowns as nuisance parameters. Let $\Omega$ denote the parameter space of the unknown quantities. We consider three cases: (1) $\Omega = \Theta$, corresponding to $\gamma$ *known*, $\theta$ *unknown*, (2) $\Omega = \Gamma$, corresponding to $\gamma$ *unknown*, $\theta$ *known*, and (3) $\Omega = \Gamma \times \Theta$, corresponding to $\gamma$ *unknown*, $\theta$ *unknown*.

### 4.1. Adaptive Tests

We seek a test that performs well for any value of the unknown parameter $\omega \in \Omega$. To this end, we study the existence of adaptive tests, defined below.

**Definition.** Given constants $\{\alpha_\omega\}_{\omega\in\,\Omega}$, an adaptive test: (i) is independent of $\omega$, (ii) has false alarm satisfying $P_F \le \alpha_\omega, \forall\omega \in \Omega$, and (iii) achieves the best possible miss probability exponent, i.e.,

$$\lim_{N\to\infty} -\frac{1}{N}\log P_M = K_\omega,\ \forall\omega \in \Omega,$$

where $K_\omega$ is given by (5) (the subscript $\omega$ is used for emphasis).

It follows from (iii) that an adaptive test requires the same asymptotic detection performance as if the nuisance parameter were known a priori, and that the false alarm constraint of such a test can depend on $\omega$. These features distinguish it from the well known but often less tractable uniformly most powerful (UMP) test [5]. A necessary and sufficient condition for the existence of an adaptive test is given by the following theorem:

**Theorem [2].** An adaptive test exists i.f.f. $\forall\omega_0, \omega_1 \in \Omega : \omega_0 \ne \omega_1$,

$$K_{\omega_1} \le K_{\omega_0,\omega_1}, \tag{7}$$

where

$$K_{\omega_0,\omega_1} \triangleq \lim_{N\to\infty}\frac{1}{N}\log\frac{p_0(s_1^N \mid \omega=\omega_0)}{p_1(s_1^N \mid \omega=\omega_1)} \quad \text{(a.s. in } \mathcal{H}_0\text{)}. \tag{8}$$

Therefore, an adaptive test exists i.f.f. each of the hypothesis testing problems corresponding to a fixed value of $\omega$ is at least as difficult as when the null hypothesis has a different value.

### 4.2. Results On the Existence of Adaptive Tests

The existence of adaptive tests in this first scenario can be determined from an analysis of (8). We have the following result.

**Result 1 (R1)**. For the hypothesis testing problem of (3) and (4), let $\theta$ and $\gamma$ denote the true values of the target angle and distance, respectively. Let $\Omega$ be the parameter space of the nuisance parameter. We have the following result:

**R1.1**. If $\Omega = \Theta$, no adaptive test exists,

**R1.2**. If $\Omega = \Gamma$, an adaptive test exists i.f.f. $R$=$R^*$,

**R1.3**. If $\Omega = \Gamma \times \Theta$, no adaptive test exists.

**Proofs.** The formal proofs of R1.1-R1.3 are omitted due to lack of space. However, see the commentary below for brief outlines.

*Commentary on R1.1.* Define the function

$$\begin{aligned}&F(\gamma_0, \gamma_1, \theta_0, \theta_1)\\ &\triangleq \frac{1}{2}\log\left[R\frac{\gamma_1}{\gamma_0}\frac{1-\mu_1^2}{1-\mu_0^2}\right] + \frac{1}{2}\left[\frac{1}{R}\frac{\gamma_0}{\gamma_1}\frac{1-2\mu_0\mu_1+\mu_1^2}{1-\mu_1^2} - 1\right],\end{aligned}$$

where $\mu_i \triangleq \exp\{-A_i d\cos(\theta_i)\}$. Note that $K_{\omega_1} = F(\gamma, \gamma, \omega_1, \omega_1)$. Applying the SLLN in a manner similar to the derivation of (5), we obtain $K_{\omega_0,\omega_1} = F(\gamma, \gamma, \omega_0, \omega_1)$. The proof that there exists $\omega_0, \omega_1 \in \Omega$ such that (7) does not hold proceeds as follows: Fix $\omega_1 = \overline{\omega}_1$. Define $\mathcal{F}(\omega_0) \triangleq K_{\overline{\omega}_1} - K_{\omega_0,\overline{\omega}_1}$. It can be verified that there always exists a $\overline{\omega}_1 \in \Theta$ such that $\mathcal{F}(\omega_0)$ is a continuous function with *non-zero* derivative at $\omega_0 = \overline{\omega}_1$, and such that $\overline{\omega}_1$ is not a boundary point of $\Theta$. Since $\mathcal{F}(\omega_0 = \overline{\omega}_1) = 0$, there exists an $\epsilon$, $|\epsilon| > 0$ and arbitrarily small, such that if we fix $\omega_0 = \overline{\omega_0} = \overline{\omega}_1 + \epsilon$, we get $\mathcal{F}(\overline{\omega}_0) = \mathcal{F}(\overline{\omega}_1 + \epsilon) > 0$. Thus, there always exists $\omega_0, \omega_1 \in \Theta$ such that (7) is violated.

*Commentary on R1.2.* It can be verified that $K_{\omega_1} = F(\omega_1, \omega_1, \theta, \theta)$ and $K_{\omega_0,\omega_1} = F(\omega_0, \omega_1, \theta, \theta)$. Substituting into (7), the proofs of the necessity and sufficiency of $R = R^*$ follow from algebraic manipulation. An intuitive explanation for this result follows. Suppose $R = R^*$. Note that the right-hand side of (7) corresponds to a fictional hypothesis test; the same hypothesis test as in (3), but with an *effective* ratio of variances given by $\overline{R} = \frac{\omega_1}{\omega_0}\ R^*$. But, by definition, the test (3) is most difficult when $R = R^*$. Thus, the test represented by the right hand side of (7) can only be less difficult for any value of $\omega_0, \omega_1 \in \Omega$ $(\omega_0 \ne \omega_1)$, and the condition of the theorem is satisfied. On the other hand, if $R \ne R^*$, then there exists $\omega_0, \omega_1 \in \Omega$ that can violate the condition in (7), e.g., choose $\omega_0, \omega_1$ such that $\overline{R} = R^*$. Therefore, an adaptive test does not exist.

*Commentary on R1.3.* This result follows from R1. In componentwise notation, let $\omega_j = (\gamma_j, \theta_j)$. We show that there exists $\omega_0, \omega_1 \in \Omega$ such that (7) is violated. Fix $\omega_j = (\overline{\gamma}, \overline{\theta}_j)$, for some $\overline{\gamma} \in \Gamma$ and $\overline{\theta}_0, \overline{\theta}_1 \in \Theta$. By the result of R1, there always exists $\overline{\theta}_0, \overline{\theta}_1 \in \Theta$ such that (7) is violated. Thus, $\exists\omega_0, \omega_1 \in \Gamma \times \Theta$ such that (7) is violated.

Results R1.1-R1.3 hold for *any* choice of the intervals $\Gamma$ and $\Theta$ that have positive length. Thus, even as we shrink the parameter space of these unknowns, reflecting an increased a priori knowledge of the unknown parameters, adaptive tests do not exist (except for the special case given in R1.2). Adaptive tests exist only in the limit that the parameter space shrinks to zero, i.e., the known case.

## 5. ERROR EXPONENT FOR UNKNOWN ENEMY TARGET

We show that under a change of assumptions, adaptive tests exist for all cases of interest. As a generalization of the model presented in Section 2, let $(\gamma_i, \theta_i)$ denote the location of the target when $\mathcal{H}_i$ is true, where $\gamma_i \in \Gamma$ and $\theta_i \in \Theta$. It is often reasonable to assume that the location of the target under $\mathcal{H}_0$ is known at the fusion center, if present, but that the location of the target under $\mathcal{H}_1$ is unknown. For example, suppose that $\mathcal{H}_0$ corresponds to a friendly target and $\mathcal{H}_1$ to an enemy target in a military application. Below, we assume that the friendly location $(\gamma_0, \theta_0)$ is known at the fusion center (if it is present), whereas the enemy location $(\gamma_1, \theta_1)$ is either unknown or partially known (if present). In parallel with the development of Section 4, let $\Omega$ denote the parameter space of the unknown parameter(s). We consider three cases: (1) $\Omega = \Theta$, corresponding to $\gamma_1$ known, $\theta_1$ unknown, (2) $\Omega = \Gamma$, corresponding to $\gamma_1$ unknown, $\theta_1$ known, and (3) $\Omega = \Gamma \times \Theta$, corresponding to $\gamma_1$ unknown, $\theta_1$ unknown. An upper bound on performance is provided by the case where $(\gamma_1, \theta_1)$ is perfectly known. A derivation of (2) with $p_j(s_1^N|\gamma, \theta)$ replaced by $p_j(s_1^N|\gamma_j, \theta_j), j \in \{0, 1\}$, reveals that

$$K = F(\gamma_0, \gamma_1, \theta_0, \theta_1).$$

We are interested to see if there exists a test that can achieve this error exponent. We can prove that:

**Result 2 (R2)**. Consider the hypothesis testing problem of (3) and (4) with the generalization that the target distance and angle are hypothesis dependent, i.e., $(\gamma_i, \theta_i)$ denotes the target location under $\mathcal{H}_i$. Let $(\gamma_0, \theta_0)$ be known, and let $\Omega$ be the parameter space of the nuisance parameter. We have the following result:

**R2.1**. If $\Omega = \Theta$, an adaptive test exists,

**R2.2**. If $\Omega = \Gamma$, an adaptive test exists,

**R2.3**. If $\Omega = \Gamma \times \Theta$, an adaptive test exists.

**Proof.** It is sufficient to show the result for $\Omega = \Gamma \times \Theta$. Note that the nuisance parameter is present only under $\mathcal{H}_1$. It follows that $K_{\omega_1} = K_{\omega_0,\omega_1}$, and so condition (7) is satisfied $\forall \omega_0, \omega_1 \in \Omega$.

*Commentary.* Comparing R1 and R2, we see that while adaptive tests do not exist when the target's location is partially unknown under both hypothesis (with the exception one case, stated in R1.2), they *do* exist when the target location is fully known under $\mathcal{H}_0$, for all degrees of partial knowledge under $\mathcal{H}_1$. Suppose that location knowledge were reversed, so that target location was fully known under $\mathcal{H}_1$, but only partially known under $\mathcal{H}_0$. It can be verified that adaptive tests exist only in rare cases. Again, suggesting that existence of adaptive tests is sensitive to the modeling assumptions.

## 6. SUMMARY AND FUTURE WORK

We have studied the Neyman-Pearson miss probability error exponent for target-class detection in a sensor network. When the target location ($\gamma$, $\theta$) is known and common to both hypothesis, we proved several properties of the error exponent (see P1-P6). When at least one component of ($\gamma$, $\theta$) is unknown under both hypothesis, we proved that an adaptive test exists in one case of interest (see R1.1-R1.3). Generalizing to the scenario where the target location may be different under each hypothesis, i.e., given by $(\gamma_i, \theta_i)$ under $\mathcal{H}_i$, we considered the case where $(\gamma_0, \theta_0)$ is known perfectly, but where $(\gamma_1, \theta_1)$ is only partially known. We proved that adaptive tests exist for any level of partial knowledge of $(\gamma_1, \theta_1)$ (see R2.1-R2.3).

As future research, we plan to address the existence of adaptive tests when the target location is not fully known in a more general framework than presented here. We also plan to study the exact form and implementation of such tests.

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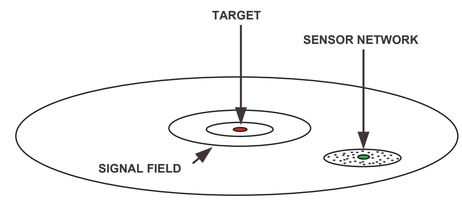


**Fig. 1**. Target class detection. The target belongs to one of two classes and produces a stochastic signal field with class-dependent power and correlation structure. Sensors collect $N$ samples of the signal field with the objective of identifying the target's class.

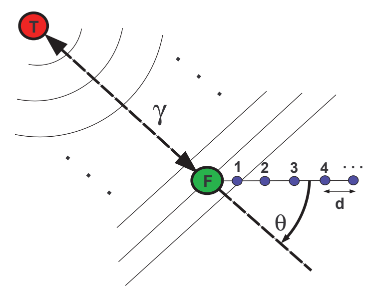


**Fig. 2**. System model. Each sensor $(1, \ldots, N)$ takes a sample of the signal field. Samples are collected on a straight line with spacing $d$ and delivered to the fusion center $F$. The sensor network is in the far-field of the target $T$ at a relative distance $\gamma$, and contours of the signal intersect the collection line at an angle $\pi/2 - \theta$.

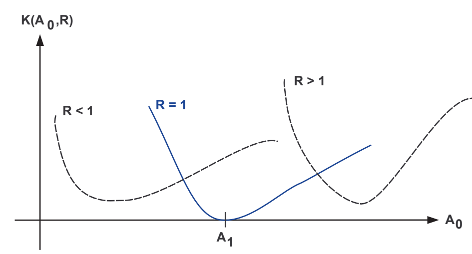


**Fig. 3**. The error exponent $K(A_0, R)$ versus $A_0$ for fixed $A_1$ and $R = 1$, $R > 1$, and $R < 1$. Note that $f_0(R) = \arg\min_{A_0} K(A_0, R)$ is increasing in $R$, and $f_0(1)$=$A_1$.

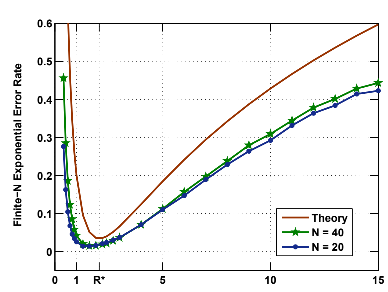


**Fig. 4**. The finite-$N$ exponential error rate for the LRT versus the number of signal samples collected $N$. The parameters are $A_0 = 0.5$, $A_1 = 0.2$, $d = 1$, $\theta = 0$, $\alpha = 0.01$, $\sigma_1^2 = R$, and $\sigma_0^2 = 1$.