COST-BASED MONTE CARLO SAMPLING APPROACHES FOR SENSOR SELF-LOCALIZATION UNDER BEACON POSITION UNCERTAINTY

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ABSTRACT

Sensor localization methods based on Monte Carlo sampling approximate the sensor position distributions by a weighted set of samples. These approaches traditionally require complete knowledge of the probabilistic distributions of the uncertainties in the sensor system. In this paper, we propose alternative sampling-based methods which do not require complete knowledge of the probabilistic distributions. The sensor position distributions are represented by a set of samples and costs which are described by spatial parametric regions. Few parameters are needed to characterize these regions, and therefore the amount of information to be transmitted to the rest of the sensors to self-localize is simplified. Computer simulations show that the proposed methods are more robust and less computationally intensive than standard sampling approaches.

Index Terms— Multisensor systems, Monte Carlo methods, self-localization.

1. INTRODUCTION

In many sensor networking applications like target tracking the sensed signals are functions of the distance between the target and the sensor. These signals are then processed to obtain the dynamics of the target. This processing requires precise information of the sensors' locations. In other scenarios, sensors are randomly deployed or subject to mobility due to climatic conditions. To obtain location information, advanced positioning technologies like GPS can be integrated into the sensor system. However, this infrastructure increases the overall cost of the sensors and prevents from their dense deployment and ubiquitous use [1]. To avoid such situations, it is desirable that sensors localize themselves. The process by which sensors collaborate to obtain information about their positions is known as self-localization (SL).

Sensor SL is commonly addressed using beacon nodes also known as anchor nodes, leader nodes or access points. We use the term beacon nodes when referring to sensors which have some initial location information about their positions. Maximum likelihood-based methods for SL in a centralized sensor systems have been proposed in [1, 2, 3]. Distributed Bayesian methods for SL have been also presented in [4, 5]. There, each sensor node computes its 'belief' about its location and broadcasts its marginal position distribution to its neighbors. An advantage of the Bayesian techniques is that they provide a principled way of dealing with location uncertainty and multi-sensor fusion [6]. A more comprehensive survey of SL in sensor networks is provided in [1, 7] and the references therein.

In this paper we address the problem of distributed SL in sensor networks using Monte Carlo-based sampling methods with beacon position uncertainty and the absence of complete knowledge of the distribution of the sensor measurement noise. Under these constraints we attempt to design algorithms that can deal with position uncertainty and fuse data from multiple sensors. In brief, the algorithm initiates with beacon nodes broadcasting their location information. Sensor nodes with unknown positions utilize this beacon location information and the characteristics of the received signals to resolve their locations. A sample-based representation of the sensor location distribution is obtained and each sample is associated with a cost, which reflects the "importance" of the sample.

The organization of the paper is as follows. In Section 2, we state the sensor SL problem; in Sections 3 and 4 we describe probabilistic and novel cost-based sampling algorithms for SL, respectively. We provide some simulation results in Section 5 and conclude the paper with Section 6.

2. PROBLEM STATEMENT

We motivate the problem of sensor SL by Fig. 1. There, beacon nodes 1, 2 and 3 broadcast their position information using known pilot signals or reference signals. Sensor 1 gathers this information and uses the characteristics of the received signal to resolve its sensor position. Sensor 2, being far away from beacon 2, does not receive any of its prior

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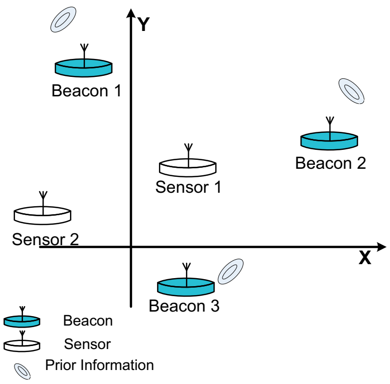

Fig. 1: Sensor network

information and therefore is unable to resolve its position uniquely. In the next iteration, sensor 1 also transmits its position information which is utilized by sensor 2 along with the signals from beacons 1 and 3 to resolve its position information.

The problem of distributed sensor SL is mathematically expressed as follows. Beacon nodes broadcast the prior information about their positions $\boldsymbol{\ell}_b \in \mathbb{R}^2, b = 1, 2, \cdots N_b$. We represent this prior information as $p(\boldsymbol{\ell}_b)$, which represents a standard probabilistic distribution or a parametric spatial distribution. The signal received by the sensor can be modeled as

$$y_{s,b} \quad = \quad f(\boldsymbol{\ell}_s, \boldsymbol{\ell}_b) + v_{s,b}, \qquad (1)$$

where $y_{s,b}$ is the received signal characteristic by sensor s from beacon b; $\boldsymbol{\ell}_s, \boldsymbol{\ell}_b \in \mathbb{R}^2$ are the positions of the nodes s and b in the two dimensional Cartesian coordinate system ($\boldsymbol{\ell}_s = [l_{s,x}, l_{s,y}]^\top$ and $\boldsymbol{\ell}_b = [l_{b,x}, l_{b,y}]^\top$); and $v_{s,b}$ is a noise process. Commonly used signal characteristics for localization are (a) received signal strength (RSS), (b) time of arrival (TOA), (c) time difference of arrival (TDOA), and (d) angle of arrival (AOA) of the signal received at the sensor nodes [1, 7]. The corresponding form of function $f(\cdot)$ of these modalities is shown in Table 1. There Ψ is the power received at a known reference distance; α is the path-loss attenuation parameter; c is the velocity of the transmitted signal; and $|\cdot|$ denotes norm of a vector. When the distribution of the measurement noise process is known, can we obtain the sensors' location posterior distribution using the measurements and beacon prior location information and in absence of such knowledge, can we still obtain the sensors' location distributions? We attempt to answer these questions in the following sections.

3. SENSOR SL UNDER KNOWN NOISE PROBABILISTIC DISTRIBUTIONS

Here we briefly review the algorithm presented in [5]. Consider the scenario where a sensor s receives signals from three

$y_{s,b}$	$f(\boldsymbol{\ell}_s, \boldsymbol{\ell}_b)$
RSS	$\Psi - 10\alpha \log_{10}(\lvert\boldsymbol{\ell}_s - \boldsymbol{\ell}_b\rvert)$
TDOA	$\frac{(\lvert\boldsymbol{\ell}_s - \boldsymbol{\ell}_b\rvert)}{c}$
AOA	$\arctan\left(\frac{l_{s,y} - l_{b,y}}{l_{s,x} - l_{b,x}}\right)$

Table 1: Sensing modalities

beacon nodes[1]. The beacons transmit their location prior information, $p(\boldsymbol{\ell}_1), p(\boldsymbol{\ell}_2)$ and $p(\boldsymbol{\ell}_3)$. The joint posterior density of the beacons and sensor's locations, $\boldsymbol{\ell} = [\boldsymbol{\ell}_s, \boldsymbol{\ell}_1, \boldsymbol{\ell}_2, \boldsymbol{\ell}_3]^\top$, can be written as

$$p(\boldsymbol{\ell} \mid y_{s,1}, y_{s,2}, y_{s,3}) \propto p(\boldsymbol{\ell}_s) \prod_{b=1}^{3} p(y_{s,b} \mid \boldsymbol{\ell}_s, \boldsymbol{\ell}_b) p(\boldsymbol{\ell}_b), \qquad (2)$$

where we have assumed independence among the prior distributions. As it can be seen from Table 1, the received reference signals are nonlinear functions of the beacon and sensor locations. In [5], a Monte Carlo approximation of the posterior distribution was obtained as

$$p(\boldsymbol{\ell}) \approx \sum_{m=1}^{M} \omega^{(m)} \delta\left(\boldsymbol{\ell}_s - \boldsymbol{\ell}_s^{(m)}\right) \prod_{b=1}^{3} \delta\left(\boldsymbol{\ell}_b - \boldsymbol{\ell}_b^{(m)}\right), \qquad (3)$$

$$\text{where } \omega^{(m)} \propto \frac{p(\boldsymbol{\ell}_s^{(m)})}{q(\boldsymbol{\ell}_s^{(m)})} \prod_{b=1}^{3} \frac{p(y_{s,b} \mid \boldsymbol{\ell}_s^{(m)}, \boldsymbol{\ell}_b^{(m)}) p(\boldsymbol{\ell}_b^{(m)})}{q(\boldsymbol{\ell}_b^{(m)})}. \qquad (4)$$

A set of M samples representing the beacons and sensors' locations are drawn from proposal functions, $q(\boldsymbol{\ell}_b), b = 1, 2,$ and 3 and $q(\boldsymbol{\ell}_s)$ which do not require knowledge of the sensor measurement noise process [5]. Using the drawn samples marginal distributions of the sensor location, $\boldsymbol{\ell}_s$, can be easily obtained from (3).

4. SENSOR SL UNDER UNKNOWN NOISE PROBABILISTIC DISTRIBUTIONS

In the above sampling algorithm, computation of the weights in (4) requires a complete knowledge of the probabilistic distributions of the measurement noise. In many scenarios such probabilistic knowledge may not be available. Therefore, we propose an alternative Monte Carlo sampling algorithm for obtaining sensor location distributions. These algorithms only require that the noise terms in (1) be zero mean distributed .

We consider the problem stated in Section 2, where a particular sensor s receives location information from beacon nodes 1, 2, and 3 and attempts to obtain its location information. Under assumption of zero mean noise, a typical cost

[1] We simplified the problem to one sensor resolving its position from three beacons' information. The extension of the problem for a complete network is straightforward.

$$\rho_a, \rho_b = \tau \left(\frac{\tilde{\sigma}_{\boldsymbol{\ell}_s,xx} + \tilde{\sigma}_{\boldsymbol{\ell}_s,yy} \pm \sqrt{(\tilde{\sigma}_{\boldsymbol{\ell}_s,xx} + \tilde{\sigma}_{\boldsymbol{\ell}_s,yy})^2 + 4 \times \tilde{\sigma}^2_{\boldsymbol{\ell}_s,xy}}}{2} \right)^{1/2} \quad \text{and} \quad \phi = \frac{1}{2} \arctan \left(\frac{2\tilde{\sigma}_{\boldsymbol{\ell}_s,xy}}{\tilde{\sigma}_{\boldsymbol{\ell}_s,yy} - \tilde{\sigma}_{\boldsymbol{\ell}_s,xx}} \right) \quad (5)$$

criterion for obtaining the sensor location given the measurements $y_{s,1}, y_{s,2}, y_{s,3}$ can be formulated as $y_{s,1}, y_{s,2}, y_{s,3}$ can be formulated as

$$\hat{\boldsymbol{\ell}}_s = \underset{\boldsymbol{\ell}_s}{\operatorname{argmin}} \left\{ \mathcal{C}(\boldsymbol{\ell}_s) = \sum_{b=1}^{3} ||y_{s,b} - f(\boldsymbol{\ell}_s, \boldsymbol{\ell}_b)||^2 \right\}. \quad (6)$$

Taking as a starting point this optimization formulation we propose a sampling procedure for SL with the following steps:

• *Generation of samples:* Beacons' location samples are drawn from the prior distributions and the sensor location samples are drawn from a proposal function as in [5]. A set $\{\boldsymbol{\ell}_s^{(m)}, \boldsymbol{\ell}_1^{(m)}, \boldsymbol{\ell}_2^{(m)}, \boldsymbol{\ell}_3^{(m)}\}_{m=1}^M$ of M samples is thus obtained.

• *Computation of costs*: Using the signal measurements, we obtain the residuals $\epsilon_{s,b}^{(m)} = y_{s,b} - f(\boldsymbol{\ell}_s^{(m)}, \boldsymbol{\ell}_b^{(m)})$. Each sample in the set is associated with a cost using these residuals. The costs, $\mathcal{C}^{(m)}$, are computed using user-defined cost functions, $\varrho(\cdot)$, as

$$\mathcal{C}^{(m)} = \sum_{b=1}^{3} \varrho(\epsilon_{s,b}^{(m)}), \quad (7)$$

such that samples with small residuals have small costs and samples with large residuals have large costs. We utilized the following cost functions in our experiments:

1. L2 Cost Function: $\varrho(\epsilon) = |\epsilon|^2$
2. L1 Cost Function: $\varrho(\epsilon) = |\epsilon|$
3. "Fair" Cost Function: $\varrho(\epsilon) = 2k^2 \left[\frac{|\epsilon|}{k} - \log(1 + \frac{|\epsilon|}{k}) \right]$ with $k = 1.3998$ [8].

We form a pseudo-probability mass function (pmf) with random measure $\boldsymbol{\Omega}^{(m)} \equiv \{\tilde{\pi}(\boldsymbol{\ell}^{(m)}), \boldsymbol{\ell}_s^{(m)}, \boldsymbol{\ell}_1^{(m)}, \boldsymbol{\ell}_2^{(m)}, \boldsymbol{\ell}_3^{(m)}\}$ where

$$\tilde{\pi}(\boldsymbol{\ell}^{(m)}) \propto \frac{1}{\mathcal{C}^{(m)}} \quad \text{with} \quad \sum_m \tilde{\pi}(\boldsymbol{\ell}^{(m)}) = 1. \quad (8)$$

• *Calculation of the spatial sensor location distributions*: Using the measure $\{\boldsymbol{\Omega}^{(m)}\}_{m=1}^M$, the sensor location distribution can be described by a simple spatial distribution, $\hat{\mathcal{R}}$, e.g., a square, circular or elliptical region. One way of obtaining the parameters of these spatial distributions is through the mean and covariance matrix of the sensor location distribution which are computed as

$$\begin{aligned} \tilde{\boldsymbol{\mu}}_{\boldsymbol{\ell}_s} &\approx \sum_{m=1}^{M} \tilde{\pi}(\boldsymbol{\ell}^{(m)}) \boldsymbol{\ell}_s^{(m)} \\ \tilde{\boldsymbol{\Sigma}}_{\boldsymbol{\ell}_s} &\approx \sum_{m=1}^{M} \tilde{\pi}(\boldsymbol{\ell}^{(m)}) \left(\boldsymbol{\ell}_s^{(m)} - \tilde{\boldsymbol{\mu}}_{\boldsymbol{\ell}_s} \right) \left(\boldsymbol{\ell}_s^{(m)} - \tilde{\boldsymbol{\mu}}_{\boldsymbol{\ell}_s} \right)^\top \\ &= \begin{pmatrix} \tilde{\sigma}_{\boldsymbol{\ell}_s,xx} & \tilde{\sigma}_{\boldsymbol{\ell}_s,xy} \\ \tilde{\sigma}_{\boldsymbol{\ell}_s,yx} & \tilde{\sigma}_{\boldsymbol{\ell}_s,yy} \end{pmatrix}. \end{aligned} \quad (9)$$

Once these parameters are computed, the spatial regions are defined as follows:

○ Square region: It is completely characterized by its center, $\boldsymbol{\mu}_r$, and the length of a side, ρ_r. We represent this region, $\hat{\mathcal{R}}$, as $\mathcal{S}q(\boldsymbol{\mu}_r, \rho_r)$ with $\boldsymbol{\mu}_r = \tilde{\boldsymbol{\mu}}_{\boldsymbol{\ell}_s}$ and $\rho_r = \tau \max(\sqrt{2\tilde{\sigma}_{\boldsymbol{\ell}_s,xx}}, \sqrt{2\tilde{\sigma}_{\boldsymbol{\ell}_s,yy}})$, where τ is a scaling factor. We have chosen $\tau = 2$ in all our experiments.

○ Circular region: It is specified by its center, $\boldsymbol{\mu}_r = \tilde{\boldsymbol{\mu}}_{\boldsymbol{\ell}_s}$, and radius, ρ_r. We represent the region as $\hat{\mathcal{R}} = \mathcal{C}i(\boldsymbol{\mu}_r, \rho_r)$ with $\boldsymbol{\mu}_r = \tilde{\boldsymbol{\mu}}_{\boldsymbol{\ell}_s}$ and $\rho_r = \tau \max(\sqrt{2\tilde{\sigma}_{\boldsymbol{\ell}_s,xx}}, \sqrt{2\tilde{\sigma}_{\boldsymbol{\ell}_s,yy}})$.

○ Elliptical region: It is described by its center, $\boldsymbol{\mu}_r = \tilde{\boldsymbol{\mu}}_{\boldsymbol{\ell}_s}$, the lengths of its major and minor axis, ρ_a and ρ_b, and the angle of inclination of the major axis with respect to the horizontal direction, ϕ. The region $\hat{\mathcal{R}}$ can be represented as $\mathcal{E}l(\boldsymbol{\mu}_r, \rho_a, \rho_b, \phi)$ and its parameters are obtained using (5).

Other kinds of inclined spatial regions can be similarly constructed. Also, our method can be implemented with any computable cost function and not the conventional 'L2' cost function only.

Once a sensor resolves its position, it broadcasts the parameters describing its location. In the next iteration other sensors will be able to localize themselves and the procedure continues repeating.

5. SIMULATIONS AND DISCUSSION

We considered a network as shown in Fig. 2, with 48 randomly distributed sensors and 16 beacons. The prior location distributions of the beacon nodes were modeled using Gaussian distributions $\mathcal{N}(\boldsymbol{\mu}_{l_b}, \sigma^2 I)$ with $\boldsymbol{\mu}_{l_b} = [l_{b,x} + \theta, l_{b,y} + \theta]^\top$ where θ represents a bias in the beacon location information. In all our simulations we assumed $\theta = 1$ and

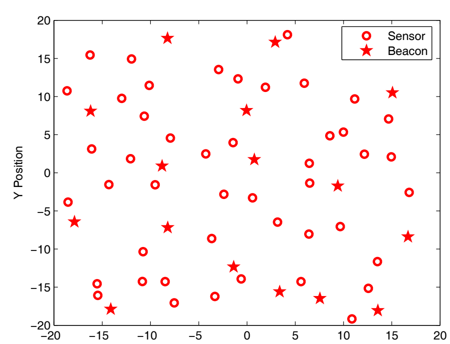

Fig. 2: Sensor network

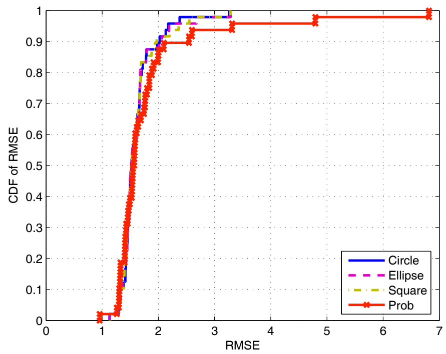

Fig. 3: Performance of the methods in terms of the CDF of the RMSEs

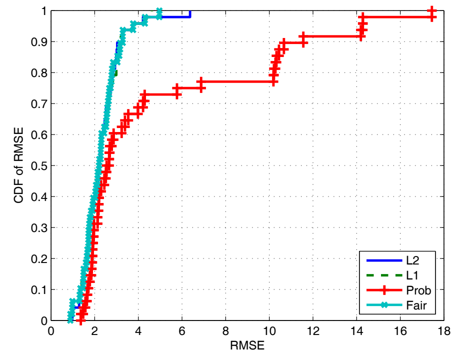

Fig. 4: Robustness of the proposed methods in terms of the CDF of the RMSEs

$\sigma^2 = 0.5$. In our experiments we considered the RSS signal model with $\Psi = -50$dB and $\alpha = 2.5$.

We performed two sets of simulations. In the first simulation experiment we modeled the measurement noise across all the sensors as a Gaussian process with $\mu_v = 0$ and $\sigma_v = 1$. The algorithms ran for 5 iterations and the root mean square errors (RMSEs) of all the sensors were computed for 25 such runs. Fig. 3 shows the cumulative distribution function (CDF) of the RMSEs of all the sensors with the probabilistic algorithm (labeled as 'Prob') and the cost-based sampling algorithm with the L2 Cost Function and different spatial distributions (labeled as 'Circle', 'Ellipse', 'Square'). In all the simulations the total number of samples drawn for estimating the costs and weights was M=2000. We can see that all the methods had similar performance.

We conducted another set of simulations to study the robustness of the algorithms when we do not have information about the distributions. The probabilistic algorithm assumed wrong measurement noise distribution $\hat{p}(v) = \mathcal{N}(0, 0.2^2)$ while the true distribution of the noise was $p(v) = 0.8\mathcal{N}(0,1) + 0.2\mathcal{N}(3, 0.2^2)$. In Fig. 4, the CDF of the RMSEs averaged over all the sensors is shown for all the considered algorithms. It can be seen that the performance of all the cost-based methods is similar and 90% of the sensors' locations are within 3m. With the probabilistic algorithm with wrong noise statistics 90% of the sensors' locations were within 12m. Clearly this plot shows the robustness of the proposed algorithms and the sensitivity of the probabilistic algorithm to the knowledge of the distributions of the noises.

6. CONCLUSIONS

In this paper we have introduced simple sampling-based algorithms for determining sensor location distributions. Simulation results show the robustness of the proposed algorithms to outliers and modelling errors. The algorithms only require the mean of the measurement noise to be zero. Future work involves the removal of this assumption and design of completely blind algorithms for sensor self-localization.

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