LOW-COMPLEXITY METHOD FOR TRANSMIT BEAMFORMING IN MIMO RADARS

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ABSTRACT

MIMO radar is a new concept in which radar employs multiple waveforms to improve its performance. Previously, a transmit beamforming method was proposed for MIMO radars. This method allows optimization of the beampattern by altering the cross-correlation matrix of the transmitted waveforms. The optimization is based on minimization of a cost function, but the use of numerical methods in the algorithm leads to high computational complexity. Here we propose a new cost function for the beampattern optimization. For linear arrays and typical beampatterns, this cost function can be evaluated in closed form, thus reducing the computational complexity considerably. Simulation examples demonstrate that the proposed cost function also leads to faster convergence and lower approximation error.

Index Terms— Array signal processing, radar, MIMO systems

1. INTRODUCTION

MIMO communication systems have been studied actively for some time, and it is known well that the diversity in these systems can be exploited to increase capacity or to improve reliability. Recently, several techniques extending the MIMO concept to radars have been proposed[1, 2, 3, 4, 5, 6]. There are several different approaches to the MIMO radar, but they all aim at exploiting waveform diversity to enhance the radar. MIMO radar could potentially improve the performance of radars as dramatically as MIMO methods can improve the rate and reliability of communication systems.

What sets MIMO radars apart from ordinary array radars is the use of signals. It was shown in [1] that by transmitting independent signals from separate transmitters, the available degrees of freedom can be increased and the angular resolution improved. Transmitting orthogonal waveforms from different antennas enabling receiver array aperture synthesis is proposed in [2]. These approaches assume that all the transmitted signals scatter from the target identically.

A totally different view is taken in [3], which introduced a concept called statistical MIMO radar. In this concept, the idea is to make the signals scatter from a target independently in order to overcome the scintillations in the target RCS, which normally degrade the performance of radar. It is claimed that this leads to improved accuracy in DOA estimation[3] and, in some cases, target detection[4].

Yet another approach is taken in [5] in which a beamforming method using partial signal correlation was introduced. Beamforming is useful in array radars, which need to focus the array to maximize the gain in the direction of a target. The beamforming method proposed in [5] allows much higher control over the beampattern than ordinary beamforming. For example, this method can be applied to do beamspoiling, which is useful in reducing backscatter clutter and scanning time[6]. Transmit beamforming using partial signal correlation offers great flexibility in synthesizing the beampattern, and it fills the gap between the traditional phased array techniques and MIMO radars which use completely uncorrelated signals. The ability to form a wide focus is useful especially when transmitting fully correlated waveforms would result in a beam that is too narrow.

The beampattern is controlled by changing the correlation matrix of the transmitted signals so that a cost function measuring the difference between the desired and the actual beampattern is minimized. However, the computational complexity of the algorithm is very high to be used in real-time applications because numerical integration has to be used in the process of minimizing the error criterion given in [5]. In this paper, we propose a new cost function that can be integrated in closed form. Consequently, computationally expensive numerical integration is no longer needed. The proposed cost function also leads to faster convergence and lower approximation error of the desired beampattern. This makes the transmit beamforming in MIMO radars feasible in practice.

This paper is organized as follows: The transmit beampattern synthesis method originally presented in [5] is reviewed in Section 2. The new cost function is introduced in Section 3, and numerical examples are given in Section 4. Final conclusions are made in Section 5.

2. BEAMPATTERN OPTIMIZATION

The transmit beampattern synthesis considered here is based on modifying the correlation matrix of the transmitted signals. It was shown in [5] that if the far-field and narrowband assumptions hold, the power per steradian radiated by a linear array of N elements is

$$P(\theta,\phi) = \frac{1}{4\pi}\mathbf{v}^H(\theta)\mathbf{R}\mathbf{v}(\theta), \tag{1}$$

where $\mathbf{v}(\theta)$ is an $N \times 1$ steering vector and $\mathbf{R}$ is the $N \times N$ cross-correlation matrix of the signals being transmitted from the array elements. The steering vector is defined as

$$\mathbf{v}(\theta) = \left[\exp(-j2\pi\tfrac{z_1}{\lambda}\sin\theta) \quad \ldots \quad \exp(-j2\pi\tfrac{z_N}{\lambda}\sin\theta)\right]^T, \tag{2}$$

where z_i's are the positions of the elements relative to a reference point in the array. The problem is how to find signals with such an $\mathbf{R}$ that the beampattern $P(\theta,\phi)$ would have a desired shape. Because the steering vector of a linear array depends only on θ, the same is true for the beampattern P.

If standard beamforming is applied at the transmitter, each element of the array transmits the signal with such a phase-shift that

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the waves interfere constructively in a desired direction. In this case, $\mathbf{R} = \mathbf{v}(\theta_0)\mathbf{v}^H(\theta_0)$, where θ_0 is the direction of interest. Therefore,

$$P(\theta_0) = \frac{1}{4\pi}\|\mathbf{v}(\theta_0)\|^4 = \frac{N^2}{4\pi}, \tag{3}$$

so there is N-fold beamforming gain in power, but this gain is smaller for all $\theta \neq \theta_0$. On the other hand, if MIMO radar transmitting independent signals is used, $\mathbf{R} = \mathbf{I}$ and $P(\theta) = \frac{N}{4\pi}$ for all θ. These methods both lack the flexibility in controlling the beampattern. However, it is possible to select the waveforms and consequently the cross-correlation matrix such that a high quality approximation of a desired beampattern is obtained.

In order to get a beampattern of a desired shape, an appropriate cross-correlation matrix $\mathbf{R}$ has to be found. This can be done by minimizing the difference between the desired and the actual beampatterns according to some error criterion that depends on $\mathbf{R}$. However, $\mathbf{R}$ cannot be chosen freely, as a correlation matrix has to be positive-semidefinite. One way to achieve positive-semidefiniteness is to construct $\mathbf{R}$ in a product form [5]

$$\mathbf{R} = \mathbf{L}\mathbf{L}^H, \tag{4}$$

where $\mathbf{L}$ is a complex-valued $N \times N$ Hermitian square root of $\mathbf{R}$. $\mathbf{L}$ can also be a Cholesky factor, but it does not necessarily have to be triangular.

The available power imposes another constraint on $\mathbf{R}$. If all antenna elements are assumed to transmit at the same maximum power, all the diagonal elements of $\mathbf{R}$ must be equal. The diagonal elements can be set to unity without loss of generality. The total power of the transmitter would be N in this case, where N is the number of the antenna elements.

Requiring the diagonal elements of $\mathbf{R}$ to be equal to one means that if $\mathbf{L}$ consists of row vectors $\mathbf{l}_n$, $n = 1 \dots N$, then $\|\mathbf{l}_n\|^2 = 1$. In other words, the vectors $\mathbf{l}_n$ must be on the surface of an N-dimensional complex hypersphere[5].

Due to these constraints on $\mathbf{R}$, it is not usually possible to achieve the desired beampattern exactly. Nevertheless, $\mathbf{R}$ can be adjusted to get an accurate approximation of the desired beampattern. The quality of the approximation depends on several factors, such as the desired beampattern, the properties of the array used, as well as the optimization method and cost function that are used for adjusting $\mathbf{R}$.

The beampattern can be optimized by choosing $\mathbf{L}$ to minimize a cost function C that measures the difference between the desired and the actual beampattern. The optimization method proposed in [5] is based on an algorithm in [7]. This algorithm works by evaluating the gradient of the cost function, $\mathbf{G}(\mathbf{L}) = \nabla C(\mathbf{L})$, and then moving to the direction opposite to the gradient. The search starts from an initial guess $\mathbf{L}^{(0)}$, which is typically an identity matrix.

Since the rows of $\mathbf{L}^{(i)}$ must be on the unit hypersphere for any iteration i, a rotational update is used for the row vectors of $\mathbf{L}^{(i)}$. For the n^{th} row vector this update is

$$\mathbf{l}_n^{(i+1)}(a) = \cos(a\psi_n^{(i)})\mathbf{l}_n^{(i)} + \sin(a\psi_n^{(i)})\mathbf{p}_n^{(i)}/\|\mathbf{p}_n^{(i)}\|, \tag{5}$$

where a is a scalar between zero and one, ψ_n is an upper bound for the rotation, and $\mathbf{p}_n$ is a projection of the n^{th} row of the gradient matrix $\mathbf{G}(\mathbf{L}^{(i)})$ such that it is orthogonal to $\mathbf{l}_n$.

A line search is performed to find the value of a that minimizes $C(\mathbf{L}^{(i+1)}(a))$, and this value is used for fixing $\mathbf{L}^{(i+1)}$. Then the gradient is computed again and the procedure iterated until a sufficiently accurate approximation of the desired beampattern is obtained. The performance and the complexity of this algorithm are largely influenced by the choice of the cost function, which is discussed in the next section.

3. COST FUNCTIONS

The cost function used in the optimization algorithm has to depend on the signal correlation matrix and measure the difference between the achieved beampattern and the desired beampattern $P_d(\theta)$. The cost function used in [5] is

$$C_1(\mathbf{L}) = \int (\sqrt{P_d(\theta)} - \|\mathbf{v}^H(\theta)\mathbf{L}\|)^2 d\theta, \tag{6}$$

where the integration is done over a region of interest.

Here an alternative cost function of the form

$$C_2(\mathbf{L}) = \int \left(P_d(\theta) - \|\mathbf{v}^H(\theta)\mathbf{L}\|^2\right)^2 \cos\theta d\theta, \tag{7}$$

is proposed. Multiplying by $\cos\theta$ can be seen as weighting by the solid angle the array sees at the angle θ, which is

$$\int_0^{2\pi} \cos\theta d\theta d\phi = 2\pi\cos\theta d\theta \tag{8}$$

because the array steering vector is independent of ϕ. In the coordinate system used here, θ is elevation, not the polar angle.

A uniform linear array (ULA) with half-wavelength interelement spacing is considered in this paper. We note that

$$\|\mathbf{v}^H(\theta)\mathbf{L}\|^2 = \sum_{m=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{N} l_{mk} l_{nk}^* v_m^*(\theta) v_n(\theta), \tag{9}$$

so for an ULA with $v_n(\theta) = e^{-j\pi(n-1)\sin\theta}$, the cost function in (7) can be written as

$$C_2(\mathbf{L}) = \int \left(P_d(\theta) - \sum_{k,m,n=1}^{N} l_{mk} l_{nk}^* e^{j\pi(m-n)\sin\theta}\right)^2 \cos\theta d\theta. \tag{10}$$

It is clear that this integration can be done in closed form if P_d does not depend on θ, which means that $C_2(\mathbf{L})$ can be evaluated in closed form if the desired beampattern is piecewise constant. This reduces the computational complexity of the beampattern optimization algorithm significantly. Closed-form integration is possible because of the structure of the steering vector $\mathbf{v}(\theta)$. For arrays that are not linear, closed-form evaluation of the cost function C_2 is more difficult.

Evaluation of the integrand in C_1 involves the product $\mathbf{v}^H(\theta)\mathbf{L}$, which requires N^2 complex multiplications. The integrand has to be evaluated a number of times to approximate the integral, and typically, the number of evaluations needed is much higher than N.

Evaluating C_2 in closed form seems very complex, as it requires integrating $\|\mathbf{v}^H(\theta)\mathbf{L}\|^4 \cos\theta$ because of the square in the integrand. However, the costliest part of C_2 is computing $\mathbf{L}\mathbf{L}^H$, which requires $\frac{1}{2}N^2(N-1)$ complex multiplications since the result is hermitian with the diagonal elements equal to one. The complexity of the remaining computations is $\mathcal{O}(N^2)$ due to the structure of $\mathbf{R}$ and $\mathbf{v}(\theta)$. It follows from the definitions of $\mathbf{L}$ and $\mathbf{v}(\theta)$ that

$$\|\mathbf{v}^H(\theta)\mathbf{L}\|^2 = \mathrm{Tr}(\mathbf{R}) + 2\sum_{k=1}^{N-1}\mathrm{Re}\left(e^{-jk\pi\sin\theta}\sum_{i=1}^{N-k}(\mathbf{R})_{i,i+k}\right). \tag{11}$$

Similar terms can be collected the same way in case of $\|\mathbf{v}^H(\theta)\mathbf{L}\|^4$. The reduction in complexity brought about by the closed-form solution is amplified by the need to evaluate the cost function numerous times in the optimization algorithm.

The optimization algorithm requires evaluating the gradient of the cost function with respect to $\mathbf{L}$. However, the cost function considered here depends on $\mathbf{L}$ and its complex conjugate (transpose), so the cost function is not analytic. Nevertheless, a gradient can be found by using a generalized definition of differentiation[8].

It can be shown that the gradient is obtained when the cost function is differentiated with respect to $\mathbf{L}^*$. When differentiating a function with respect to a complex variable, the variable and its complex conjugate can be considered independent[8]. Applying this to (9), one sees that

$$\frac{\partial}{\partial l_{pq}^*} \|\mathbf{v}^H(\theta)\mathbf{L}\|^2 = \sum_{m=1}^{N} l_{mq} v_m^*(\theta) v_p(\theta), \tag{12}$$

and thus,

$$\frac{\partial}{\partial \mathbf{L}^*} \|\mathbf{v}^H(\theta)\mathbf{L}\|^2 = \mathbf{v}(\theta)\mathbf{v}^H(\theta)\mathbf{L}. \tag{13}$$

After switching the order of integration and differentiation and substituting (13), the gradient of the cost function (7) can be written as

$$\mathbf{G}(\mathbf{L}) = \int -2\left(P_d(\theta) - \|\mathbf{v}^H(\theta)\mathbf{L}\|^2\right)\mathbf{v}(\theta)\mathbf{v}^H(\theta)\mathbf{L}\cos\theta d\theta. \tag{14}$$

This integration can be done in closed form as well. Moreover, also $\frac{\partial}{\partial a}C_2(\mathbf{L}(a))$ and $\frac{\partial^2}{\partial a^2}C_2(\mathbf{L}(a))$ could be evaluated in closed form to facilitate the search for the minimum of $C_2(\mathbf{L}(a))$.

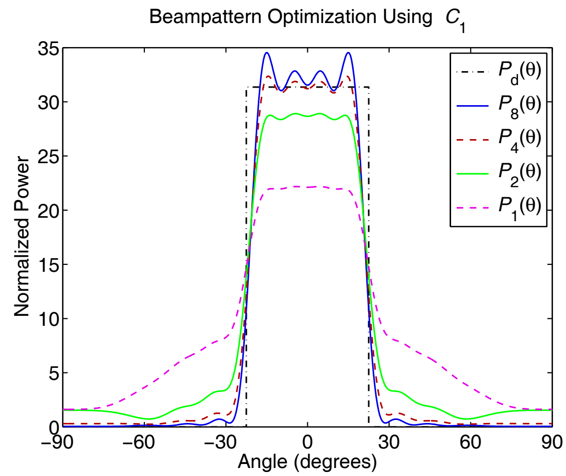
4. EXAMPLES

In this section, we show the results of optimizing the beampattern for 12-element ULA with half-wavelength interelement spacing. Two different desired beampatterns are considered to compare the cost functions.

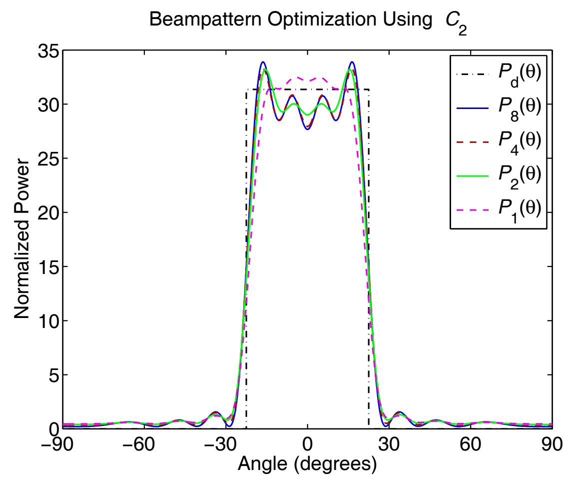
Standard beamforming use phase-shifted replicas of a single signal to control the beampattern. With standard beamforming methods, beampatterns with a narrow focus can be achieved within certain resolution limit that depends on the size of the array. Forming beampatterns with wide and flat areas is more difficult. Therefore, to get the benefit of multiple signals, the beampattern optimization method was tested with a wide focus region. Figure 1 displays the achieved beampatterns after several iterations in the case of 45° focus. The cost function (6) was used in Figure 1(a) and the proposed cost function (7) in Figure 1(b). The resulting beampatterns are quite similar in shape with the proposed cost function having slightly larger ripple but much faster convergence.

The results in Figure 2 show that the novel cost function C_2 works well even for beampatterns that are not symmetric about the broadside of the array ($\theta = 0°$). When cost function C_1 is used, the beampattern overshoots significantly and remains too focused, which can be seen in Fig. 2(a). The proposed cost function C_2 leads to good approximation of the desired beampattern, as shown by Fig. 2(b).

A more quantitative analysis of convergence is shown in Figure 3, which shows the square error of the approximated beampatterns as a function of the number of iterations. It can be seen that using C_2 leads to lower squared error and significantly faster convergence. The performance achieved by using cost function C_1 is particularly poor if the focus is not near the broadside of the array.



(a) Cost function C_1



(b) Cost function C_2

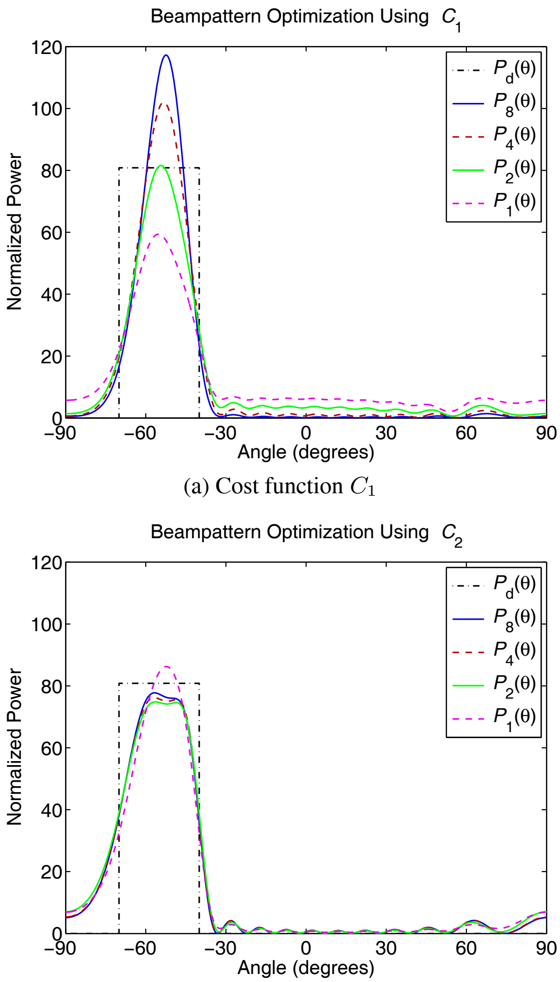
Fig. 1. Convergence of the beampattern in case of a centered 45° focus. P_i is the beampattern at i^{th} iteration and P_d is the desired beampattern. Beampatterns are similar in shape, but convergence is faster when C_2 is used.

5. CONCLUSIONS

Beamforming is used in radars to maximize the gain in the direction of the target. A transmit beamforming method exploiting partial signal correlation was proposed in [5]. However, this method suffers from very high computational complexity.

In this paper, we have proposed a new cost function for transmit beamforming. It was shown that the proposed cost function can be integrated in closed form when a linear array is used and the desired beampattern is piecewise constant. This decreases the computational complexity of the beampattern optimization algorithm considerably.

Numerical examples in Section 4 demonstrate that a good approximation of the desired beampattern can achieved with the new cost function. In addition to the reduced computational complexity, the proposed cost function provides faster convergence and lower squared approximation error.



(a) Cost function C_1

(b) Cost function C_2

Fig. 2. Convergence of an asymmetric beampattern with a 30° focus. P_i is the beampattern at i^{th} iteration and P_d is the desired beampattern. The beampattern remains too focused if C_1 is used, whereas convergence is good with C_2.

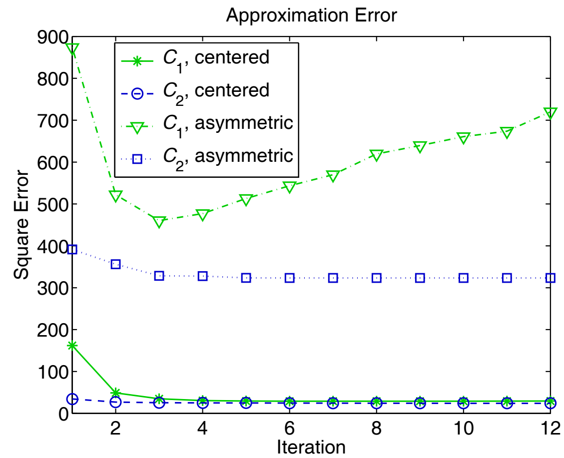


Fig. 3. Convergence of approximation error. The word 'centered' refers to the desired beampattern in Fig. 1 and 'asymmetric' to that in Fig. 2. Using C_2 results in faster convergence and lower squared error.

6. REFERENCES

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