### IMPROVING ROBUSTNESS OF CDM SPREAD SPECTRUM WATERMARKING

Joceli Mayer and José Carlos M. Bermudez

LPDS: Digital Signal Processing Research Laboratory Department of Electrical Engineering, Federal University of Santa Catarina, UFSC Florianópolis, SC, Brazil, CEP 88040-900

# ABSTRACT

This paper proposes improvements to the Code Division Modulation and Multiplexing (CDM) spread spectrum watermarking. The improved CDM technique exploits all the available information at the embedder and determines the required energy for the spreading sequences to achieve a specified bit error probability. This approach considers an additive noise in the channel and all the interferences from the host signal, cross-correlation between spreading sequences, perceptual masking and interferences from collusion attacks. We provide results to illustrate the improvements achieved and also comparisons with traditional CDM watermarking.

*Index Terms*— Data hiding, digital watermarking, Spread spectrum watermarking.

# 1. INTRODUCTION

Code Division Modulation and Multiplexing (CDM) has been widely employed for watermarking embedding in both the spatial and frequency domains [1-3]. By spreading the information over the chosen domain, CDM can help to provide secure embedding with robustness to non-malicious attacks such as compression, filtering, equalization and A/D and D/A conversions, as well as to tampering attacks [1, 4-6]. CDM-based embedding can be made robust to geometric operations such as rotation, cropping and scaling by employing a pre-processing tailored to the specific attack [5,7]. The computational complexity required for detection is linear (O(N)) for embedding N bits). In contrast, techniques such as basic message coding and orthogonal modulation [6] have computational complexities of  $O(2^N)$  required for detections, pattern generation and storage of these patterns. On the other hand, the perceptual impact of CDM embedding on the host signal increases with the number of bits embedded. This impact can be mitigated by using perceptual masking [8,9], M-ary modulation [10] (which increases the detection complexity) or by embedding in transformed domains [1, 2].

Many approaches in the literature propose to embed the watermark into reduced length sequences [11]. Unfortunately, however, short sequences produced by pseudo random generators tend to be highly cross-correlated. The degrading effect of cross-correlated sequences on the CDM detector performance becomes specially important in applications that require small bit error rates (*BER*), as we illustrate in Section 4.

In this paper we address the design of CDM watermarking systems which are also robust to collusion attacks. Depending on the number of colluders involved, such attacks can considerably reduce the energy of the embedded watermarks, hence degrading the detection performance and improving the fidelity of the resulting illegal copy [12]. The improved CDM watermarking is based on the informed multi-bit approach proposed in [9], but provides robustness to collusion attacks [13]. Similarly to [9], it also compensates for interferences from the host signal, perceptual masking and crosscorrelation between spreading sequences.

#### 2. IMPROVED INFORMED EMBEDDING

Consider a channel subject to additive noise and collusion attacks. Assume that T watermarked signals of the same Work are to be distributed to T users. The Work is represented by  $c_o$ , and the watermarked copies by  $c_i = c_o + w_i$ , i = 1, ..., T, where  $w_i$  is the watermark used for the *i*-th user. Using CDM, NT spreading sequences  $p_j^{(i)}$ , i = 1, ..., T, j = 1, ..., N, are required to embed a unique N-bit message for each of the T users. Each spreading sequence  $p_j^{(i)}$  has M samples, which are assumed to have zero average. These sequences are usually created by using a pseudo random generator. The colluded watermark w can be expressed as [9, 12]:

$$\boldsymbol{w} = \frac{1}{T} \sum_{i=1}^{T} \boldsymbol{w}_{i} = \frac{1}{T} \sum_{i=1}^{T} \sum_{j=1}^{N} \alpha_{j}^{(i)} b_{j}^{(i)} \boldsymbol{p}_{j}^{(i)} * \boldsymbol{x}.$$
 (1)

In (1),  $\boldsymbol{x}$  is the perceptual shaping mask vector [9] and the operator \* represents element-by-element vector multiplication. The coefficients  $b_j^{(i)}$  are the antipodal bits that carry the watermark information;  $\boldsymbol{p}_j^{(i)}$  is the spreading sequence associated with the *j*-th information bit that is used for the *i*-th user. The factor  $\alpha_j^{(i)}$  controls the energy of the sequence  $\boldsymbol{p}_j^{(i)}$ . In traditional CDM,  $\alpha_j^{(i)}$  is a constant  $\alpha, \forall i, j$ , used to adjust the total energy of the watermark. Our approach, based on [9], is to adjust each sequence individually by determining each factor  $\alpha_j^{(i)}$  in order to mitigate the interferences from host, sequences crosscorrelation, perceptual masking and collusion. We expand the work proposed in [9], by considering collusion attacks.

The watermarked signal  $c_W$  obtained after collusion of T users is received by the detector as  $c_W = c_o + w$ . Notice that we are disregarding for the moment any channel noise generated by the colluders or already existing in the signal path. The channel noise will be considered in Section 3.

The detection of the  $\ell$ -th message bit is done by linear correlation. In the absence of noise and after collusion of T copies of the Work, the detection statistics  $d_{\ell}^{(k)}$  of the  $\ell$ -th message bit for the k-th

This work was partly supported by CNPq under grant No. 308095/2003-0. E-mails: mayer@eel.ufsc.br and j.bermudez@ieee.org

user is given by:

$$d_{\ell}^{(k)} = \frac{1}{M} \left\langle \boldsymbol{p}_{\ell}^{(k)}, \boldsymbol{c}_{W} - \bar{\boldsymbol{c}}_{W} \right\rangle$$
$$= \frac{1}{M} \left[ \left\langle \boldsymbol{p}_{\ell}^{(k)}, \boldsymbol{c}_{o} - \bar{\boldsymbol{c}}_{W} \right\rangle + \left\langle \boldsymbol{p}_{\ell}^{(k)}, \boldsymbol{w} \right\rangle \right]$$
$$= \frac{1}{M} \left( r_{\ell}^{(k)} + \frac{1}{T} \sum_{i=1}^{T} \sum_{j=1}^{N} \alpha_{j}^{(i)} b_{j}^{(i)} \left\langle \boldsymbol{p}_{j}^{(i)} * \boldsymbol{x}, \boldsymbol{p}_{\ell}^{(k)} \right\rangle \right)$$
(2)

where the average vector  $\bar{s}$  of a vector s with elements  $s_k$ ,  $k = 1, \ldots, M$  is given by  $\bar{s} = \left(\frac{1}{M}\sum_{k=1}^M s_k\right)\mathbf{1}_M$ , where  $\mathbf{1}_M$  is an  $M \times 1$  vector of ones.  $\langle s, r \rangle = \frac{1}{M}\sum_{k=1}^M s_k r_k$  is the zero-lag cross-correlation<sup>1</sup> of the two sequences  $\{s_k\}$  and  $\{r_k\}$  composed of the elements of vectors s and r, respectively.  $r_{\ell}^{(k)} = \left\langle p_{\ell}^{(k)}, c_o - \bar{c}_W \right\rangle$  contains the host signal interference at the detection, where  $\bar{c}_W = \bar{c}_o + \bar{w}$ . Notice that the non-blind detection approach is a special case of our model when  $r_{\ell}^{(k)} = \left\langle p_{\ell}^{(k)}, -\bar{w} \right\rangle$ .

Let  $\hat{d}_{\ell}^{(k)} = \beta b_{\ell}^{(k)}$  be the specified decision level at the detector. Then, the detection mismatch between the desired and achieved detection levels for bit  $\ell$  and user k is given by  $e_{k,l} = \hat{d}_{\ell}^{(k)} - d_{\ell}^{(k)}$ , and the total squared detection error is defined as  $\xi = \sum_{k=1}^{T} \sum_{\ell=1}^{N} e_{k,\ell}^2$ :

$$\xi = \sum_{k=1}^{T} \sum_{\ell=1}^{N} \left( \hat{d}_{\ell}^{(k)} - \frac{r_{\ell}^{(k)}}{M} - \frac{1}{TM} \sum_{i=1}^{T} \sum_{j=1}^{N} \alpha_{j}^{(i)} b_{j}^{(i)} q_{j,\ell}^{(i,k)} \right)^{2}$$
(3)

where  $q_{j,\ell}^{(i,k)} = \left\langle \mathbf{p}_j^{(i)} * \mathbf{x}, \mathbf{p}_\ell^{(k)} \right\rangle$ . The double sums in (3) can be simplified to single sums by the changes of variable m = (j-1)T+i and  $n = (\ell-1)T + k$ , yielding<sup>2</sup>

$$\xi = \sum_{n=1}^{NT} \left[ \tilde{d}_n - \left( \frac{\tilde{r}_n}{M} + \frac{1}{TM} \sum_{m=1}^{NT} \tilde{\alpha}_m \psi_{m,n} \right) \right]^2$$
$$= \sum_{n=1}^{NT} \left[ \tilde{d}_n - \left( \frac{\tilde{r}_n}{M} + \frac{1}{TM} \left[ \Psi^t \phi \right]_n \right) \right]^2 \tag{4}$$

where  $\tilde{d}_n = \hat{d}_{\ell}^{(k)}$ ,  $\tilde{r}_n = r_{\ell}^{(k)}$ ,  $\tilde{\alpha}_m = \alpha_j^{(i)}$ ,  $\psi_{m,n} = b_j^{(i)} q_{j,\ell}^{(i,k)}$ ,  $\phi = [\tilde{\alpha}_1, \dots, \tilde{\alpha}_{NT}]^t$  and  $\Psi$  is an  $NT \times NT$  matrix with elements  $\psi_{m,n}$ . Defining the vectors  $\tilde{d} = [\tilde{d}_1, \dots, \tilde{d}_{NT}]^t$  and  $\tilde{r} = [\tilde{r}_1, \dots, \tilde{r}_{NT}]^t$ , (4) can be written as a vector inner product:

$$\xi = \left[\tilde{\boldsymbol{d}} - \left(\frac{1}{M}\tilde{\boldsymbol{r}} + \frac{1}{TM}\boldsymbol{\Psi}^{t}\boldsymbol{\phi}\right)\right]^{t} \left[\tilde{\boldsymbol{d}} - \left(\frac{1}{M}\tilde{\boldsymbol{r}} + \frac{1}{TM}\boldsymbol{\Psi}^{t}\boldsymbol{\phi}\right)\right]$$
(5)

The vector  $\phi$  that minimizes  $\xi$  for a given desired detection level  $\hat{d}_{\ell}^{(k)} = \beta b_{\ell}^{(k)}$  (a given vector  $\tilde{d}$ ) satisfies

$$\frac{\partial \xi}{\partial \phi} = 2\Psi \left( \frac{\Psi^t \phi}{TM} - \tilde{d} + \frac{1}{M} \tilde{r} \right) = 0.$$
 (6)

and thus is the solution of

$$\Psi^{t}\phi = TM\left(\tilde{\boldsymbol{d}} - \frac{1}{M}\tilde{\boldsymbol{r}}\right)$$
(7)

<sup>1</sup>Note that in this paper  $\langle s, r \rangle$  corresponds to a *normalized* inner product of vectors s and r.

Assuming that  $\Psi$  has full rank,

$$\phi = TM(\Psi^t)^{-1}g \tag{8}$$

where  $\boldsymbol{g} = \tilde{\boldsymbol{d}} - \frac{1}{M}\tilde{\boldsymbol{r}}$ . It is easy to verify that  $\phi$  given by (8) leads to  $\xi = 0$  and thus to  $d_{\ell}^{(k)} = \hat{d}_{\ell}^{(k)} = \beta b_{\ell}^{(k)}$ .

Now consider an  $M \times 1$  additive vector channel noise  $\eta$ , whose elements are drawn from a zero-mean independent, identically distributed (iid) noise process with even probability density function (pdf). For a sequence embedding using the factors  $\tilde{\alpha}_m$  determined from (6), the resulting detection levels will be:

$$d_{\ell}^{(k)} = \beta b_{\ell}^{(k)} + \left\langle \boldsymbol{p}_{\ell}^{(k)}, \boldsymbol{\eta} \right\rangle$$
(9)

In terms of index n,

$$d_n = \beta b_n + v_n \tag{10}$$

where  $v_n = \langle p_{\ell}^{(k)}, \eta \rangle$ ,  $d_n = d_{\ell}^{(k)}$  and  $b_n = b_{\ell}^{(k)}$  with  $n = (\ell - 1)T + k$ . Note that  $v_n$  models the additive noise interference.

# 3. DETERMINING $\beta$ FOR ADDITIVE WHITE NOISE $\eta$

In the following, we estimate the parameter  $\beta$  to attain a given specified bit error probability  $P_e$  in the noisy case. Recall that using  $\phi$  obtained from (8) and the notation in (10),  $d_n = \tilde{d}_n = \beta b_n$  for the noiseless case. Comparing this expression with (10), a detection error occurs when: a)  $b_n = 1$  and  $d_n = \beta + v_n \leq 0$ ; b)  $b_n = -1$  and  $d_n = -\beta + v_n \geq 0$ . Thus, the resulting error probability  $P_{e_n}$  for bit<sup>3</sup> n is given by  $P_{e_n} = \Pr\{v_n \geq \beta | b_n = -1\} + \Pr\{v_n \leq -\beta | b_n = 1\}$ . Assuming that the message bits are independent of the noise source,

$$P_{e_n} = \Pr\{v_n \ge \beta\} \Pr\{b_n = -1\} + \Pr\{v_n \le -\beta\} \Pr\{b_n = 1\}$$
(11)

To determine the value of  $\beta$  necessary to achieve a specified  $P_{e_n}$ , we need to calculate the statistics of  $v_n$ . Since the spreading sequence  $p_{\ell}^{(k)}$  is statistically independent of the zero-mean noise vector  $\boldsymbol{\eta}$ ,  $E[v_n] = 0$ , where  $E[\cdot]$  stands for statistical expectation. To determine the variance of  $v_n$  we first define  $p_{\ell}^{(k)}(m)$  as the *m*-th binary pattern in the spread sequence  $p_{\ell}^{(k)}$ , with  $m = 1, \ldots, M$ , and  $\eta(m)$  as the *m*-th element of vector  $\boldsymbol{\eta}$ . Then, assuming that  $p_{\ell}^{(k)}(m) \in \{-P, P\}$  and using the property  $E[\eta_i \eta_j] = 0$  for  $i \neq j$ ,

$$\sigma_v^2 = E[v_n^2] = E\left[\left(\frac{1}{M}\sum_{m=1}^M p_\ell^{(k)}(m)\eta(m)\right)^2\right]$$
  
=  $\frac{1}{M^2}\sum_{m=1}^M E\left[\left(p_\ell^{(k)}(m)\right)^2\eta^2(m)\right]$  (12)  
=  $\frac{P^2}{M^2}\sum_{m=1}^M E[\eta^2(m)] = \frac{P^2}{M}\sigma_\eta^2.$ 

Now, since we have assumed  $\eta(m)$  to be white and with even pdf,  $v_n$  is a sum of independent random variables with even pdfs and thus has an even pdf itself which tends to a Gaussian for M large. Hence,  $\Pr\{v_n \ge \beta\} = \Pr\{v_n \le -\beta\}$ . Using this property in (11) and the fact that  $\Pr\{b_n = 1\} + \Pr\{b_n = -1\} = 1$  yields

$$P_{e_n} = \Pr\{v_n \ge \beta\} \tag{13}$$

<sup>&</sup>lt;sup>2</sup>These changes of variables change matrices into column vectors and the fourth-dimensional array with elements  $q_{j,\ell}^{(i,k)}$  into a matrix with elements  $\psi_{m,n}$ .

<sup>&</sup>lt;sup>3</sup>Here, bit n is a simplified terminology to refer to bit  $\ell$  of the information sent to user k, where  $n = (\ell - 1)T + k$ .

Finally,  $\sigma_v^2$  in (12) is not dependent on the bit value  $b_n$  or on the specific binary patterns  $p_\ell^{(k)}(m)$ . Thus,  $P_{e_n}$  in (13) is equal to the average probability of error  $P_e$ .

Assuming  $v_n$  to be zero-mean Gaussian, (13) yields

$$P_e = \frac{1}{\sqrt{2}\sigma_v} \int_{\beta}^{\infty} e^{-\frac{t^2}{2\sigma_v^2}} dt = \frac{1}{2} \operatorname{erfc}\left(\frac{\beta}{\sqrt{2}\sigma_v}\right) \quad (14)$$

where  $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$  is the complementary error function.

Hence, the value of  $\beta$  required to achieve a specified bit error rate  $P_e$  can be determined from:

$$\beta = \sqrt{2}\sigma_v \operatorname{erfc}^{-1}(2P_e) \tag{15}$$

with  $\sigma_v^2$  given by (12).

Note that it is not difficult to estimate the noise power  $\sigma_{\eta}^2$  for most applications. This power can also be related to the maximum distortion power the watermarking system should survive to. Thus, given an acceptable probability of error  $P_e$  and an estimate of the channel noise power, (15) can be used to determine the value of  $\beta$ required to compensate for the noise correlation. Then, using this value of  $\beta$  to determine the vector  $\tilde{d}$  from  $\hat{d}_{\ell}^{(k)} = \beta b_{\ell}^{(k)}$ , (8) leads to the vector  $\phi$  of factors  $\alpha_j^{(i)}$  to be employed in (1). For instance,  $P_e = 0.0013$  requires  $\beta = 3\sigma_v = 3\frac{P}{\sqrt{M}}\sigma_{\eta}$ , while  $P_e = 0.1587$ requires  $\beta = \sigma_v = \frac{P}{\sqrt{M}}\sigma_{\eta}$ . In the absence of poise, the meeting

In the absence of noise, the most transparent embedding requires  $\beta = 0$ . In practice, however,  $\beta$  cannot be too small due to roundoff and clipping effects. Under noisy conditions, transparency will be limited by the noise variance, by the amount of host interference, by the number of users in collusion, by the cross-correlations between different spread sequences and by perceptual mask interferences. In both the noiseless and noisy cases, the proposed embedding approach provides the required embedding energy for the sequences given a specified bit error probability and the noise variance. Perceptual transparency is considerably improved by using a proper shaping mask x. Notice that the mask interferences are also compensated by our approach. The compensation of these interferences is required to achieve the specified  $P_e$  since they affect the orthogonality of different spread sequences.

### 4. EXPERIMENTS

In order to verify the accuracy of the design equation (15) for  $\beta$ , we realized experiments with specified  $P_{e_1} = 0.0013$  and  $P_{e_2} = 0.1587$ , and for several values of noise power  $\sigma_{\eta}^2$ . In these experiments we used N = 40 (insertion of 40 bits) and T = 5 (5 users, all of them colluding). The resulting bit error rates shown in Table 1 were obtained from Monte Carlo simulations with 300 realizations. These results indicate that we are properly compensating for the aforementioned interferences and properly setting the strength factor of the sequences to achieve the specified performance.

To illustrate the degrading effects of sequence cross-correlations on the detection performance we made T = 1 (no collusion attacks) and compared the performances obtained using the proposed and the traditional CDM embeddings [5]. Orthogonality among sequences is assumed by most spread spectrum based embedders in the literature. However, orthogonality rarely occurs in practice due to residual cross-correlations existing in sequences generated by practical pseudo-random generators or due to perceptual masking. The results (for T = 1) are shown in Figs. 1 and 2. Fig. 1 illustrates the effects of  $P_e$  and the image size. Images of sizes  $16 \times 16, 64 \times 64$ and  $128 \times 128$  were used. For the traditional CDM, these cases are labelled SS16, SS64 and SS128, respectively. The sequence length M is equal to the number of image pixels. The label "Prop" refers to the proposed technique. In all cases,  $N = 10, \sigma_{\eta} = 64$ . The signalto-watermark ratio is kept the same for both competing techniques by adjusting the energy of the sequences used in the traditional CDM technique.  $P_e$  is the specified (or desired) bit error probability. Notice from Fig. 1 that the BER performance of the traditional approach depends on the value of  $P_e$  and on the sequence length M. The improved performance of the traditional approach as M increases is due to the reduction of the sequence cross-correlations for larger spreading sequences. The performance of the proposed approach is clearly independent of these parameters. Fig. 2 illustrates the effect of the noise power on the detection performance. This figure was generated for  $M = 64 \times 64, N = 10, P_e = 3.17 \times 10^{-5}$ and the signal-to-watermark ratio (SWR) was kept the same for both techniques. Once again, the proposed technique leads to a performance that is independent of the noise power (provided it can be estimated). In Fig. 3 we consider a variable number of users in collusion, namely, T = 10, 20 and 40 users. It can be verified that the proposed approach provides a significantly better embedding than the traditional approach.

In general, the experiments show that the improvement over the traditional CDM is more significant for larger number of users T, smaller  $P_e$ , smaller sizes M, smaller noise deviation  $\sigma_{\eta}$  and larger number of bits N. We did not employ any perceptual masking in these examples, which would tend to improve the performance of the proposed technique, as compared to traditional CDM embedding.

Regarding the necessary resources, the proposed CDM improvement requires the solution of (7) at the embedding. It also requires an amount of memory proportional to the number of bits N, to the number of users T and to the host signal size M. Thus, in general, the approach may be limited to hundreds of users and few bits per user due to memory limitations. A numerically optimized routine to solve (7) may significantly alleviate this limitation. Regarding detection, the proposed method presents the same order of complexity as the traditional CDM spread spectrum technique, namely, O(N).

**Table 1.** Monte Carlo simulations (300 realizations each) for specified  $P_{e_1} = 0.0013$ ,  $P_{e_2} = 0.1587$  and various  $\sigma_{\eta}$ .  $M = 144 \times 95$ , N = 40 and T = 5. BER are given by the number of errors divided by  $40 \times 5 \times 300$ .

	$P_{e_1} = 0.0013$	$P_{e_2} = 0.1587$
$\sigma_{\eta}$	BER	BER
5	0.001417	0.1592
10	0.001317	0.1594
21	0.001233	0.1588
43	0.001234	0.1571

#### 5. CONCLUSIONS

We proposed improvements to CDM spread spectrum by compensating for the host interference, collusion attack, cross-correlation of the spreading sequences and perceptual masking interferences. The new approach enforces a specified robustness, expressed as error probability, to additive noise in the channel for a given number of users



**Fig. 1**. *BER* performance as a function of the specified  $P_e$  and the sequence length M. T = 1, N = 10,  $\sigma_{\eta} = 64$  and the signal-to-watermark-ratio is kept the same for both approaches.



Fig. 2. *BER* performance as a function of the noise deviation  $\sigma_{\eta}$ .  $M = 64 \times 64, T = 1, N = 10, P_e = 3.17E - 05.$ 

working in a collusion to defeat the watermarking system. Experiments were provided which verify the accuracy of the analysis and the performance when compared to traditional CDM embedding.

# 6. REFERENCES

- Santi P. Maity and Malay K. Kundu, "A blind cdma image watermarking scheme in wavelet domain," in *Intl. Conf. on Image Processing, ICIP'04*, 2004, pp. 2633–2636.
- [2] Joseph J.K.Ó. Ruanaidh and Gabriella Csurka, "A bayesian approach to spread spectrum watermark detection and secure copyright protection for digital image libraries," in *IEEE Comp. Society Conf. on Comp. Vision and Pattern Recognition*, 1999, pp. 207–212.
- [3] Yongqing Xin and Miroslaw Pawlak, "Multibit data hiding based on cdma," in *Canadian Conf. on Electrical and Computer Engineering*, 2004, vol. 2, pp. 935–938.



Fig. 3. Performances of traditional CDM and the proposed approach for N = 10, T = 10, 20, 40 and  $\sigma_n = 4$ .

- [4] I.J. Cox, M.L. Miller, A.L. Mckellips, "Watermarking as communications with side information," *Proc. of the IEEE*, vol. 87, no. 7, pp. 1127–1141, 1999.
- [5] I. J. Cox, J. Kilian, F. T. Leighton, T. Shamoon, "Secure spread spectrum watermarking for multimedia," *IEEE Trans. on Im*age Processing, vol. 6, no. 12, pp. 1673–1687, 1997.
- [6] Z. Jane Wang, Min Wu, Hong Vicky Zhao, Wade Trappe, K. J. Ray Liu, "Anti-collusion forensics of multimedia fingerprinting using orthogonal modulation," *IEEE Trans. on Image Processing*, vol. 14, no. 6, pp. 804–821, June 2005.
- [7] Yanmei Fang, Jiwu Huang, Shaoquan Wu, "Cdma-based watermarking resisting to cropping," in *Proc. of the 2004 Intl. Symposium on Circuits and Systems, ISCAS'04*, 2004, vol. 2, pp. 25–28.
- [8] Martin Kutter and Stefan Winkler, "A vision-based masking model for spread-spectrum image watermarking," *IEEE Trans.* on Image Processing, vol. 11, no. 1, pp. 16–25, January 2002.
- [9] Joceli Mayer and José C. M. Bermudez, "Multi-bit informed embedding watermarking with constant robustness," *IEEE Intl. Conf. on Image Processing, ICIP2005*, vol. 1, pp. 804–821, 2005.
- [10] Martin Kutter, "Performance improvement of spread spectrum based image watermarking schemes through m-ary modulation," *Lecture Notes in Computer Science, Springer Verlag*, vol. 1768, pp. 238–250, 1999.
- [11] Paulo V. K. Borges, Joceli Mayer, "Informed positional embedding for multi-bit watermarking," in *IEEE Intl. Conf. on Acoustics, Speech, and Signal Processing, ICASSP*, March 2005, vol. 2, pp. 809–812.
- [12] Wade Trappe, Min Wu, Jane Wang, K. J. Ray Liu, "Anticollusion fingerprinting for multimedia," *IEEE Trans. on Signal Processing*, vol. 51, no. 4, pp. 1069–1087, April 2003.
- [13] Min Wu and Bede Liu, "Data hiding in image and video: Part i-fundamental issues and solutions," *IEEE Trans. on Image Processing*, vol. 12, no. 6, pp. 685–695, June 2003.