DCT SIGN-ONLY CORRELATION WITH APPLICATION TO IMAGE MATCHING AND THE RELATIONSHIP WITH PHASE-ONLY CORRELATION

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ABSTRACT

A close relationship between the sign information of Discrete Cosine Transform (DCT) coefficients and the phase term of Discrete Fourier Transform coefficient is explained. From this relationship, the DCT sign-only correlation is proposed and its relationship with the phase-only correlation (POC) is considered. As a result, the proposed correlation can be applied to applications based on the POC, and also provide a theoretical explanation of target image search and retrieval using the sign information of DCT.

Index Terms— Discrete cosine transforms, Correlation, Image matching, Image registration, Image coding

1. INTRODUCTION

The Discrete Cosine Transform (DCT) is a key technology for coded images, such as those using JPEG and MPEG compressions. Since the images are encoded using the DCT, it is convenient to study image processing and image registration in the DCT domain. Also, the sign information of the DCT coefficients (called DCT signs) is robust against scalar quantization noise because positive signs do not change to negative signs and vice versa. Moreover, the concise expression of DCT sings saves physical space to calculate and store them. Because of these DCT properties, target image search and retrieval taking advantage of the DCT signs in coded image is being studied[1]-[3].

Phase-only correlation (POC) is a kind of limited correlation and can estimate translational displacements, rotation and scaling with subpixel accuracy between two images [4]-[10]. The study of its application in areas such as biometrics [6]-[8], image motion analysis [9] and video retrieval [10] has advanced. If POC is to be applied to coded images, they should first be decoded. Moreover, because a phase term is a complex number, the use of the POC requires more physical space and time.

In this paper, the DCT sign only correlation is proposed. Moreover its relationship with the POC is considered. From this consideration, it is explained that the proposed correlation can be applied to applications based on the POC[6]-[10]. Also, the correlation provide a theoretical explanation of target image search and retrieval using DCT signs[1]-[3]. Through simulations of image matching, it is demonstrated that the DCT sign-only correlation is efficient with correlation based applications. In some applications, the correlation has an advantage over the POC due to its robustness against scalar quantization in JPEG compression.

2. PRERIMINARY

The phase-only correlation, the phase-only image and the DCT sign-only image are described as single dimensional notation for clarity.

2.1. DCT signs and DFT phase term

Let $f(n)$ be an N-point sequence, and the DCT of $f(n)$ is denoted by $F_C(k)$. $F_C(k)$ is a real value and can be expressed in terms of the absolute value, $|F_C(k)|$, and the DCT signs, $F'_C(k)$, as

$$F_C(k) = |F_C(k)|F'_C(k). \tag{1}$$

The Discrete Fourier Transform (DFT) of $f(n)$ is denoted by $F(k)$. $F(k)$ is a complex value and can be expressed in terms of the magnitude, $|F(k)|$, and the phase term, $F'(k)$, as

$$F(k) = |F(k)|F'(k). \tag{2}$$

2.2. POC

The phase-only correlation (POC) is a kind of limited cross correlation. The cross spectrum, $R(k)$, of $F'(k)$ with $G'(k)$ is given as follows:

$$R(k) = F'(k) \cdot \overline{G'(k)} \tag{3}$$

where $G(k) = |G(k)|G'(k)$ is the DFT of an N-point sequence, $g(n)$, and $\overline{G'(k)}$ denotes the complex conjugate of $G'(k)$.

The POC, $r(n)$, between $f(n)$ and $g(n)$ is given as the inverse DFT of $R(k)$, as follows:

$$r(n) = \frac{1}{N} \sum_{k=0}^{N-1} R(k) W_N^{-nk}, \quad n = 0, 1, \cdots, N-1 \tag{4}$$

where $W_N = exp(-j2\pi/N)$.

2.3. DCT sign-only image and phase-only image

There are some types of DCTs from their symmetric properties. The DCT-II is used in many image compression methods because of its properties[12]. In this paper, DCT refers to DCT-II unless otherwise indicated.

The N-point phase-only sequence, $f'(n)$, is defined as the inverse DFT of $F'(k)$ as

$$f'(n) = \frac{1}{N} \sum_{k=0}^{N-1} F'(k) W_N^{-nk} \quad n = 0, 1, \cdots, N-1. \tag{5}$$

The N-point DCT sign-only sequence, $f'_C(n)$, is defined as the inverse DCT of $F'_C(k)$. Namely,

$$\begin{aligned} f'_C(n) &= \frac{2}{\sqrt{N}} \sum_{k=0}^{N-1} C_k F'_C(k) \cos\left(\frac{2\pi(n+\frac{1}{2})k}{2N}\right) \\ n &= 0, 1, \cdots, N-1 \end{aligned} \tag{6}$$

where

$$C_k = \begin{cases} \frac{1}{\sqrt{2}}, & k = 0 \\ 1, & k \neq 0 \end{cases}. \tag{7}$$

The DCT sign-only image in Fig.1(a) is given as the two-dimensional expression of the DCT sign-only sequence. Similarly, the phase-only image in Fig. 1(b) is yielded as that of the phase-only sequence. The intensity of both images is adjusted for easier visibility. From Fig. 1, we can see that the DCT sign-only image is similar to the phase-only one.

Therefore, the purposes of this paper are to propose the correlation using the DCT signs and to explain that the correlation can be applied to applications based on the POC.

3. DCT SIGN-ONLY CORRELATION AND ITS PROPERTIES

The DCT sign-only correlation is proposed and its relationship with POC is considered in this section.

3.1. DCT sign-only correlation

The DCT sign product, $R_C(k)$, of $F'_C(k)$ with $G'_C(k)$ is given as

$$R_C(k) = F'_C(k) \cdot G'_C(k). \tag{8}$$

(a) DCT sign-only image (b) phase-only image

Fig. 1. DCT sign-only image and phase-only image

The DCT sign-only correlation, $r_C(n)$, is defined as

$$r_C(n) = \frac{1}{N} \sum_{k=0}^{N} F_k R_C(k) \cos\left(\frac{\pi n k}{N}\right) \tag{9}$$

where, $R_C(N) = 0$ and

$$F_k = \begin{cases} \frac{1}{2}, & k = 0, N \\ 1, & k = 1, 2, \cdots, N-1 \end{cases}. \tag{10}$$

This correlation uses the DCT-II as the forward transform and the DCT-I as the inverse transform[12], as in Eq.(9). A close relationship with the POC will be explained in the next section.

3.2. Relationship between DCT sign-only correlation and POC

A. DCT signs and DFT phase term

The 2N-point sequence, $\hat{f}(n)$, is defined as the sequence extended symmetrically from an N-point sequence, $f(n)$, as

$$\hat{f}(n) = \begin{cases} f(n), & n = 0, 1, \ldots, N-1 \\ f(2N-n-1), & n = N, N+1, \ldots, 2N-1. \end{cases} \tag{11}$$

$\hat{F}(k)$, the 2N-point DFT of $\hat{f}(n)$, is expanded as follows:

$$\begin{aligned} \hat{F}(k) &= \sum_{n=0}^{N-1} f(n) W_{2N}^{nk} + \sum_{n=N}^{2N-1} f(2N-n-1) W_{2N}^{nk} \\ &= \sum_{n=0}^{N-1} f(n) W_{2N}^{nk} + \sum_{n=0}^{N-1} f(n) W_{2N}^{(2N-n-1)k} \\ &= \sum_{n=0}^{N-1} f(n) \left(W_{2N}^{nk} + W_{2N}^{-(n+1)k}\right). \end{aligned} \tag{12}$$

By multiplying both sides of Eq.(12) by $\sqrt{\frac{2}{N}}C_k\frac{1}{2}W_{2N}^{\frac{k}{2}}$, it follows that

$$\begin{aligned}
&\sqrt{\frac{2}{N}}C_k\frac{1}{2}W_{2N}^{\frac{k}{2}}\hat{F}(k)\\
&= \sqrt{\frac{2}{N}}C_k\sum_{n=0}^{N-1} f(n)\frac{1}{2}\left(W_{2N}^{(n+\frac{1}{2})k}+W_{2N}^{-(n+\frac{1}{2})k}\right)\\
&= \sqrt{\frac{2}{N}}C_k\sum_{n=0}^{N-1} f(n)\cos\left(\frac{2\pi(n+\frac{1}{2})k}{2N}\right). \qquad (13)
\end{aligned}$$

Note that the right hand side of Eq.(13) expresses the definition of the N-point DCT-II[12], $F_C(k)$, of $f(n)$. Therefore, for $k = 0, 1, \cdots, N-1$, Eq.(13) is written as

$$\sqrt{\frac{2}{N}}C_k\frac{1}{2}W_{2N}^{\frac{k}{2}}\hat{F}(k) = F_C(k). \qquad (14)$$

From Equations (1) and (2), Eq.(14) is normalized by $|F_C(k)|$. Therefore, we obtain

$$F'_C(k) = W_{2N}^{\frac{k}{2}}\hat{F}'(k) \quad k = 0, 1, \cdots, N-1. \qquad (15)$$

Equation (15) shows that the DCT signs, $F'_C(k)$, is equal to the phase term, $\hat{F}'(k)$, multiplied by $W_{2N}^{\frac{k}{2}}$.

From Equations (3), (8) and (15), the cross spectrum, $\hat{R}(k)$, of $\hat{F}'(k)$ with $\hat{G}'(k)$ is expanded as follows:

$$\begin{aligned}
\hat{R}(k) &= \hat{F}'(k)\cdot\overline{\hat{G}'(k)}\\
&= W_{2N}^{-k/2}F'_C(k)\cdot\overline{W_{2N}^{-k/2}G'_C(k)}\\
&= W_{2N}^{-k/2}F'_C(k)\cdot W_{2N}^{k/2}G'_C(k)\\
&= F'_C(k)\cdot G'_C(k)\\
&= R_C(k) \quad k = 0, 1, \cdots, N-1 \qquad (16)
\end{aligned}$$

where $\hat{F}'(k)$ and $\hat{G}'(k)$ denote the DFT phase term of the 2N-point sequences, $\hat{f}(n)$ and $\hat{g}(n)$, and $F'_C(k)$ and $G'_C(k)$ denote the DCT signs of the N-point sequences, $f(n)$ and $g(n)$.

B. Relationship with POC

$\hat{r}(n)$, the 2N-point Inverse DFT of $\hat{R}(k)$, is expanded as follows:

$$\begin{aligned}
\hat{r}(n) &= \frac{1}{2N}\left(\sum_{k=0}^{N-1}\hat{R}(k)W_{2N}^{-nk}+\sum_{k=N}^{2N-1}\hat{R}(k)W_{2N}^{-nk}\right)\\
&= \frac{1}{2N}\left(\sum_{k=0}^{N-1}\hat{R}(k)W_{2N}^{-nk}+\sum_{k=1}^{N}\hat{R}(-k)W_{2N}^{nk}\right)\\
&= \frac{1}{2N}\left(\sum_{k=0}^{N-1}\hat{R}(k)W_{2N}^{-nk}+\sum_{k=1}^{N-1}\overline{\hat{R}(k)}W_{2N}^{nk}\right)\\
&\quad +\frac{1}{2N}\sum_{k=N}^{N}\overline{\hat{R}(k)}W_{2N}^{nk} \qquad (17)
\end{aligned}$$

From Eq.(16) and $\hat{R}(N) = 0$,

$$\begin{aligned}
\hat{r}(n) &= \frac{1}{2N}\left(\sum_{k=0}^{N-1}R_C(k)W_{2N}^{-nk}+\sum_{k=1}^{N-1}\overline{R_C(k)}W_{2N}^{nk}\right)\\
&= \frac{1}{2N}\sum_{k=0}^{N-1}F_kR_C(k)(W_{2N}^{-nk}+W_{2N}^{nk})\\
&= \frac{1}{N}\sum_{k=0}^{N}F_kR_C(k)\cos\left(\frac{\pi nk}{N}\right) = r_C(n) \qquad (18)
\end{aligned}$$

where $R_C(N) = 0$. From Eq.(18), we conclude that $\hat{r}(n)$ is equivalent to $r_C(n)$. Thus, the DCT sign-only correlation, $r_C(n)$, is a special case of the POC, where each sequence is extended symmetrically, as in Eq.(11).

3.3. Relationship with Similarity

The relationship between the DCT sign-only correlation and the similarity is explained below.

Target image search and retrieval in DCT domain [1]-[3] uses a similarity, s_C , in practice, as

$$s_C = \frac{1}{K}\sum_{k=0}^{N-1}F'_C(k)\cdot G'_C(k) \qquad (19)$$

where K is a constant. In the DCT sign-only correlation, on the other hand, when $n = 0$ and $R_C(N) = 1$, we obtain

$$r_C(0) = \frac{1}{N}\sum_{k=0}^{N}F_kR_C(k) = \frac{1}{N}\sum_{k=0}^{N-1}R_C(k) = s_C. \qquad (20)$$

where the weight of the sum $1/N = 1/K$. Therefore, calculating similarity s_C is calculating the DCT sign-only correlation for n = 0. In other words, the similarity is a special case of the DCT sign-only correlation. K in Eq.(19) is given as the sum of nonzero $R_C(k)$ values because of the robustness against scalar quantization in JPEG compression in [1] [2].

4. SIMULATION

4.1. Translational displacement

We estimate the translational displacement between two images, an original image and an object one. We use the grayscale image, 'lena', whose size is 512 by 512 pixels. The object image is created from the original image, shifted 20 pixels in the right and upward directions respectively. Figure 2 shows that the location $(n_1, n_2) = (20, 20)$ has a peak in the DCT sign-only correlation, and expresses the translational displacement as well as the POC. If the direction of the translational displacement is desired, it is possible to obtain it with additional steps[11]. As space is limited, the details are omitted.

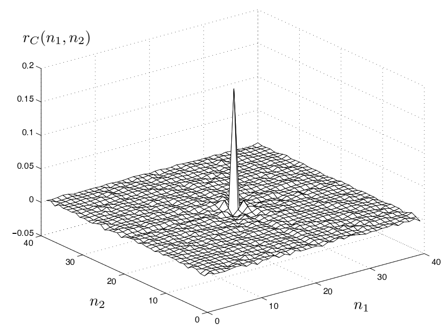

Fig. 2. DCT sign-only correlation

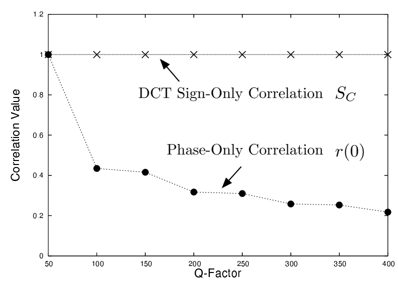

Fig. 3. Robustness against quantization in JPEG compression

4.2. Robustness against scalar quantization in JPEG

Two JPEG-coded images are generated from the same grayscale image, 'lena', whose size is 512×512 pixels. Let the Q-factor of one image be 50 and that of the other image vary. In JPEG compression, images are tiled into blocks whose sizes are 8×8 pixels. In the DCT sign-only correlation, similarity of each block is calculated, as in Eq.(19), where the weight is K, and the correlation value is the average similarity of all blocks. In the POC, on the other hand, after the coded blocks are decoded and the whole image is obtained, the POC is applied to the whole image and the correlation value is calculated.

Figure 3 shows that in the DCT sign-only correlation, taking advantage of its robustness against scalar quantization, the correlation value is constant, 1. Whereas in the POC, it decreases as the compression ratio increases, because it is affected by scalar quantization.

5. CONCLUSION

The DCT sign-only correlation has been proposed, and the relevant theory with POC has been explained. As a result of this, the proposed correlation using only sign information in DCT domain can be applied to POC based applications. Also, the correlation has provided a theoretical explanation of similarity using the DCT signs. From simulations, we have seen that the DCT sign-only correlation is efficient and has an advantage over the POC due to its robustness against scalar quantization in JPEG compression.

6. REFERENCES

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