DETECTION OF CURVILINEAR OBJECTS IN NOISY IMAGE USING FEATURE-ADAPTED BEAMLET TRANSFORM

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ABSTRACT

This paper addresses the problem of detecting features running along lines or piecewise constant curves. Our method is adapted either for common image features like edges or ridges as well as any kind of features that can be designed by *a priori* knowledge. The main contribution of this paper is to unify the well-known Beamlet transform, introduced by Donoho *et al.* [1], with linear filtering technique in order to define what we call the Feature-adapted Beamlet transform. If the desired feature is chosen to belong to the class of steerable filters, our method can be achieved in linear time and can be easily implemented on a parallel machine. We present some experimental results both on edge- and ridge-like features that demonstrate the substantial improvement over classical feature detectors.

***Index Terms*—** Beamlet transform, steerable filters, features detection, curvilinear objects, biology.

1. INTRODUCTION

The problem of detecting curvilinear objects in images arises in various areas of image processing and computer vision, since such kind of objects occur in every natural and synthetic images, like contours of objects, roads in aerial imaging or DNA filaments in biological microscopy.

Commonly, curvilinear objects are considered as 1-dimensional manifolds that have a specific profile running along a smooth curve. The shape of this profile may be an edge- or a ridge-like feature. It can also be represented by more complex designed features. For example, in the context of DNA filament analysis in fluorescent microscopy, it is acceptable to consider the transverse dimension of a filament to be small relative to the PSF width of the microscope. Hence, the shape of the profile may be accurately approximate by a PSF model. A recent study of such models for various types of microscopes can be found in [2].

One way to detect curvilinear objects is to track locally the feature of the curve-profile; linear filtering or template matched filtering are well-known techniques for doing so. Classical Canny edge detector [3] and more recently detectors designed in [4] are based on such linear filtering techniques. They involve the computation of inner-products with shifted and/or rotated version of the feature template at every point in the image. High response at a given position in the image means that the considered area has a similarity with the feature template. Filtering is usually followed by a non-maxima suppression and a thresholding step in order to extract the objects. The major drawbacks of such approaches come from the fact that linear filtering is based on local operators: it is highly sensitive to noise but not sensitive to the underlying smoothness of the curve, which is a typical non-local property of curvilinear objects.

Alternativaly, the Radon transform is a powerful non-local technique which may be used for line detection. Also known as the Hough transform in the case of discrete binary images, it performs a mapping from the image space into a line parameter space by computing line integrals. Formally, given an image f defined on a sub-space of $\mathbb{R}^2$, for every line parameter (ρ, θ), it computes

$$\varphi(\rho,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(\rho - x\cos(\theta) - y\sin(\theta))dxdy. \quad (1)$$

Peaks in the parameter space reveals potential lines of interest. This is a very reliable method for detecting lines in noisy images. However, there are severals limitations. First, direct extension of that method to detect more complex curves is unfeasible in practice for it increases the complexity exponentially by adding one dimension to the parameter space. In addition, Radon transform computes line integrals on lines that pass through the whole image domain and does not provide information on small line segments.

Given an image of $N \times N$ pixels, the number of possible line segments defined is in $O(N^4)$. Direct evaluation of line integrals upon the whole set of segments is practically infeasible due to the computational burden. One of the methodologies proposed to address this problem is the Beamlet transform [1, 5, 6]. It defines a set of dyadically organized line segments occupying a range of dyadic locations and scales, and spanning a full range of orientations. This system of line segments, called beamlets, have both their end-points lying on dyadic squares that are obtained by recursive partitionning of the image domain (see [1] for complete details). The collection of beamlets has a $O(N^2 \log(N))$ cardinality. The underlying idea of the Beamlet transform is to compute line integrals only on this smaller set, which is an efficient substitute of the entire set of segments for it can approximate any segment by a finite chain of beamlets. Beamlet chaining technique also provides an easy way to approximate piecewise constant curves.

Formally, given a beamlet $b = (x, y, l, \theta)$ centered at position (x, y), with a length l and an orientation θ, the coefficient of b computed by the Beamlet transform is given by

$$\Phi(f, b) = \int_{-l/2}^{l/2} f(x + \gamma\cos(\theta), y + \gamma\sin(\theta))d\gamma. \quad (2)$$

Equation (2) is closely related to equation (1) since Beamlet transform can be viewed as a multiscale Radon transform; they both integrate image intensity along line segments. However, they do not take into account any line-profile. It implies that the Radon and Beamlet transforms are not well-adapted to represent curvilinear objects carrying a specific line-profile.

The contribution of this paper is to unify the Beamlet transform with linear filtering technique in order to introduce what we call the Feature-adpated Beamlet Transform, which is able to incorporate knowledge about the desired line-profile running along curves. We will see that, if the profile is designed as a steerable filter, our methodology leads to an efficient implementation. Section 2 presents our contribution while section 3 presents the methodology we use for detecting of curvilinear objects in noisy images. Finally, in section 4, we present some experimental results both on edge- and ridge-like features detection that demonstrate the substantial improvement of our method over classical feature detectors.

2. FEATURE-ADAPTED BEAMLET TRANSFORM

Consider a filter h representing a 2-dimensional line-profile. Let h^θ be a rotated version of h in the direction θ:

$$h^\theta(x, y) = h(\mathbf{R}_\theta(\mathbf{x}, \mathbf{y})), \quad (3)$$

where $\mathbf{R}_\theta$ is the 2-dimensional rotation matrix of angle θ. In a first step, we filter our image f with h^θ before computing the beamlet coefficient from equation (2).We have:

$$\Psi(f, b) = \int_{-l/2}^{l/2} f * h^\theta(x + \gamma\cos(\theta), y + \gamma\sin(\theta))d\gamma. \quad (4)$$

A high coefficient means that the local feature runs significantly along b. We call this transform the Feature-adapted Beamlet transform. In general, the computation of all beamlet coefficients is not conceivable, since it requires to convolve the image as many times as the number of θ's. For the special case where h is selected to be within the class of steerable filters [7], we can write h^θ as a linear combination of basis filters:

$$h^\theta(x, y) = \sum_{j=1}^{M} k_j(\theta) h^{\theta_j}(x, y), \quad (5)$$

where k_j's are interpolation functions that only depend on θ. The basis filters h^{θ_j}'s are independant of θ. A convolution of an image with a steerable filter of arbitrary orientation is then equal to a finite weighted sum of convolution of the same image with the basis filters. Hence, equation (4) can be written as

$$\begin{aligned}\Psi(f, b) &= \sum_{j=1}^{M} k_j(\theta) \int_{-l/2}^{l/2} f^{\theta_j}(x + \gamma\cos(\theta), y + \gamma\sin(\theta))d\gamma \\ &= \sum_{j=1}^{M} k_j(\theta)\Phi(f^{\theta_j}, b), \quad (6)\end{aligned}$$

where $f^{\theta_j} = f * h^{\theta_j}$ and $\Phi(f^{\theta_j}, b)$ corresponds to the beamlet coefficient of b computed over f^{θ_j} using equation (2). As a result, in order to compute equation (4) for every beamlet coefficient, we do as follows: we first convolve the image as many times as the number of basis filters composing our filter h. This number is typically very small. On each filtered image, we compute the standard Beamlet transform. Finally, for each beamlet, we compute its coefficient using equation (6). Inspired from the scheme defined in [7], Fig.1 shows the Feature-adapted Beamlet transform diagram.

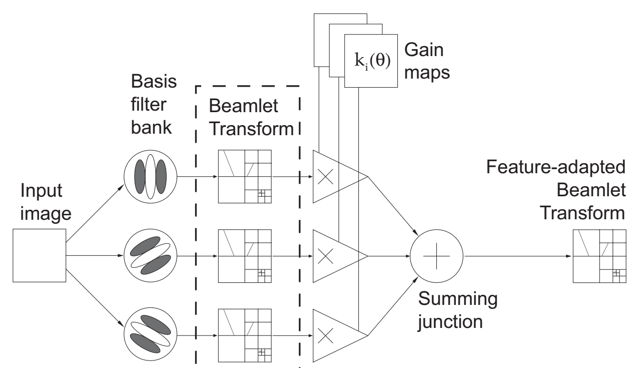


Fig. 1. Feature-adapted Beamlet transform diagram.

All steps relative to a single basis filter can be simultaneously computed on a parallel machine. All these steps have a $O(N^2)$ complexity. In this scheme, the evaluation of beamlet coefficients consumes most of the computation time. To speed up this step, we set an efficient cache strategy to precompute most of the computation and use an approximation of beamlet coefficients based on the two-scale recursion technique [8]. This strategy is quite fast at the expense of a significant memory load. For a 1024×1024 image, our implementation of the standard Beamlet transform takes approximately $1s$ on a dual processors based computer.

3. DETECTION OF CURVILINEAR OBJECTS

In this section, we present a detection method using the Feature-adpated Beamlet transform. This method provides a list of beamlets that best represent curvilinear objects carrying a specific line-profile in an image. It is based on a multiscale coefficient thresholding technique directly taken from [9] so we refer the reader to this paper for more details.

A Recursive Dyadic Partition (RDP) of the image domain is any partition, starting from the whole image domain, obtained by recursively choosing between replacing any square of the partition by its decomposition into four dyadic squares or leaving it unsplit. This concept is very similar to the *quadtree* decomposition technique. A beamlet-decorated RDP (BD-RDP) is a RDP in which terminal nodes of the partition are associated with at most one beamlet. By construction, BD-RDP provides a list of non-overlapping beamlets. In order to select the list of beamlets that best represent curvilinear objects in the image, we maximize over all beamlet-decorated recursive dyadic partitions $P = \{S_1, S_2, ..., S_n\}$ the following complexity penalized residual sum of square:

$$E(P) = \sum_{S \in P} C_S^2 - \lambda^2 \# P, \tag{7}$$

where

$$C_S = \max_{b \in S} \frac{FBT(f, b)}{\sqrt{l}}$$

measures the energy required to model the region S of the image f by the beamlet b and λ is a MDL-like criteria that controls the complexity of the model. A high value of λ yields to a coarse representation of curvilinear structures; a small value leads to a quite complex model with potentially a significant number of false alarms. Equation (7) can be solved very efficiently by a recursive tree-pruning algorithm due to additivity of the cost function. See [9] for complete details.

4. EXPERIMENTS & RESULTS

4.1. Edge detection

In this section, we compare our methodology with a linear filtering technique described in [4] which convolves the image with a steerable filter and resolves for each image point a polynomial equation in order to find the optimal orientation maximizing the filter response. This step is followed by a non-maxima suppression and a thresholding step. This class of steerable filters are designed as a combinaision of Gaussian-based filters which are optimized under Canny-like criteria. We use a 3^{rd} order filter for both experiments (see [4] for details). Fig.2 shows results on a noisy image corrupted by Gaussian white noise with standard deviation $\sigma_{noise} = 50$. For our method, we use the well-known Bresenham algorithm to highlight pixels traversed by meaningful beamlets. In both cases, we determine the threshold value to keep 2,000 pixels only. As we can see in Fig.2.d, the number of false positives is highly reduced with our mehtod.

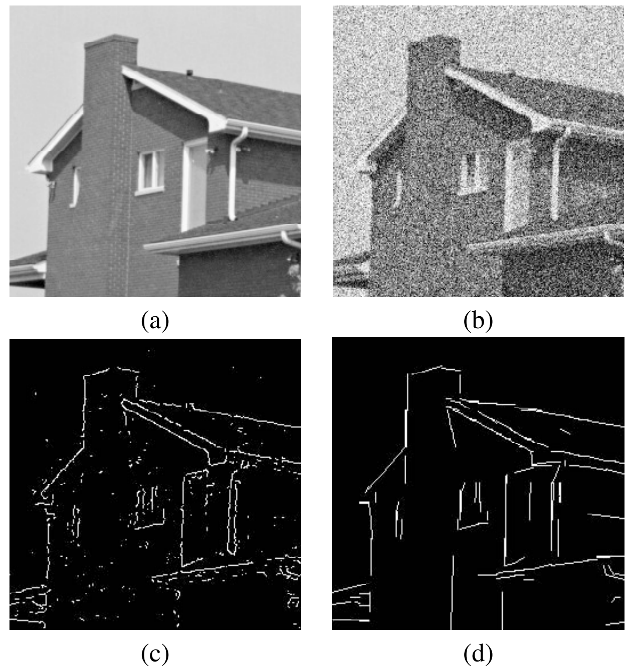
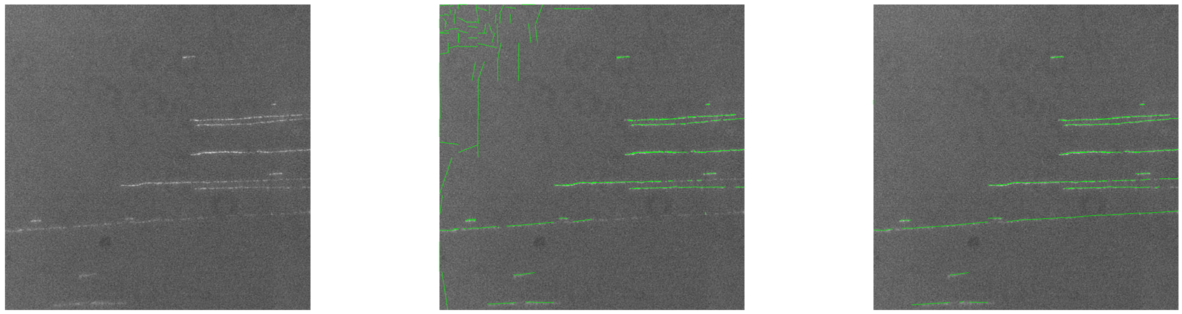


Fig. 2. Edge detection: (a) original image (b) corrupted image with Gaussian white noise $\sigma_{noise} = 50$. (c) Detection using 3^{rd} order edge detector defined in [4]. (d) Detection using feature adapted beamlet transform carrying the same 3^{rd} order filter.

4.2. Ridge detection

We evaluate the performance of the Feature-adapted Beamlet transform compared to the standard Beamlet transform for the detection of multiple lines segments in noisy images. We test these two techniques on images of DNA filaments obtained by fluorescent microscopy. These filaments have a ridge-like profile. For the choice of h, we choose a 2^{nd} order filter defined in [4]. We use the same algorithm described in section 3 for both transforms with $\lambda = 100$. Notice that standard Beamlet transform behaves like a low-pass filter and hence, is sensitive to the background intensity, as opposed to the Feature-adpated Beamlet transform which can cancel constant or more complex background, depending on the vanishing moments of h. In the following experiment, in order to get these two transforms comparable between each other, we suppose the background to be constant and substract it from the image before computing the beamlet coefficients. To do so, we estimate the background mean intensity from the me-



(a) (b) (c)

Fig. 3. Ridge detection: (a) image of DNA filaments obtaining by fluorescent microscopy. (b) Detection using standard Beamlet transform. (c) Detection using Feature-adapted Beamlet transform carrying a 2^{nd} order filter. Notice that spurious detections are reduced with our method.

dian of the image. Fig 3 presents the results. Notice that in the top left corner of Fig 3.b, the spurious detections are due to the fact that real background is not constant over the whole image domain. As can be seen in Fig 3.c, this is not the case for our method.

5. CONCLUSION

In this paper, we have presented a method for detecting features running along lines or piecewise constant curves. Our contribution unifies the Beamlet transform with steerable filtering technique. It leads to an original and efficient implementation of the Feature-adapted Beamlet transform. This transform is very general for representing curves carrying any kind of features designed by *a priori* knowledge. Preliminary results on both edge- and ridge-like profile have shown significant improvements over linear detector techniques and multiscale detection techniques based on traditional Beamlet transform. This work is a first step towards a more in-depth investigation of the method. We point out that statistical hypothesis tests can also be easily incorporated in the coefficient thresholding, so that our detection method is able to control the number of false alarms.

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