DISTANCE MEASUREMENT FROM SINGLE IMAGE BASED ON CIRCLES

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ABSTRACT

Computing distance from captured images is a common task in image analysis and scene understanding. Distance serves as building block for calculating other geometry information, such as area and volume. Besides directly measuring distance on the spot, it can be computed using projective geometry from scene images. Existing work on distance measurement consider point or line constraints. Since circles are common in daily life, we propose two methods of measuring distance using circles in one captured image. The first method deals with two separate circles, which can be coplanar or in parallel planes. The second one handles two concentric circles on the same plane, which offers higher accuracy than directly solving equations of concentric circles. Both methods are verified by experiments with simulated data and real images.

Index Terms— image analysis and processing, single view metrology, projective geometry

1. INTRODUCTION

Measuring world distance from single or multiple images has attracted much attention in the past decade [1, 8, 9]. It is useful in many applications, such as 3D reconstruction [1, 9], plane metric rectification [8], indoor and outdoor measurement [1], and forensic measurement (e.g., investigation of traffic accident).

Distance can be measured manually on the spot using active devices, such as ultrasonic device, laser range finder and structured light device. The results obtained from active devices are heavily affected by unexpected interferences. Hence, these methods are error prone and time consuming. Another measurement approach uses projective geometry [1, 8, 9]. This approach is more accurate, efficient and maintainable. The first step is taking one or more images using digital camera, then measurement method is chosen to obtain distances. With user-friendly GUI, the entire procedure is easy and fast, even for a novice user.

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Previous projective geometry based methods calculate distances using point or line in the scene [1, 8, 9] . However, point and line may be unavailable in some circumstances and could be noisy. Circles are quite common and widely exist in the environment, e.g., manholes on road, cups, wheels, CD disks, etc. They can be fitted robustly from image [7]. In this paper, we propose two methods for distance measurement by taking advantage of circles. The first method is based on two separate parallel circles (separate circles in one plane, or in two parallel planes). The second one is based on two concentric circles on one plane.

The paper is organized as follows. Section 2 gives a brief description of related work about projective geometry based measurement methods. Our methods are elaborated in Section 3. The proposed methods are validated from experiments with simulated data and real images in Section 4. Conclusions and future work are pointed out in Section 5.

2. RELATED WORK

Projective geometry based methods exploit scene constraints on one plane (reference plane) for measuring distance. The reference plane is the plane specified by user and serves as the base for deriving geometry information. Researchers proposed many distance measurement methods that consider different constraints in the scene [1, 8, 9].

Points and lines are common features in the scene, most methods exploit these constraints for distance measurement [1, 8, 9]. Researchers used at least four points on reference plane and their corresponding points on image plane to determine homography between them [4]. Once homography is known, distance can be easily measured. Since computing homography is difficult in some cases only using feature points, several methods are proposed without fully recovering homography using other constraints. These constraints can be vanishing line of reference plane, vanishing point of reference direction [1], and known length ratio or known angles [8, 9].

In some cases, points and lines may be unavailable and could be noisy. Circles are commonly available in the scene and can be fitted robustly from image [7]. Researchers use them for 3D reconstruction, plane metric rectification and camera calibration [2, 3, 5, 6, 7, 10]. However, they overlook using circles for distance measurements. Although the metric rectification method based on circles [7] could be extended to distance measurement by adding reference information, it would generate four possible results. Our two methods generate one unique result without necessity of choosing the correct one. Recently, a unified framework of recovering Euclidean structure from N (N≥2) parallel circles has been proposed [3]. However, the method is complicated, and no measurement results were reported. Our proposed methods are more straightforward and can be implemented easily with high accuracy.

3. TWO MEASUREMENT METHODS BASED ON CIRCLES

3.1. Distance Measurement Based on Two Separate Parallel Circles

In projective geometry, a plane is mapped to its image by a projective transformation. That is, point $\mathbf{X}$ in 3D space is projected to image point $\mathbf{x}$ via a 3×3 non-singular matrix $\mathbf{H}$ as $\mathbf{X} \approx \mathbf{H}\mathbf{x}$, where $\mathbf{X}$, $\mathbf{x}$ are homogeneous 3-vectors and $\approx$ denotes the equality up to a scale. $\mathbf{H}$ is homography and can be decomposed into a concatenation of three matrices, $\mathbf{S}$, $\mathbf{A}$ and $\mathbf{P}$ [4]. They denote similarity (metric), affine and pure projective transformation, respectively.

$$\mathbf{H} = \mathbf{SAP} \ . \tag{1}$$

where

$$\mathbf{S} = \begin{pmatrix} s\mathbf{R} & \mathbf{t} \\ \mathbf{0}^\intercal & 1 \end{pmatrix} \ \mathbf{A} = \begin{pmatrix} \frac{1}{\beta} & -\frac{\alpha}{\beta} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \ \mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{pmatrix}.$$

$\mathbf{R}$ is a rotation matrix, $\mathbf{t}$ is a translation vector, and s is a nonzero isotropic scaling. α and β are affine parameters. $\mathbf{l}_\infty = (l_1, l_2, l_3)^\intercal$ is the vanishing line of reference plane (the image of the infinite line of this plane [4]). Since both $\mathbf{R}$ and $\mathbf{t}$ are only involving coordinates rotation and translation for all points in the entire plane, they do not matter during distance measurement. When $\mathbf{A}$, $\mathbf{P}$ and s are determined, distance between 3D points $\mathbf{X}_1$ and $\mathbf{X}_2$ on the reference plane can be measured from their corresponding 2D image points $\mathbf{x}_1$ and $\mathbf{x}_2$ using the following equation:

$$d(\mathbf{X}_1, \mathbf{X}_2) = s\|\mathbf{AP}(\mathbf{x}_1 - \mathbf{x}_2)\| \ . \tag{2}$$

In projective geometry, α, β and $\mathbf{l}_\infty$ are associated with the image position of two points known as the circular points $\mathbf{I}(1, i, 0)^\intercal$ and $\mathbf{J}(1, -i, 0)^\intercal$. They are the intersection points between any circles in the parallel planes and the infinite line of these planes [4]. They are projected to the image via $\mathbf{H}^{-1}$ in (1) as

$$\begin{cases} \mathbf{m_I} = ((\alpha - i\beta)l_3, l_3, -\alpha l_1 - l_2 + i\beta l_1)^\intercal \ . \\ \mathbf{m_J} = ((\alpha + i\beta)l_3, l_3, -\alpha l_1 - l_2 - i\beta l_1)^\intercal \ . \end{cases} \tag{3}$$

where $\mathbf{m_I}$, $\mathbf{m_J}$ are images of $\mathbf{I}$, $\mathbf{J}$ respectively and referred as the ICPs below. Once the ICPs are obtained, $\mathbf{A}$ and $\mathbf{P}$ in (1) can be determined. For distance measurement on the reference plane, the isotropic scaling s in (1) need to be determined via a known line segment length on this plane.

The ICPs can be computed directly from the images of parallel circles (a circle is projected to an ellipse in the image in general case). If there are two parallel circles, the ICPs are dependent on the relative positions of two circles [10]. They can be identified by quasi-affine invariance when the two parallel circles are separate [10]. In theory, the number of common solutions of two ellipse equations is four. When they are separate or enclosed (but not concentric), it is impossible to directly determine which pair are the ICPs from two pairs of complex solutions. For separate case, the two complex solution pairs form two real associated lines of two circles, respectively. One lies between the images of two circles and the other one does not. If the camera lies in front of the two parallel planes containing the two circles, the associated line not lying between the images of two circles is the vanishing line passing through the ICPs.

Instead of using multiple images of two parallel circles for camera calibration, we employ the same techniques [10] to compute the ICPs for distance measurement. Our measurement method of handling parallel separate circles is outlined as following steps.

1. Establish two ellipse equations as e_1: $\mathbf{x}^\intercal\mathbf{c}_1\mathbf{x} = 0$, and e_2: $\mathbf{x}^\intercal\mathbf{c}_2\mathbf{x} = 0$ by edge detection and ellipse fitting.
2. Calculate the common solutions of e_1, e_2 in complex field.
3. Identify the ICPs $\mathbf{m_I}$, $\mathbf{m_J}$ from two pairs of solution as described in [10].
4. Compute the vanishing line $(l_1, l_2, l_3)^\intercal = \mathbf{m_I} \times \mathbf{m_J}$ for $\mathbf{P}$ in (1).
5. Compute affine parameters α and β for $\mathbf{A}$ in (1) using (3).

Without loss of generality, we take line segment $\mathbf{L}$ with length l as the reference line segment to compute isotropic scaling s. Reformulate $\mathbf{S}$ in (1) as below:

$$\mathbf{S} = \begin{pmatrix} s\mathbf{R} & \mathbf{t} \\ \mathbf{0}^\intercal & 1 \end{pmatrix} = s\begin{pmatrix} \mathbf{R} & \mathbf{t}/s \\ \mathbf{0}^\intercal & 1/s \end{pmatrix} = s\mathbf{S}' \ . \tag{4}$$

where $\mathbf{S}'$ denotes Euclidean transformation which does not change the vector's norm. By referring end points of $\mathbf{L}$ as $\mathbf{P}_1$, $\mathbf{P}_2$ and their corresponding image points $\mathbf{p}_1$ and $\mathbf{p}_2$, s can be computed from following equation:

$$s = \frac{l}{\|\mathbf{AP}(\mathbf{p}_1 - \mathbf{p}_2)\|} \ . \tag{5}$$

Since $\mathbf{A}$, $\mathbf{P}$ and s are known, distance between any two points on the reference plane can be obtained with (2).

3.2. Distance Measurement Based on Two Concentric Circles

In this section, we investigate a measurement approach based on two concentric circles, denoted by $\mathbf{C}_1$ and $\mathbf{C}_2$ with radii R_1 and R_2 ($R_1 > R_2$), respectively. Their image ellipse equations are denoted as $\mathbf{c}_1$ and $\mathbf{c}_2$.

In principle, solving $\mathbf{c}_1$ and $\mathbf{c}_2$ will obtain two identical pairs of solution in complex field. They are the ICPs and can be used to perform distance measurement by the method described in Sect. 3.1. Unfortunately, we can not obtain two identical pairs due to numerical calculation, image noise, edge detection and ellipse fitting precision problems. Two different solution pairs will be generated. Experimental results illustrate that any one of two pairs or the average value of two pairs are not accurate enough to approximate the true ICPs for measurement (Fig.3). The accuracy was low compared with our method described below. Our proposed method is based on pole-polar relationship, and compute the ICPs uniquely from concentric circles. There exists method of computing the ICPs uniquely using SVD decomposition [6], it has comparable performance with ours (Fig. 3). Our method is derived geometrically and easy to understand. It is also beneficial to the work in [7].

A conic $\mathbf{C}$ and a point $\mathbf{X}$ in the plane containing $\mathbf{C}$ define a line $\mathbf{L} = \mathbf{CX}$ via pole-polar relationship. When $\mathbf{C}$ is a circle and $\mathbf{X}$ is its center, $\mathbf{L}$ is the infinite line of the plane containing $\mathbf{C}$ [4] and its image is vanishing line $\mathbf{l}_\infty$. $\mathbf{C}$ intersects $\mathbf{L}$ with circular points $\mathbf{I}$ and $\mathbf{J}$.

Since pole-polar relationship is a projective invariance, we can obtain vanishing line $\mathbf{l}_\infty$ via the image of circle and the image of concentric circle center. Then the ICPs can be uniquely determined by intersecting the image of circle $\mathbf{c}_1$ or $\mathbf{c}_2$ and $\mathbf{l}_\infty$. Similar to the first method, distance measurement can be taken between any two points on the reference plane by knowing a reference length.

For conducting measurement, the remaining unknown variable is the image of concentric circle center $\mathbf{pc}$ (Fig. 1). The image of concentric circle center is not the center of imagery ellipse center, but always lies on a line defined by the ellipses centers [5]. Our method to determine the image of concentric circle center is similar to [2]. We assume that the radii of concentric circles are known. It is easily adjusted when radius *prior* is unavailable [5].

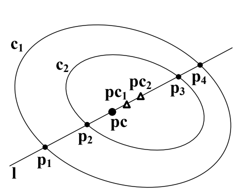

Fig. 1. Projected circle center $\mathbf{pc}$ is on a line $\mathbf{l}$ defined by the centers of $\mathbf{c}_1$ and $\mathbf{c}_2$.

In Fig. 1, the two centers $\mathbf{pc}_1$ and $\mathbf{pc}_2$ of ellipses $\mathbf{c}_1$, $\mathbf{c}_2$ define a line $\mathbf{l} = \mathbf{pc}_1 \times \mathbf{pc}_2$. $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ and $\mathbf{p}_4$ are intersection points between $\mathbf{l}$ and two ellipses. The cross-ratio [4] formed by $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ and $\mathbf{pc}$ is

$$Cr2D = \frac{\Delta \mathbf{p}_1\mathbf{pc} \cdot \Delta \mathbf{p}_2\mathbf{p}_3}{\Delta \mathbf{p}_1\mathbf{p}_3 \cdot \Delta \mathbf{p}_2\mathbf{pc}} \ . \quad (6)$$

where $\Delta \mathbf{p}_i\mathbf{p}_j$ means pixel distance between image points $\mathbf{p}_i$ and $\mathbf{p}_j$. In 3D space, the corresponding scene points of $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ and $\mathbf{pc}$ satisfy

$$Cr3D = \frac{2R1}{R1 + R2} \ . \quad (7)$$

Because the cross-ratio is a projective invariant, we have

$$Cr2D = Cr3D \ . \quad (8)$$

We also have

$$\mathbf{pc}^\intercal \cdot \mathbf{l} = 0 \ . \quad (9)$$

since $\mathbf{pc}$ is on the line $\mathbf{l}$. The image coordinates of the concentric circle center can be computed from (6) to (9).

The measurement method of dealing with two concentric circles is outlined as follows.

1. Establish two ellipse equations as e_1: $\mathbf{x}^\intercal \mathbf{c}_1 \mathbf{x} = 0$, and e_2: $\mathbf{x}^\intercal \mathbf{c}_2 \mathbf{x} = 0$ by edge detection and ellipse fitting.
2. Determine the image of the concentric circle center $\mathbf{pc}$ by the method described above.
3. Calculate vanishing line $\mathbf{l}_\infty = \mathbf{c}_1 \cdot \mathbf{pc}$ according to pole-polar relationship.
4. Compute the ICPs by intersecting vanishing line $\mathbf{l}_\infty$ with ellipse $\mathbf{c}_1$.
5. Determine $\mathbf{P}$ and $\mathbf{A}$ in (1) from vanishing line $\mathbf{l}_\infty$ and the ICPs via (3).
6. Use known radius $R1$ or $R2$ to obtain isotropic scaling factor s.
7. Use (2) to measure distance on the reference plane.

4. EXPERIMENTS

4.1. Experiment with Simulated Data

The simulated camera's internal parameters are $f_u = 1200$, $f_v = 1000$, $s = 0$, $u_0 = 512$, $v_0 = 384$. Rotation axis $\mathbf{r} = (30, 40, 20)^\intercal$, rotation angle $\theta = \pi/6$ and translation vector $\mathbf{t} = (10, -6, -160)^\intercal$. The image size is 1024×768 pixels. We simulate the world scene with a reference plane $Z = 0$ and three circles $\mathbf{C}_1 : (X-20)^2 + (Y-20)^2 = 16^2, Z = 0$; $\mathbf{C}_2 : (X-20)^2 + (Y-20)^2 = 4^2, Z = 0$ and $\mathbf{C}_3 : (X-60)^2 + (Y-20)^2 = 4^2, Z = 5$. Gaussian noise with mean 0 and standard deviation ranging from 0 to 4 pixels is added to the image points of three circles. For each noise level, 50 space lines on reference plane are randomly generated and 50 times independent trials are performed. We validate the first method using $\mathbf{C}_1$ and $\mathbf{C}_3$. The average result is shown in Fig. 2. The second method is verified using $\mathbf{C}_1$ and $\mathbf{C}_2$. We also compare it with four ICPs computation methods in Fig. 3. From statistic results, both methods proposed are accurate enough even with high degree of noise.

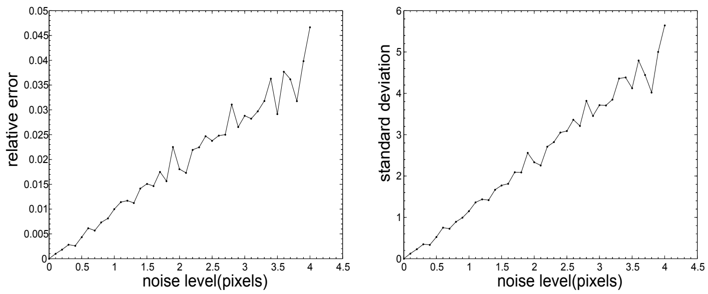

Fig. 2. The first method handling separate parallel circles: relative error and standard deviation versus pixel noise level.

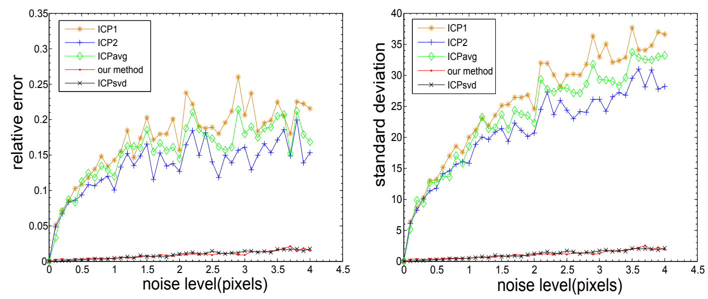

Fig. 3. The second method dealing with two concentric circles: relative error and standard deviation versus pixel noise level. It is compared with four ICPs computation methods (taking any one of two solution pairs of two ellipse equations, the average value of them as approximation of the ICPs, and the method based on SVD decomposition [6], referred as ICP1, ICP2, ICPavg and ICPsvd, respectively (Sect.3.2)).

4.2. Experiment with Real Images

We took two pictures with Canon EOS 20D digital camera for real tests. The resolution of image is 2544×1696 pixels. Figure 4 shows the test images. The circles chosen for measurement are marked by red color, and one of them is partially occluded. The left image is used to verify the first method, and the length of reference line segment R is 50cm. The right one is used to verify the method handling two concentric circles. The radii of outer and inner circles of CD are $R_1 = 6cm, R_2 = 1.5cm$, respectively. The estimated lengths of six line segments are listed in Table 1.

Fig. 4. Two experimental real images.

Table 1. Measurement results from real images.

Line segment in image	S_1	S_2	S_3	S_4	S_5	S_6
Real distance(cm)	65.0	65.0	1812.0	60.0	23.5	30.0
Measured distance(cm)	64.12	64.52	1746.50	59.22	23.62	29.66
Relative error(%)	1.35	0.74	3.61	1.30	0.51	1.13

5. CONCLUSIONS AND FUTURE WORK

In this paper, we proposed two methods of measuring distance based on circles from single image. The first method handles two separate parallel circles while the second one deals with two concentric circles. Simulated and real data experiments verify that our proposed methods offer high accuracy and robustness. They are useful for directly measuring distance between two points on reference plane from single uncalibrated image. In the future work, we will generalize our methods to handle more complex cases, such as enclosed circles, circles in non-parallel planes and so on.

6. REFERENCES

[1] A. Criminisi, I. Reid and A. Zisserman, "Single view metrology," *IJCV*, 40(2):123-148, 2000.

[2] V. Fremont and R. Chellali, "Direct Camera Calibration Using Two Concentric Circles from a Single View," In *Proc. Int. Conf. Artificial Reality and Telexistence*, pp. 93-98, 2002.

[3] P. Gurdjos, P. Sturm and Y.H. Wu, "Euclidean Structure from N≥2 Parallel Circles," In *Proc. ECCV*, 2006.

[4] R. Hartley and A. Zisserman, *Multiple View Geometry in Computer Vision*, Cambridge Univ. Press, 2000.

[5] J-S. Kim, H-W. Kim and I.S. Kweon, "A Camera Calibration Method using Concentric Circles for Vision Applications," In *Proc. ACCV*, pp. 515-520, 2002.

[6] J-S. Kim, P. Gurdjos and I.S. Kweon, "Geometric and Algebraic Constraints of Projected Concentric Circles and Their Applications to Camera Calibration," *IEEE Trans. PAMI*, 27(4):637-642, 2005.

[7] M.P. Kumar, C.V. Jawahar and P.J. Narayanan, "Geometric Structure Computation from Conics," In *Proc. Indian Conference on Computer Vision Graphics and Image Processing*, pp. 1-6, 2004.

[8] D. Liebowitz and A. Zisserman, "Metric rectification for perspective images of planes," In *Proc. CVPR*, pp. 482-488, 1998.

[9] D. Liebowitz, A. Criminisi and A. Zisserman, "Creating architectural models from images," In *Proc. Eurographics*, pp. 39-50, 1999.

[10] Y.H. Wu, H.J. Zhu, Z.Y. Hu and F.C. Wu, "Camera Calibration from the Quasi-affine Invariance of Two Parallel Circles," In *Proc. ECCV*, Vol.1, pp. 190-202, 2004.