WAVELET-BASED DESPECKLING OF MEDICAL ULTRASOUND IMAGES WITH THE SYMMETRIC NORMAL INVERSE GAUSSIAN PRIOR

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ABSTRACT

A major problem in medical ultrasonography is the inherent corruption of ultrasound images with speckle noise that severely hampers the diagnosis and automatic image processing tasks. In this paper, an efficient wavelet-based method is proposed for despeckling medical ultrasound images. A closed-form Bayesian wavelet-based maximum a posteriori denoiser is developed in a homomorphic framework, based on modelling the wavelet coefficients of the log-transform of the reflectivity with a symmetric normal inverse Gaussian (SNIG) prior. A simple method is presented for obtaining the parameters of the SNIG prior using local neighbors. Thus, the proposed method is spatially adaptive. Experiments are carried out using synthetically speckled and real ultrasound images, and the results show that the proposed method performs better than several other existing methods in terms of the signal-to-noise ratio and visual quality.

Index Terms— Ultrasound image, speckle noise, wavelet transform, symmetric normal inverse Gaussian distribution, Bayesian maximum a posteriori estimator.

1. INTRODUCTION

Ultrasonography is a popular diagnostic tool used for medical investigations. It is noninvasive, cost-effective, accurate and practically harmless to human body. Unfortunately, ultrasound images are inherently corrupted with speckle noise that makes it difficult to discriminate diagnostically important details such as cysts in breast imagery, and complicates the task of automatic image processing. Thus, reduction of speckle is a major problem in the processing of medical ultrasound images. Various spatial domain filters using local statistics have been proposed in the literature in order to reduce speckle noise especially in synthetic aperture radar and medical ultrasound images [1, 2]. However, these filters often suppress speckle at the expense of blurring the image details. Jain [3] proposed a homomorphic approach wherein the multiplicative speckle noise is converted into an additive noise via logarithmic transformation of the noisy image. Next, a Wiener filter is applied on the log-transformed image. The despeckled image is obtained by applying an exponential operation on the filtered output. However, the process essentially being low-pass filtering, blurs many important details of the image. In recent years, a number of multi-scale wavelet-based methods have been developed for despeckling medical ultrasound images [4–7]. Achim *et al.* [4] proposed a homomorphic method in which the wavelet coefficients of the log-transformed ultrasound image are denoised by a Bayesian minimum mean absolute error (MMAE) estimator which is developed using a symmetric alpha-stable probability density function (PDF) for modelling the coefficients of the log-transformed reflectivity. However, the lack of a closed-form expression for the alpha-stable PDF hampers parameter estimation from the noisy data and increases the complexity of the Bayesian estimation process [8]. In [5], a Bayesian maximum a posteriori (MAP) estimator is developed by using a Gaussian PDF for locally modelling the signal coefficients, whereas the log-transformed speckle with a Rayleigh PDF. Gupta *et al.* [6] have developed a homomorphic method for simultaneous compression and denoising of ultrasound images by modelling the signal coefficients using the generalized Gaussian (GG) PDF. Pizurica *et al.* [7] have proposed a robust multi-scale method for despeckling the ultrasound images by employing a novel generalized likelihood ratio that makes use of the local neighbors in deriving the ratio. In this paper, an efficient wavelet-based method for despeckling medical ultrasound images is proposed. The wavelet coefficients of the log-transformed reflectivity are modelled with a symmetric normal inverse Gaussian (SNIG) PDF while those of the log-transformed noise are assumed to be Gaussian distributed. A Bayesian MAP estimator is obtained in closed-form using the assumed statistics. In order to incorporate the spatial dependency of the wavelet coefficients, the parameters of the SNIG prior are estimated using local neighbors. A simple method is presented for estimating the SNIG parameters.

2. PROBLEM FORMULATION

Let $G(k,l)$, $X(k,l)$ and $N(k,l)$ denote the (k,l)-th pixel of an ultrasound image, the corresponding tissue-reflectivity and the speckle noise. Assuming the speckle noise to be fully developed and independent of X, we can write

$$G(k,l) = X(k,l)N(k,l) + \xi(k,l) \tag{1}$$

where $\xi(k,l)$ is an additive noise (such as sensor noise) [3]. In practice, the additive noise can be ignored [4], and thus, (1) becomes

$$G(k,l) = X(k,l)N(k,l) \tag{2}$$

A fully developed speckle is often modelled by a Rayleigh distribution [6]. The speckle noise can be simulated by low-pass filtering a complex Gaussian random field and then taking the magnitude of the filtered output. A common practice is to perform the filtering with a 3×3 window, since such a short-term correlation is sufficient for modelling real ultrasound images [7]. To convert the multiplicative noise to an additive one, the noisy image is log-transformed yielding

$$G_l(k,l) = X_l(k,l) + N_l(k,l) \tag{3}$$

where G_l, X_l and N_l are the logarithms of G, X, and N, respectively. The distribution of the log-transformed noise can be sufficiently approximated by a Gaussian distribution for a fully developed speckle [6]. Even if the noise is not fully developed, the corresponding distribution can still be considered Gaussian for practical purposes [6].

The conventional multi-scale critically sampled discrete wavelet transform (DWT) is not shift-invariant, thus leading to pseudo-Gibbs phenomena in the denoised image such as undershoots and overshoots at the locations of sharp signal transitions, and lacks good directionality [9]. Recently, the dual-tree complex wavelet transform (DT-CWT) [9] has been proposed in order to overcome these problems. The DT-CWT consists of two parallel real DWTs where the first DWT gives the real part and the second one the imaginary part of the complex coefficient. A single stage decomposition of an image provides seven subbands with one approximation subband, and six detail subbands, three containing the real parts whereas the other three the corresponding imaginary parts of the complex wavelet coefficients. For a J level decomposition, the corresponding detail subbands at level q are denoted as LH_q^{Re}, LH_q^{Im}, HL_q^{Re}, HL_q^{Im}, HH_q^{Re}, and HH_q^{Im}, where $q = 1, 2, \cdots, J$. The DT-CWT is redundant, nearly shift-invariant, provides better directional and rotational selectivity, and is computationally efficient. Due to these advantages, the DT-CWT is employed in the proposed method. Since the wavelet transform is a linear operation, after applying the DT-CWT on (3), we obtain

$$g_q(k,l) = x_q(k,l) + \eta_q(k,l) \tag{4}$$

where $g_q(k,l)$, $x_q(k,l)$ and $\eta_q(k,l)$ denote the (k,l)-th wavelet coefficient at level q of a particular detail subband of the DT-CWT of G_l, X_l and N_l, respectively. For notational simplicity, we will drop the subscripts and use only g, x and η. After the wavelet decomposition, the problem is to reduce the noise term η and preserve the signal, x as much as possible.

3. THE BAYESIAN MAP ESTIMATOR

In order to reduce the noise in the wavelet domain, a Bayesian MAP estimator is developed using the symmetric inverse Gaussian (SNIG) PDF for modelling the signal coefficients. The SNIG PDF is expressed as

$$P_x(x) = A\frac{K_1(\alpha\sqrt{\delta^2 + x^2})}{\sqrt{\delta^2 + x^2}} \tag{5}$$

where $A = \frac{\alpha\delta\exp(\delta\alpha)}{\pi}$, and K_1 is the modified Bessel function of the second kind with index 1 [10]. Among the two parameters, α controls the shape of the distribution and δ is a scale parameter. In order to illustrate the efficacy of the proposed prior, the GG and SNIG PDFs are fitted to the wavelet coefficients of the subband HH_1^{Re} for a log-transformed ultrasound image previously denoised by the method of [7]. The ultrasound image and the denoising software are obtained from http://www.telin.tue.nl/ sanja/. Fig. 1 (a) shows the empirical PDF of the wavelet coefficients along with the fitted GG and NIG PDFs. The corresponding values of the Kolmogorov-Smirnov (KS) statistic [11] are also obtained as 0.0429 and 0.0614 for the NIG and GG PDFs, respectively. From Fig. 1 as well as from the values of the KS statistic, it is clear that the NIG prior provides a better fit to the empirical distribution then that achieved by the GG PDF. Since the DT-CWT consists of two real orthogonal DWTs, we assume the distribution of the noise coefficients in each DWT is Gaussian with zero mean and standard deviation of σ_η, and denote

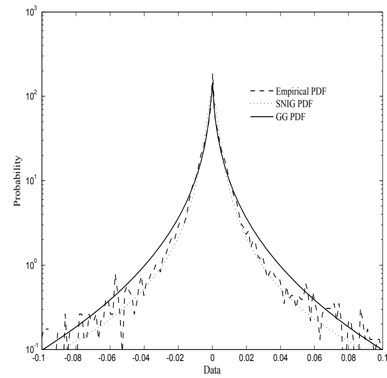

Fig. 1. Empirical PDF of the wavelet coefficients in the subband HH_1^{Re} of the log-transform of a previously denoised ultrasound image (broken line), and the corresponding fitted NIG (dotted line), and GG PDFs (solid line).

it by $P_\eta(\eta)$. The Bayesian MAP estimator is given by [8]

$$\widehat{x}(g) \quad = \quad \arg\max P_\eta(g-x)P_x(x) \tag{6}$$

To obtain the MAP estimate, the derivative of the logarithm of the argument in (6) is set to zero resulting in

$$\frac{x-g}{\sigma^2} + p'(x) = 0 \tag{7}$$

where $p(x) = -\ln P_x(x)$ and $p'(x) = \frac{\partial}{\partial x}p(x)$. Using the approach proposed by Hyvarinen [12], an approximate solution of (7) is obtained as

$$\widehat{x}(g) = sign(g)\max(|g| - \sigma_\eta^2|B|, 0) \tag{8}$$

where

$$B = \frac{2g}{\delta^2 + g^2} + \frac{\alpha g}{\sqrt{\delta^2 + g^2}}\frac{K_0(\alpha\sqrt{\delta^2 + g^2})}{K_1(\alpha\sqrt{\delta^2 + g^2})} \tag{9}$$

We need to estimate the parameters α, δ and σ_η to obtain the MAP estimates. In order to take speckle correlation into account, for each real DWT tree of the DT-CWT, the corresponding value of σ_η is obtained using the coefficients in the corresponding finest subbands of diagonal orientation as

$$\sigma_\eta = C\frac{D1 + D2}{2} \tag{10}$$

where $D1 = MAD(g(k,l))/0.6745), \;\; g(k,l) \in HH_1$, and $D2 = MAD(g(k,l))/0.6745), \;\; g(k,l) \in HH_2$, and C is a smoothing factor. To obtain the SNIG parameters for the (k,l)-th coefficient, the estimates of the second and fourth order signal moments, denoted by $\widehat{m_2}(k,l)$ and $\widehat{m_4}(k,l)$, respectively, are obtained as

$$\begin{aligned} \widehat{m_2}(k,l) &= \max((m_2(k,l) - \sigma_\eta^2), 0) \qquad (11)\\ \widehat{m_4}(k,l) &= \max((m_4(k,l) - 6\widehat{m_2}(k,l)\sigma_\eta^2 - 3\sigma_\eta^4), 0) \end{aligned}$$

The values of $m_2(k,l)$ and $m_4(k,l)$ are obtained using a $D \times D$ square window as

$$m_2(k,l) = \frac{1}{D^2}\sum_{i=-(M)/2}^{(M)/2}\sum_{j=-(M)/2}^{(M)/2} g(k-i, l-j)^2 \tag{12}$$

$$m_4(k,l) = \frac{1}{D^2} \sum_{i=-(M)/2}^{(M)/2} \sum_{j=-(M)/2}^{(M)/2} g(k-i, l-j)^4 \qquad (13)$$

where $M = D-1$. Next, the corresponding second and fourth order cumulants, denoted by $\widehat{K_2}$ and $\widehat{K_4}$, respectively, are obtained as

$$\begin{aligned} \widehat{K_2} &= \widehat{m_2} \\ \widehat{K_4} &= \max((\widehat{m_4} - 3\widehat{m_2}^2), 0) \end{aligned} \qquad (14)$$

The parameters α and δ are estimated as

$$\alpha = \sqrt{3\frac{\widehat{K_2}}{\widehat{K_4}}}, \quad \delta = \alpha\widehat{K_2} \qquad (15)$$

The proposed method can be summarized as follows:

1. Perform log-transformation of the ultrasound image.
2. Apply the DT-CWT on the log-transformed image.
3. Obtain the Bayesian MAP estimates using (8).
4. Obtain the inverse-transform of the MAP estimates.
5. Perform an exponential transformation of the quantities obtained in Step 4.

4. SIMULATION RESULTS

Simulations are carried out in MATLAB in order to study the performance of the proposed method using synthetically speckled and real medical ultrasound images. The performance of the proposed method is compared with those of [7], Bayes-shrink [13], and homomorphic Wiener filter [3]. The value of D in (12) and (13) is set to 9 and 13 for the subbands at levels 4 and 3, respectively, and to 23 for the subbands at levels 1 and 2. The smoothing factor C in (10) is set to 1.5. The first set of synthetically speckled images is obtained by corrupting the classical *Lena* image of 256×256 pixels with noise having various standard deviations. A real ultrasound image obtained from (http://www.telin.tue.nl/ sanja/) is denoised by the method of [7], and then corrupted for various noise standard deviations to obtain the second set of synthetically speckled images. The results of [7] are obtained by using the software available in the same website with optimal parameters. Finally, experiments are carried out with a 256×256 excerpt of an ultrasound image of intraductal papilloma obtained from an online depository (ftp://wuerlim.wustl.edu/pub/dicom/images/.). The signal-to-noise ratio (SNR) [7] is used as the objective performance criterion for the synthetically speckled images. The values of SNR for various methods are provided in Tables I and II. Note that the proposed method gives better values of SNR compared to that obtained by other techniques especially at high noise levels. The denoised images obtained by applying the various methods on a synthetically speckled image from the second set (noise standard deviation of 0.5) are shown in Fig. 2. Images obtained by denoising a real ultrasound image with various methods are shown in Fig. 3. It can be observed from Figs. 2 and 3 that the proposed method not only provides an effective speckle suppression, but also preserves the diagnostically important details. In contrast, the homomorphic Wiener filter blurs the important signal features, whereas the images obtained by the Bayes-shrink [13] method are still noisy. In addition, the proposed method is computationally faster than the method in [7]. For example, in order to process an image of 256×256 pixels on a 2.4GHz Pentium IV machine, the average CPU time required by the proposed method and the method of [7] are 1.20 and 3.59 seconds, respectively.

Table 1. SNR (in dB) for the first set of synthetically speckled images

Std. of noise	Proposed method	Method in [7]	Bayes-shrink [13]	Homomorphic Wiener [3]
0.2	21.72	21.32	21.43	13.81
0.3	19.44	19.13	18.21	13.61
0.4	17.89	17.30	15.72	13.42
0.5	16.57	15.72	14.06	13.10
0.6	15.39	14.39	12.55	12.79
0.7	14.53	13.29	11.24	12.47
0.8	13.62	12.38	10.04	12.13
0.9	12.90	11.47	9.25	11.74
1	12.18	10.61	8.41	11.30

Table 2. SNR (in dB) for the second set of synthetically speckled images

Std. of noise	Proposed method	Method in [7]	Bayes-shrink [13]	Homomorphic Wiener [3]
0.2	22.41	22.18	22.58	12.67
0.3	19.65	19.39	19.20	12.56
0.4	17.71	17.26	16.83	12.42
0.5	16.29	15.59	15.07	12.26
0.6	15.06	14.16	13.56	12.06
0.7	14.16	12.88	12.30	11.82
0.8	13.29	11.91	11.32	11.60
0.9	12.57	10.99	10.32	11.32
1	11.90	10.11	9.41	11.06

5. CONCLUSION

In this paper, we have proposed a spatially adaptive wavelet-method based on the symmetric normal inverse Gaussian (SNIG) PDF for despeckling medical ultrasound images. In this method, the real and imaginary parts of the dual-tree complex wavelet coefficients of the log-transform of the underlying tissue reflectivity have been modelled by using the SNIG PDF, whereas those of the log-transformed speckle with a zero-mean Gaussian PDF. A closed-form Bayesian MAP estimator has been obtained using the assumed prior distributions. Spatial adaptation in wavelet domain is provided by estimating the parameters of the SNIG distribution using local neighbors. A simple method has been provided for obtaining the parameters of the prior distributions. The proposed technique is self sufficient in the sense that the parameters of the prior PDFs are obtained from the noisy data. Experiments have been carried out using both synthetically speckled and real ultrasound images to compare the performance of the proposed technique with some of the existing methods. The simulation results have shown that the proposed method performs better than others in terms of SNR for the synthetically speckled images. For the real ultrasound and synthetically speckled images, it has been observed that the proposed method ensures an effective suppression of speckle noise while retaining diagnostically important details.

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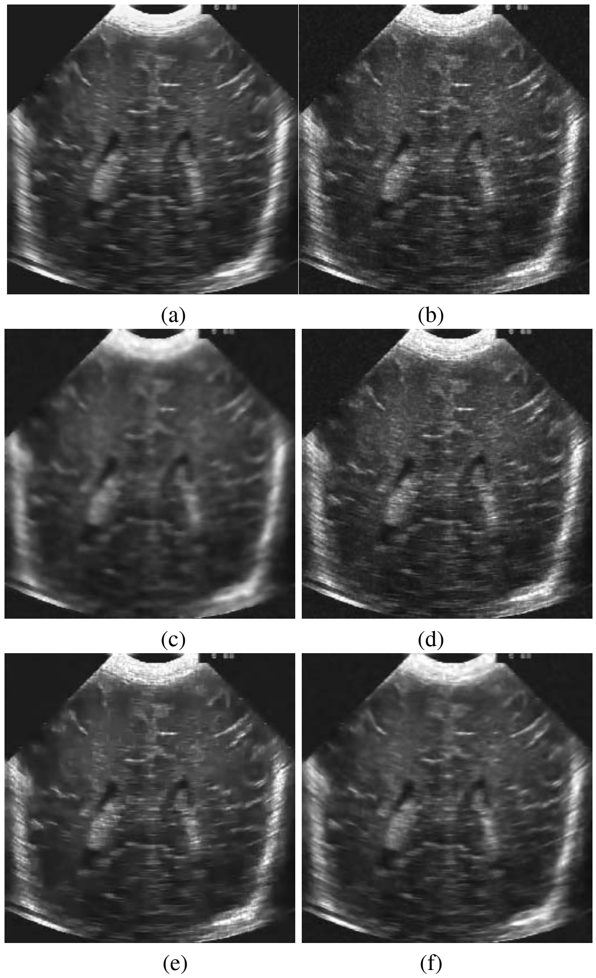

Fig. 2. (a) A clean ultrasound image of 256×256 pixels, and (b) the noisy image for a noise standard deviation of 0.5. Denoised images obtained using (c) the homomorphic Wiener [3], (d) Bayes-shrink [13], (e) the method in [7], and (f) the proposed method.

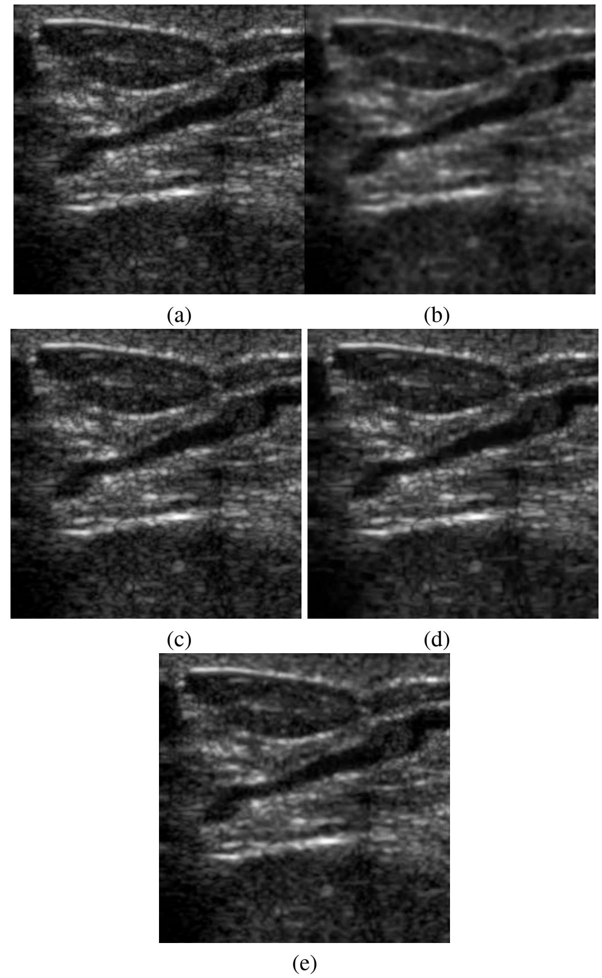

Fig. 3. (a) Noisy ultrasound image of 256×256 pixels. Denoised images obtained using (b) the homomorphic Wiener [3], (c) Bayes-shrink [13], (d) the method in [7], and (e) the proposed method.

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