ADAPTIVE RECONSTRUCTION METHOD OF MISSING TEXTURE BASED ON PROJECTION ONTO CONVEX SETS

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ABSTRACT

This paper presents a missing texture reconstruction method based on projection onto convex sets (POCS). The proposed method classifies textures within the target image into some clusters in a highdimensional texture feature space. Further, for the target missing texture, our method performs a novel approach, that monitors the errors caused by the POCS algorithm in the feature space, and adaptively selects the optimal cluster including similar textures. Then, the missing texture is restored from these similar textures by a new POCSbased nonlinear subspace projection scheme. Consequently, since the proposed method realizes the nonconventional adaptive technique using the optimal nonlinear subspace, the accurate restoration result can be obtained. Experimental results show that our method achieves higher performance than the traditional method.

Index Terms— Image restoration, image texture analysis, interpolation, nonlinear estimation.

1. INTRODUCTION

Reconstruction of missing areas is a very important topic in the fields of image restoration because it can be applied to a number of fundamental applications. For example, it is applied to removal of unnecessary objects such as superimposed text, restoration of corrupted old films, and interpolation of missing blocks transmitted in errorprone environment for video communications.

In previous work, several researchers have proposed reconstruction methods of the missing areas. Though most of them can restore missing edges accurately [1], their performance in texture areas is not sufficient for accurate reconstruction. Thus, several texture reconstruction methods have been proposed so far [2]. They reconstruct the missing textures by utilizing statistical estimation, eigenspace methods, etc. However, when some kinds of textures exist in the target image, they cannot successfully reconstruct all the missing textures.

This paper proposes a novel texture reconstruction method based on the theory of projections onto convex sets (POCS). The POCS algorithm has been applied to blocking artifacts reduction in coded images as a nonlinear image restoration method [3]. First, the proposed method defines a new criterion in a high-dimensional feature space [4] and classifies textures within the target image into some clusters. Further, for the missing texture, we adaptively select the optimal cluster including similar textures. In this procedure, a novel approach that monitors the errors caused by the POCS algorithm in the feature space is introduced into the selection scheme. Then, the proposed POCS algorithm reconstructs the missing texture from the selected cluster's textures by a new nonlinear subspace projection technique. Since these similar textures are correctly approximated by using the nonlinear subspace in the least-squares sense, the accurate reconstruction of the target texture can be expected. Consequently, performing the nonconventional approach, that adaptively selects the optimal nonlinear subspace for the target texture, we restore all the missing textures in the target image more accurately than the previous technique.

This paper is organized as follows. Section 2 explains the POCS algorithm. In Section 3, we present a novel texture reconstruction method using the POCS algorithm. In Section 4, experimental results are shown to verify its high performance.

2. POCS ALGORITHM

This section explains the POCS algorithm. The theory of POCS was first introduced in the field of image restoration by Youla and Webb. This algorithm estimates the original image f in the Hilbert space H from its known properties. Given n properties of the original image f, these properties generate n well-defined closed convex sets C_i ($i = 1, 2, \dots, n$). Further, the original image f should be included in all the C_i 's and also the intersection of all the C_i 's C^* , i.e.,

$$f \in C^* = \bigcap_{i=1}^n C_i.$$
⁽¹⁾

It is clear that the intersection C^* is a closed and convex set. Consequently, the estimation of f from the n properties is equivalent to finding at least one point f^* belonging to C^* . Unfortunately, C^* may be nonlinear and complex in structure so that a direct estimation of f^* is almost infeasible. However, as shown in Fig. 1, given the projection operator P_i onto C_i , the iteration

$$f_t = P_n P_{n-1} \cdots P_2 P_1 f_{t-1} \quad t = 1, 2, \cdots$$
 (2)

converges to a limiting point f^* of the intersection C^* for an arbitrary initial element f_0 in H. Note that the operator P_i satisfies

$$\|f - P_i f\| = \min_{g \in C_i} \|f - g\|$$
(3)

where $\|\cdot\|$ denotes the norm in *H*. Then, we can calculate f^* from the *n* properties of the original image by using Eq. (2).

3. POCS-BASED TEXTURE RECONSTRUCTION

This section presents a POCS-based texture reconstruction method. First, in a high-dimensional feature space, the proposed method defines a new criterion and classifies textures within the target image



Fig. 1. An illustration of the recursion in Eq. (2) at work.

into some clusters. Further, using the POCS algorithm which includes an adaptive nonlinear subspace projection approach, we reconstruct the missing textures from the classification results.

In this section, the details of the classification procedure are described in 3.1. Further, in 3.2, we present the reconstruction algorithm of the missing textures.

3.1. Texture Classification Algorithm

In this subsection, the proposed method classifies textures within the target image into *K* clusters as the preprocessing for the reconstruction of the missing textures. First, we clip *N* local images f_i ($w \times h$ pixels, $i = 1, 2, \dots, N$) not including missing textures from the target image and generate vectors \mathbf{x}_i ($i = 1, 2, \dots, N$), whose elements are the raster scanned intensities of them. Secondly, using the Gaussian kernel [4], we nonlinearly map \mathbf{x}_i into the feature space to obtain $\phi(\mathbf{x}_i)$. Finally, the proposed method regards the obtained results $\phi(\mathbf{x}_i)$ ($i = 1, 2, \dots, N$) as the texture feature vectors and performs their classification that minimizes the following new criterion:

$$D = \sum_{k=1}^{K} \sum_{j=1}^{M} ||\phi_{j}^{k} - \mathbf{U}^{k} \mathbf{U}^{k'} \phi_{j}^{k}||^{2}$$
(4)

where ϕ_j^k ($j = 1, 2, \dots, M$) is $\phi(\mathbf{x}_i)$ ($i = 1, 2, \dots, N$) included in the cluster *k*. Further, \mathbf{U}^k satisfies the following equation of the singular value decomposition:

$$\mathbf{\Xi}^{k}\mathbf{H} \cong \mathbf{U}^{k}\mathbf{\Lambda}^{k}\mathbf{V}^{k'}.$$
(5)

In the above equation, $\boldsymbol{\Xi}^{k} = [\phi_{1}^{k}, \phi_{2}^{k}, \cdots, \phi_{M}^{k}]$ and

$$\mathbf{H} = \mathbf{I} - \frac{1}{M} \mathbf{1} \mathbf{1}' \tag{6}$$

where **I** is the $M \times M$ identity matrix and $\mathbf{1} = [1, 1, \dots, 1]'$ is an $M \times 1$ vector. From Eq. (5), the following equation can be obtained.

$$\mathbf{U}^{k} \cong \mathbf{\Xi}^{k} \mathbf{H} \mathbf{V}^{k} \mathbf{\Lambda}^{k^{-1}}.$$
 (7)

Then, Eq. (4) can be rewritten as follows:

$$D \cong \sum_{k=1}^{K} \sum_{j=1}^{M} \|\phi_j^k - \mathbf{\Xi}^k \mathbf{H} \mathbf{V}^k \mathbf{\Lambda}^{k-2} \mathbf{V}^{k'} \mathbf{H} \mathbf{\Xi}^{k'} \phi_j^k \|^2.$$
(8)

Since the columns of \mathbf{U}^k in Eq. (4) are high-dimensional, we cannot calculate it directly. Therefore, we use Eq. (8) for the calculation of D.

From Eq. (5), \mathbf{U}^k is the eigenvector matrix and also the projection matrix onto the eigenspace spanned by these eigenvectors. Thus, the criterion *D* in Eq. (4) represents the sum of the approximation errors of ϕ_j^k ($j = 1, 2, \dots, M$) in their eigenspaces. Further, since we regard $\phi(\mathbf{x}_i)$ $(i = 1, 2, \dots, N)$ as the texture feature vector of \mathbf{x}_i , the squared errors in Eq. (4) correspond to the differences of textures. Therefore, the new criterion *D* is useful for the classification of the textures. Then, the proposed method can classify the textures in the target image into *K* clusters.

3.2. Texture Reconstruction Algorithm

In this subsection, the missing textures in the target image are reconstructed by the POCS-based nonlinear subspace projection approach from the classification results in the previous subsection. From the target image, we clip a local image f_0 ($w \times h$ pixels) including a missing texture and generate a vector **x** whose elements are its raster scanned intensities. Further, for the vector **x**, the proposed method calculates its reconstruction result $\hat{\mathbf{x}}$, which satisfies the following two constraints.

[Constraint 1]

In the vector $\hat{\mathbf{x}}$, the known original intensities are fixed.

[Constraint 2]

In the high-dimensional feature space, $\phi(\hat{\mathbf{x}})$ is in the eigenspace whose projection matrix is \mathbf{U}^k . Then, $\hat{\mathbf{x}}$ satisfies

$$\hat{\mathbf{x}} = \phi^{-1} \left(\mathbf{U}^k \mathbf{U}^{k'} \phi(\hat{\mathbf{x}}) \right). \tag{9}$$

In the above equation, ϕ^{-1} represents the inverse mapping (pre-image) from the feature space back to the input space. However, the exact pre-image typically does not exist [4]. Therefore, the proposed method newly introduces the linear map \mathbf{A}^k , which satisfies the following equation, into the calculation scheme of the approximate solution.

$$\mathbf{X}^{k} = \mathbf{A}^{k} \mathbf{\Xi}^{k} \tag{10}$$

where $\mathbf{X}^k = [\mathbf{x}_1^k, \mathbf{x}_2^k, \cdots, \mathbf{x}_M^k]$ and $\mathbf{x}_j^k (j = 1, 2, \cdots, M)$ is $\mathbf{x}_i (i = 1, 2, \cdots, N)$ included in the cluster k. From Eqs. (5) and (10), \mathbf{A}^k can be obtained as follows:

$$\mathbf{A}^{k} \cong \mathbf{X}^{k} \mathbf{H} \mathbf{V}^{k} \mathbf{\Lambda}^{k^{-1}} \mathbf{U}^{k'}.$$
 (11)

Thus, by using Eqs. (7) and (11), Eq. (9) can be rewritten below.

$$\hat{\mathbf{x}} \cong \mathbf{X}^{k} \mathbf{H} \mathbf{V}^{k} \mathbf{\Lambda}^{k-1} \mathbf{U}^{k'} \phi(\hat{\mathbf{x}})$$
$$= \mathbf{X}^{k} \mathbf{H} \mathbf{V}^{k} \mathbf{\Lambda}^{k-2} \mathbf{V}^{k'} \mathbf{H} \mathbf{\Xi}^{k'} \phi(\hat{\mathbf{x}}).$$
(12)

Utilizing the POCS algorithm, the proposed method calculates $\hat{\mathbf{x}}$ that satisfies the above two constraints from the initial vector \mathbf{x} . Note that in the eigenspace used at [Constraint 2], we correctly approximate ϕ_j^k ($j = 1, 2, \dots, M$) classified into the cluster k in the least-squares sense. Therefore, if we can classify $\phi(\mathbf{x})$ of the target local image f, the proposed method accurately reconstructs it by using the eigenspace of the cluster k ($k = 1, 2, \dots, K$) including $\phi(\mathbf{x})$. Unfortunately, since \mathbf{x} contains the missing intensities, $\phi(\mathbf{x})$ cannot be classified by using the algorithm shown in 3.1. Thus, in order to achieve the classification of \mathbf{x} , the proposed method utilizes the following novel criterion as a substitute for Eq. (4).

$$E^{k} = \frac{1}{\operatorname{diag}\left(\overline{\Sigma}\right)} \|\phi(\overline{\Sigma}\hat{\mathbf{x}}) - \phi(\overline{\Sigma}\mathbf{X}^{k}\mathbf{H}\mathbf{V}^{k}\boldsymbol{\Lambda}^{k^{-2}}\mathbf{V}^{k'}\mathbf{H}\mathbf{\Xi}^{k'}\phi(\hat{\mathbf{x}}))\|^{2}$$
(13)

where Σ is a diagonal matrix whose diagonal elements are zero or one and satisfies $\mathbf{x} = \Sigma \hat{\mathbf{x}}$. The criterion E^k exactly corresponds to the squared error converged by the POCS algorithm in the feature



(d)

(e)

(f)

Fig. 2. (a) Corrupted image including text regions, (b) Reconstructed image by the proposed method (28.13 dB), (c) Reconstructed image by the traditional method (25.82 dB), (d) Zoomed portion of the original image, (e) Zoomed portion of (b), (f) Zoomed portion of (c).

space. Therefore, this criterion E^k as well as *D* in Eq. (4) is applicable for the classification of the textures. Then, we can realize the selection of the optimal cluster for the target texture including the missing intensities. Further, the proposed method regards the result $\hat{\mathbf{x}}$ obtained by the eigenspace of the selected cluster as the output. Consequently, performing the nonconventional approach, that adaptively selects the optimal eigenspace for the missing texture, we can restore all the missing textures in the target image accurately.

In this way, we can reconstruct the missing texture in the target local image. The proposed method clips local images ($w \times h$ pixels) including missing textures at even interval from the upper-left of the target image and reconstructs them by using the POCS algorithm. Note that each restored pixel has multiple estimation results if the clipping interval is smaller than the size of the local images. In this case, the proposed method regards the result minimizing Eq. (13) as the final one.

4. EXPERIMENTAL RESULTS

The performance of the proposed method is verified in this section. In the experiments, Fig. 2(a) is a test texture image $(480 \times 359 \text{ pix-els}, 24\text{-bit color levels})$, which includes text "Grand Canyon". Fig. 2(b) shows the reconstruction result of the proposed method. For comparison, Fig. 2(c) shows a result by the traditional eigenspace method using the kernel principal component analysis in [4]. For better subjective evaluation, the enlarged portions around the middle of the images are shown in Figs. 2 (d)–(f). It can be observed that the proposed method has achieved noticeable improvements. In the conventional method, different kinds of textures affect the reconstruction of the target missing textures. On the other hand, selecting the optimal cluster including similar textures, the proposed method can adaptively reconstruct the missing textures from only the reliable

 Table 1. Performance comparison (PSNR) of the proposed method and the conventional method.

Image	Conventional method	Proposed method
Flowers	25.48 dB	26.25 dB
Bark	23.87 dB	24.97 dB
Brick	35.44 dB	37.69 dB

ones. Therefore, the proposed method realizes higher performance than the conventional method. Further, different experimental results are shown in Figs. 3 and 4. Compared to the conventional method, we can restore various kinds of textures accurately.

Further, in order to quantitatively evaluate the performance of the proposed method, we use several texture images [5] and perform the same simulations as Fig. 2. Table 1 shows the $PSNR^1$ of the reconstructed results. From this table, it can be seen that our method has achieved 0.77–2.25 dB improvement over the conventional method. Therefore, high performance of the proposed method was verified by the experiments.

Finally, we verify the computational complexity of the proposed method. Fig. 5 shows the converged errors versus the iteration number in our POCS algorithm. From this figure, the errors are converged in the first few times. Since the POCS algorithm guarantees the strong convergence [3], we can obtain the reconstruction results rapidly. Then, several applications of the proposed method can be expected.

 $^{^{1}\}text{PSNR} = 10 \log_{10} \frac{MAX^{2}}{MSE}$ where MAX denotes the maximum value of intensities and MSE is the mean square error between the original image and the reconstructed image.



Fig. 3. (a) Target image including text "Buildings in Tokyo", (b) Reconstructed image by the proposed method (27.48 dB), (c) Reconstructed image by the traditional method (25.91 dB).



Fig. 4. (a) Target image including text "Blue Sky and White Cloud", (b) Reconstructed image by the proposed method (37.33 dB), (c) Reconstructed image by the traditional method (31.83 dB).



Fig. 5. Errors converged by our POCS algorithm versus iteration number in the experiment of Fig. 2.

5. CONCLUSIONS

This paper has proposed a POCS-based texture reconstruction method. The proposed method classifies textures within the target image in a high-dimensional feature space. Further, for a target missing texture, we select the optimal cluster including similar textures by using the novel approach that monitors errors converged by the POCS algorithm in the feature space. Then, the proposed POCS algorithm performs the nonlinear subspace projection scheme and restores the missing texture from those similar textures. Consequently, since we can adaptively reconstruct the missing textures within the target image from the optimal nonlinear subspaces, the accurate restoration result is obtained. The simulation results show that our method realizes the high reconstruction performance subjectively and quantitatively.

6. REFERENCES

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