EXPLOITING MULTI-FRACTAL AND CHAOTIC PHENOMENA OF MOTION IN IMAGE SEQUENCES: FOUNDATIONS

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ABSTRACT

Accurate and robust image motion detection has been of substantial interest in the image processing and computer vision communities. Unfortunately, no single motion detection algorithm has been universally superior; while biological vision systems are adept at motion detection. Recent research in neural signals have shown biological neural systems are highly responsive to chaotic signals. In this paper, we analyze image sequences using frame-wise phase plots and demonstrate that the changes in pixel amplitudes due to the motion of objects in an image sequence, results in apparently chaotic behavior in phase space. We explore these chaotic phenomena in a variety of image datasets to show their repeatability, to validate the assumption of ergodicity, and to demonstrate their uniqueness from the changes due to illumination, particularly spatio-temporally varying illumination.

Index Terms-- Image motion analysis, Image segmentation, Image sequence analysis, Chaos, Nonlinearities.

1. INTRODUCTION

Detection and segmentation of objects based on their motion in images, including in the presence of spatio-temporally varying illumination has been the subject of intensive research for many years, with two more recent examples provided in [1] and [3]. Unfortunately, none of these algorithms have been shown to be generally superior. One interesting observation is that all of these methods operate on the grayscale imagery. However, a grayscale image is the result of a complex interaction of: (i) the motion of object of interest, (ii) its surface characteristics, and (iii) the external illumination [3]. Since the effects of motion and illumination are both due to underlying multiplicative processes, interrelated through the non-linear interaction of the surface normal with the direction of illumination, it is difficult to separate the effects of illumination and motion.

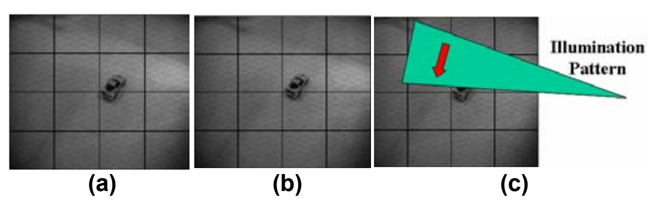
It has been shown that non-linear dynamical systems which are driven by an underlying multiplicative process often exhibit chaotic behavior [9][10]. Chaos theory has been successfully used to model many naturally occurring processes in physics, and most recently have also found success in modeling the biological neural activity [12][13][14] In this paper we demonstrate that the effects of motion in images have chaotic behavior, while the effects of illumination remain deterministic; thereby making it possible to robustly distinguish illumination changes from the changes due to the motion. Also, the demonstration of chaotic behavior in image motion sequences may have a profound impact in providing insight into the understanding why many biological vision systems are so adept at detecting motion.

2. BACKGROUND OF IMAGE MODELING

There are two elements to the motion segmentation problem that a successful algorithm must provide: (i) sensitivity to motion of the objects in the image, and (ii) insensitivity to the effects of spatio-temporal illumination changes, which are demonstrated in Figure 1. Various researchers have modeled illumination changes as multiplicative linear effects [2][4], and under the simple Lambertian model, effect the scene radiance according to[1]:

$$L_m = \rho \vec{N} \cdot \vec{I} \cdot \lambda , \tag{1}$$

where λ is the scale change to the illumination and L_m is the resulting radiance. Both the author in [1] and Cho and Kim in [2] have verified this multiplicative model. The changes in radiance due to the motion of an observed object however result in *non-linear* multiplicative effects through the product of the surface normal with the illumination source as the object moves in the scene. This has also been verified by Xu and Roy-Chowdury who state: "[the changes in the observed coordinate points] is a non-linear function of the [rotational and translational] motion variables" [5]. They developed a bi-linear model to explain the effects of motion and illumination, under the assumption of small changes to support linearizing this non-linear relationship. Rather than relying on linearization, we will utilize methods from analyzing chaotic systems to attempt to exploit these non-linearities, particularly to allow us to robustly separate the effects of motion from the effects of spatio-temporally varying illumination.



(a) (b) (c)

Figure 1: Spatio-temporal illumination: (a) first image, (b) second image, and (c) graphic showing moving light band.

3. CHAOS AND MULTI-FRACTALITY IN NEURAL SYSTEMS

There is substantial evidence that signals in biological neural pathways have a chaotic nature [12][13][14]. As stated by Nagao, et al. 'diverse types of chaos have been confirmed at several hierarchical levels in the real neural systems from single cells to cortical networks' [13]. They proposed a chaotic model to explain the problem of alternation in perception of ambiguous objects, where each possible explanation of the viewed object represents a distinct basin in a chaotic attractor, and the neural system transitions between these basins [13]. Likewise, earlier work by Freeman proposed that the olfactory system exhibits chaotic behavior, and then used this model to explain the ability of biological systems to recognize a single scent from a complex background of scents. They likewise found evidence of chaotic behavior in visual perception, where Freeman observed: 'the images [of EEG brain activity] suggest that an act of perception consists of an explosive leap of the dynamical system from the "basin" of one chaotic attractor to another' [12].

There is a strong relationship between chaos and fractals, as mentioned by Peitgen [9]: "as geometrical patterns, strange attractors are fractals; [while] as dynamical objects, strange attractors are chaotic." Consequently, measures of multi-fractality can be an indicator of possible chaotic behavior in systems [9]. For example, recently many researchers have been "using fractal theory to…characterize neuronal dynamics" [14]. Zheng et al. have demonstrated that the neural firings in the human brain "are consistent with a multi-fractal process…" and that signals emanating from these regions are distinguishable through multi-fractal analysis [14].

The following section provides evidence that the effect of motion of objects in an image sequence on pixel amplitude does indeed exhibit multi-fractal and possibly chaotic behavior, which may provide some insight into explaining the exceptional ability of most biological vision systems to detect and exploit motion.

4. CHAOTIC PHENOMENA IN IMAGE SEQUENCES

Physical systems with underlying multiplicative processes have been shown to have the potential to exhibit chaotic behavior, and as Equation (1) demonstrates, the perceived amplitude of a pixel in an image is a non-linear multiplicative process (non-linear through the vector dot product) [9]. Recent research by the author in [1] has shown that motion between image pairs exhibits interesting chaos-like behavior when viewed in the joint-histogram domain between the two images, as shown in Figure 2.

Analysis of the image pairs in the joint histogram domain was motivated by the recent popularization of mutual information for image registration. In these applications, the mutual information between two images is defined as [6]:

$$I(A;B) = \sum_{a,b} p(a,b) \cdot \log\left(\frac{p(a,b)}{p(a)p(b)}\right), \qquad (2)$$

where $p(a)$ and $p(b)$ are the distributions of images A and B, and $p(a,b)$ is the joint distribution of images A and B, and a is the intensity of a pixel in image A and b is the corresponding intensity of the same pixel in image B. Also, the joint distribution of two images can be approximated by using the joint histogram $S(a, b)$, where a and b are the grey levels in the respective images and the pair (a, b) provides the coordinate for the entry in the histogram.

Researchers have shown a tangible connection between information creation and chaos, with Eckmann and Ruelle stating, "the average rate of information production in an ergodic state is related to sensitive dependence on initial conditions", which is a well known characteristic of chaotic systems [9][10]. In particular, researchers have shown that the mutual information between data sets can be used to "provide a quantitative characterization of chaotic spatial patterns" [11]. Additionally, Peitgen, et al, states that chaotic behavior (particularly the existence of strange attractors) of dynamical systems can be detected in the phase plot of the system, which is the 2-dimensional plot of the state variable versus its velocity over time [9]. Since the key quantity in estimating the mutual information between two signals is the joint density function, which can be estimated through the joint histogram, the joint histogram image is actually an alternate representation to the phase plot.

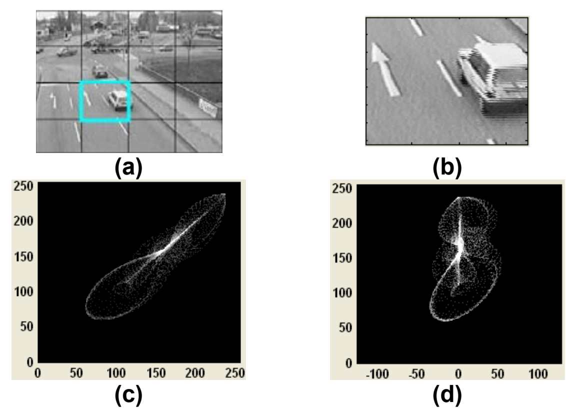


Figure 2: Scattegrams vs. phase plots, (a) start image, (b) mosaic from start image, (c), histogram of (b), and (d) phase plot of (b).

Figure 2 for a sub-sequence of the 'Reinhafen' [7] outdoor traffic sequence, where Figure 2 (b) provides a zoom of the specific the mosaic within the image being processed. It is important to process image regions (called mosaics in this paper) rather than the entire image since the phase plot of an entire image is often too complex due to the variety of simultaneously occurring events. Figure 2 (c) shows the joint histogram between the two successive image frames, and Figure 2 (d) shows the corresponding frame-wise phase plot between the two images (display of image intensity versus the change in intensity). Note the duality of the two measures is immediately apparent.

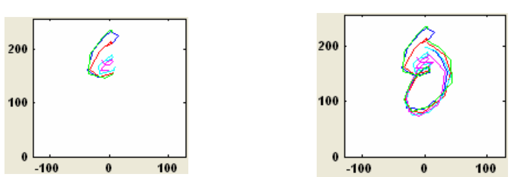
Phase plots traditionally map the trajectory of a single point over time, while here the phase plot maps the intensity-versus-change in intensity for all the pixels between two adjacent image frames. The term frame-wise phase plot is used here to differentiate it from the traditional temporal phase plot. We observe here, in particular, the non-deterministic nature of the signal, where it shows highly fractal characteristics [9].

Likewise, traditional temporal phase plots capture the trajectory of the system state variable of a dynamical system over time. Figure 3 highlights a collection of closely spaced pixels (5 pixels spacing) which will be tracked over 24 time samples as a vehicle moves through the image. Figure 4 (a) shows the resulting temporal phase plot of this set of pixels tracked between the frame in Figure 3 (a) and the frame in Figure 3 (b) (12 frames later), while Figure 4 (b) captures the complete trajectory of the pixels over the entire image sequence through the frame in Figure 3 (c) (24 frames total). Figure 5 provides a similar comparison of the frame-wise versus temporal phase plots for a sequence of 20 frames from the CAVIAR [8] image set highlighting the trajectories taken by the pixel amplitudes in two different regions in the image. The image sequence in Figure 5 is particularly interesting due to the motion of human subjects both in the region of nominal illumination and in the region of extreme, nearly saturated illumination. The set of points selected in the brighter region occupies a space where one of the subjects walks through during the time interval. Clearly the phase plots in Figure 2 and Figure 5 exhibit both a fractal and a chaotic nature, which we will demonstrate are quite distinct from the phase plots of pixel amplitudes experiencing illumination changes [10].

There are three interesting phenomena to observe from Figure 2, Figure 4, and Figure 5; (i) the parallelism between the frame-wise phase plots in Figure 2 (d) and the conventional temporal phase plots in Figure 4 (b),which is also seen by comparing Figure 5 (b) with Figure 5 (d) and (f), (ii) the sensitivity to the initial pixel location of the trajectory through phase space, and (iii) the transition of Figure 4 (b) into a different region (in comparison to Figure 4 (a)) of the phase plot, as well as localization of the phase trajectories in Figure 5, where there are two distinct regions in the phase corresponding to the two groups of moving objects in the image. These effects of local areas of attraction in the phase plot are examples of basins of attraction.

(a) (b) (c)

Figure 3: Image sequence at the beginning, middle, and end of time window; where motion is present at labeled pixels.



(a) (b)

Figure 4: Temporal phase plot of five closely spaced image points, (a) for sequence from Figure 3 (a) through (b), and (b) for entire sequence through Figure 3 (c).

The strict parallelism between the frame-wise phase plots of Figure 2 (d) and the conventional temporal phase plots in Figure 4 (b) and Figure 5 (d) and (f) have tremendous significance. This demonstrated parallelism is a manifestation of *ergodicity* where the temporal behavior of a single particle can alternatively be analyzed by viewing a spatial average over an ensemble of particles [10]. Ergodicity is a critical concept in analyzing chaotic systems, and can be expressed mathematically by [9]:

$$\lim_{m \to \infty} \frac{1}{m+1} \sum_{k=0}^{m} g(x_k) = \int_A g(x) d\mu(x) \qquad (3)$$

where m is the size of the ensemble, $g(x)$ is the ergodic quantity being computed, x is the state space variable, and μ is the measure of integration over the region A of the phase plot. While ergodicity is a well accepted assumption in many physical systems, it can rarely be visually demonstrated.

Item (ii) can be seen by following the trajectories of pixels at closely spaced locations in the images over time, particularly visible in Figure 5. The sensitivity to initial conditions of the trajectories in the phase space is a key indicator of the possible presence of chaos in a dynamical system [9][10].

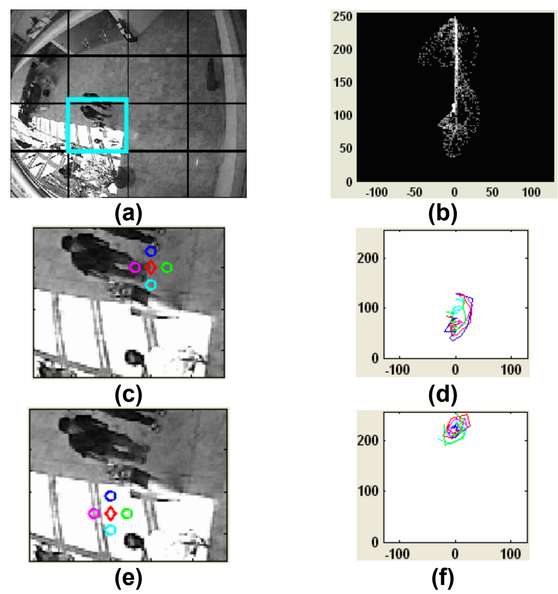


Figure 5: Scattegrams versus phase plots, (a) start image, (b) frame-wise phase plot, (c) mosaic from start image, (d) temporal phase plot of five closely spaced image points, (e) mosaic from start image, (f) temporal phase plot of five closely spaced image points.

Item (iii), addresses the phenomenon shown in Figure 4 (a) and Figure 4 (b) where the trajectory of the pixels transition from one region (or basin) of the phase plot into another basin as the pixels transition from capturing the reflectance of the vehicle, to capturing the reflectance of the shadow region. This is an interesting demonstration that may help explain Freeman's noting the presence of leaps in signals in the neural system between basins of chaotic attractors during perception [12]. Thus, it may be that the images of objects and their shadows are related through the interaction of two attractor basins.

While the key symptoms of chaos, namely sensitivity to initial conditions and distinct basins of attraction is demonstrated from the effects on the dynamics of the amplitudes of image pixels due to motion, Figure 6 shows the effects on pixel amplitudes due to illumination changes. Figure 6 (c) shows that the phase plot corresponding to illumination is tightly packed and more deterministic than the chaotic phase plot associated with motion from Figure 2 (d) and Figure 5 (b). Likewise, the pixel amplitudes captured in the temporal phase plot also clearly follow a more deterministic non-chaotic trajectory for illumination changes. Due to space limitations, only a few image examples have been provided, however, numerous standard image databases (from [7] and [8]), and custom illumination data sets have all verified the repeatability of the distinct *fractal and chaotic* nature of the phase space trajectories for motion sequences, versus the *deterministic* phase space trajectories caused by spatio-temporally varying illumination.

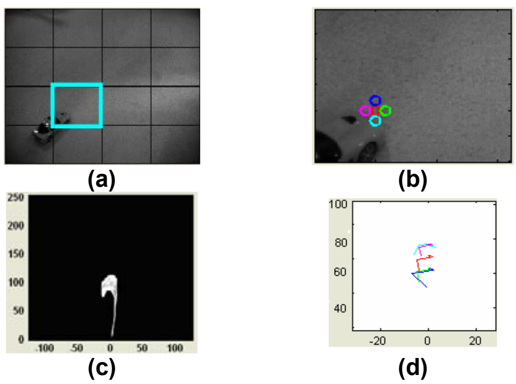


Figure 6: Effects of illumination change, (a) image with moving illumination band but no motion, (b) mosaic region to analyze, (c) graphic of spatio-temporal illumination, (d) frame-wise phase plot, and (e) temporal phase plot.

We are ultimately interested in segmenting motion in image sequences with spatio-temporal illumination changes. Consequently, Figure 7 demonstrates the effects on the phase plot of an image sequence with both motion and illumination present from the image pair shown originally in Figure 1, and the specific mosaic being processed is provided in Figure 7 (a). In Figure 7 (b) the non-chaotic characteristic of the upper portion of the phase plot is clearly visible (this region corresponds to the pixels in the image mosaic with illumination change), which is in sharp contrast to the highly chaotic nature of the lower portion of the phase plot (corresponding to the object motion).

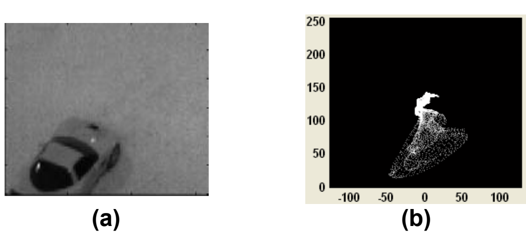


Figure 7: Example of multi-fractal image, (a) image mosaic from first frame, (b) phase plot.

5. CONCLUSIONS & FUTURE WORK

We proposed that motion in images may cause chaotic changes in the image pixel amplitudes. This chaotic nature was witnessed both in the temporal phase plots of single pixels over time and through ensemble phase plots from adjacent image frames. Hence, we demonstrated support for the ergodic nature of pixel amplitude changes in images due to motion. Ergodicity will play a key role in supporting the development of measures suitable for motion detection and segmentation. We also demonstrated that the effects due to illumination changes are non-chaotic and follow deterministic trajectories in phase space. Our ongoing research is addressing possible measures to quantify the chaotic nature of the motion signals to detect motion and differentiate motion changes from illumination changes for the purpose of motion segmentation, and once these measures are finalized, further work will support the integration of the output of the chaos-based motion detection with means for accurately estimating the actual pixel velocities.

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