WAVELET IMAGE DENOISING BASED ON IMPROVED THRESHOLDING NEURAL NETWORK AND CYCLE SPINNING

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ABSTRACT

In this paper we propose a new method for image noise reduction based on wavelet transform. In this method we introduce an improved version of thresholding neural networks (TNN) by utilizing a new class of smooth nonlinear thresholding functions as the activation function. Using this approach we will find the best thresholds in the sense of minimum mean square error (MMSE). Then using TNN with obtained thresholds, we employ a cycle-spinning-based technique to reduce image artifacts. Experimental results indicate that the proposed method outperforms several other established wavelet denoising techniques, in terms of Peak-Signal-to-Noise-Ratio (PSNR) and visual quality.

Index Terms— Image enhancement, wavelet transforms, neural networks.

1. INTRODUCTION

Noise reduction is one of the most common and important preprocessing steps in many image and video systems. The corruption of images by noise is common during its acquisition or transmission. Thus the aim of denoising is to remove the noise while keeping the important image features such as edges as much as possible.

Recently, a vast amount of papers in the literature has been published on image denoising using wavelet based nonlinear techniques [1-14]. Donoho and Johnstone [1-2] introduced a new method, known as wavelet shrinkage, which consists of transforming the noisy image into an orthogonal domain by 2-D discrete wavelet transform. The wavelet coefficients smaller than a given amplitude are suppressed (soft or hard thresholding). The 2-D inverse discrete wavelet transform is performed to get the denoised image. This approach can significantly reduce noise, due to the excellent localization property of wavelet transforms which concentrates signal energy on a few number of wavelet coefficients. However, this method exhibits visual artifacts and oscillations in the vicinity of discontinuities, called pseudo-Gibbs phenomena. Therefore, many variants of the wavelet shrinkage techniques were developed.

One of the methods to find the optimum threshold values is by a method called Thresholding Neural Network (TNN), introduced by Zhang [3-5]. In this method, wavelet coefficients of the corrupted signal are applied to TNN to perform thresholding by using a class of smooth nonlinear functions. In this method, thresholds are adaptively adjusted for a given nonlinear function.

To reduce the pseudo-Gibbs artifacts, another improved method has been proposed by Coifman and Donoho [6], known as cycle spinning. This method consists of applying the thresholding process to translated versions of the original image and averaging. As the wavelet transform is not translation invariant, this approach will result in different estimates of the original image with statistically different noises, which will be reduced by averaging.

In this paper, we propose a two step denoising scheme based on these two methods, TNN and cycle spinning. Also a new class of smooth nonlinear functions is developed as the activation function for TNN. In the first step thresholds are adaptively adjusted for the given nonlinear function. Then cycle spinning algorithm, with TNN as its thresholding parameter, will be used to improve the visual quality and reduce artifacts. Experimental results indicate that the proposed method outperforms TNN, cycle spinning and several other established wavelet denoising techniques, in terms of PSNR and visual quality.

2. WAVELET DENOISING THECHNIQUES

2.1. Thresholding Neural Network (TNN)

Zhang introduced TNN to find the optimum threshold values in the transform domain to achieve noise reduction [3-5]. The neural network structure of the TNN is shown in Fig. 1. The input of the TNN is noisy samples, $y_i = x_i + n_i$, where x is the true signal and n is additive noise. The transform shown in Fig. 1 is an orthogonal wavelet transform. Here the thresholding function $\eta(x,t)$ is employed as nonlinear activation functions of the neural network. Zhang suggested a class of activation functions as follows.

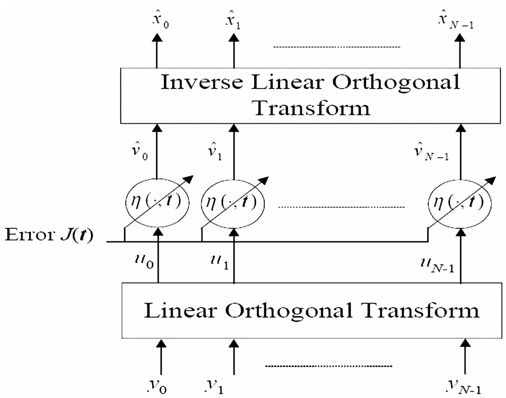


Fig. 1. Zhang's thresholding neural networks (TNN) [3].

$$\eta_\lambda(x,t) = x + \tfrac{1}{2}\left(\sqrt{(x-t)^2+\lambda} - \sqrt{(x+t)^2+\lambda}\right) \qquad (1)$$

This function is the smooth version of the soft-thresholding function which is an entire function for $\lambda > 0$. In TNN algorithm a neural network based scheme is used to obtain the estimate $\hat{v}_i$ of the true image DWT coefficients v_i, which minimize the MSE risk.

$$J(t) = E\{(\hat{v}_i - v_i)^2\} = \frac{1}{N}\sum_{i=1}^{N}[\eta_\lambda(u_i,t_i) - v_i]^2 \qquad (2)$$

where u_i and v_i denote the data stream of the 2-D DWT coefficients of the input noisy image y and the true image x, respectively. Also, t_i is the threshold used for the i^{th} wavelet coefficient, which will be adjusted by TNN to minimize the risk $J(t)$. Since we do not have information about the original image x and cannot utilize its DWT coefficients v_i as reference to estimate the risk $J(t)$, Zhang has suggested a practical approach to this problem. His suggestion is to use another noisy image y' as the reference. This image is obtained from the same true image x plus the noise term n' that is uncorrelated to n. This is a reasonable assumption, since in some applications we may have an array of sensors and obtain more than one corrupted version of the signal [3]. Zhang proved that using such noisy reference signal leads to the same optimum threshold as using the true signal [4]. Also if we have not any available reference signal, it is possible to use TNN. Zhang suggested using *Stein's Unbiased Risk Estimate* (SURE) as an estimator of the MSE [5].

In TNN, gradient-based LMS stochastic adaptive learning algorithm is used to obtain the optimum thresholds. To do so, in the j^{th} iteration, the threshold parameter t at position i is adjusted by $t_i^{j+1} = t_i^j - \Delta t_i^j$ where

$$\Delta t_i^j = \alpha_i^j . \frac{\partial \hat{v}_i^j}{\partial t} . \varepsilon_i^j \qquad (3)$$

where α_i^j is a learning parameter, $\varepsilon_i^j = \hat{v}_i^j - v_i'$ is the instantaneous error for i^{th} wavelet coefficient and v'_i denotes the data stream of the 2-D DWT coefficients of the reference image y'. Thus the optimum thresholds which minimize the risk $J(t)$ is obtained and can be used to denoise the image.

2.2. Cycle Spinning

The basic thresholding functions of Donoho [1-2] exhibits visual artifacts and oscillations in the vicinity of signal discontinuities, called pseudo-Gibbs phenomena. Therefore, Coifman and Donoho [6] proposed an improvement to basic wavelet thresholding called cycle spinning. This method utilizes the shift variant property of wavelet transform. In this algorithm by using different shifts of the noisy image, we can compute different estimates of the unknown signal, and then linearly average these estimates. As the wavelet transform is not translation invariant, this approach will result in different estimates of the original image with statistically different noises, which is reduced by averaging.

If we denote the 2-D circular shift by $S_{i,j}$, the wavelet transform by W, and the threshold operator by T, the cycle spinning will be performed as :

$$\hat{y} = \frac{1}{k_1 k_2}\sum_{i=1,j=1}^{k_1,k_2} S_{-i,-j}(W^{-1}(T(W(S_{i,j}(y))))) \qquad (4)$$

where k_1 and k_2 are the maximum number of shifts which would cause an improvement in denoising. The maximum numbers of effective shifts will be equal to the number of decomposition levels used for wavelet transform [7]. This approach reduces the image artifacts.

3. PROPOSED METHOD

Now we will describe the proposed method. In this method, first we use an improved version of thresholding neural network (TNN) with a new class of smooth nonlinear thresholding functions as the activation function. After this process we use the optimum thresholds obtained from the TNN for the thresholding step of cycle spinning.

Zhang [3] used the smooth version of the soft-thresholding function as the activation function in TNN. Also we know that basic hard-thresholding could excellently preserve the detailed characteristics of the image edges, while producing more artifacts than soft-thresholding. On the other hand, soft-thresholding yields a smoother image, but it can create distortion along the image edges [8]. Considering this fact, we present a new type of smooth differentiable hard-thresholding, which can keep the good properties of the standard hard-thresholding, i.e.,

$$\eta_b(x,t) = \begin{cases} a(e^{b|x|}-1).\mathrm{sgn}(x) & |x| \le t \\ (|x| + ce^{-b|x|}).\mathrm{sgn}(x) & |x| > t \end{cases} \qquad (5)$$

Fig. 2 shows this function for various values of b. As we see this function is the smooth version of the basic hard-thresholding and estimates two segments of hard-thresholding with two exponential functions. The parameter

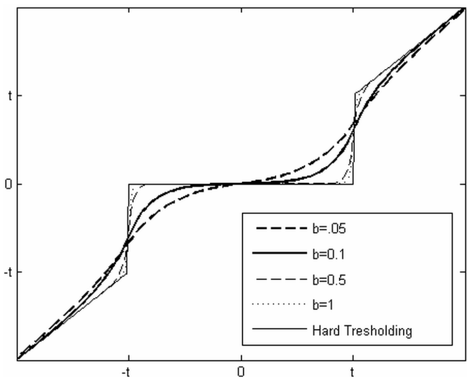


Fig. 2. Proposed thresholding function $\eta_b(x,t)$

b determines the degree of the thresholding effect. Also parameters a and c is determined such that the continuity of the thresholding function and its derivative is preserved at the threshold value t. For its similarity to hard-thresholding function, it preserves the edges.

The DWT of an image consists of four frequency bands: HH, HL, LH and LL, where "H" represents high pass filter and "L" represents low pass filter. We use four separate TNNs for these subbands. These TNN's are trained based on the algorithm shown in (3). This method will lead us to the optimum threshold values.

Finally, we use cycle spinning to improve the resulted image from the TNN step. In the thresholding step of cycle spinning we use our proposed thresholding function (5) with the optimum derived thresholds. This approach will reduce the artifacts appeared by thresholding.

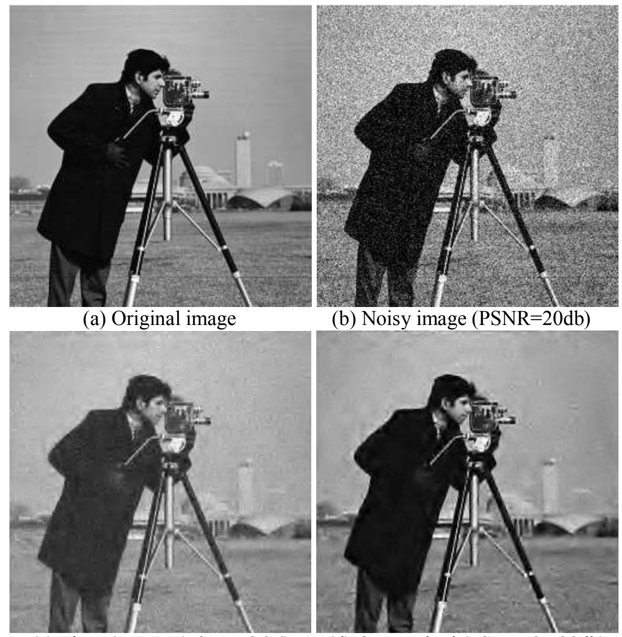
4. EXPERIMENTAL RESULTS

In this section we perform several experiments to test the proposed algorithm and compare it with other image denoising techniques. In the first experiment we compare our method with the TNN method proposed by Zhang [3]. To do so, the 256×256 *Cameraman* image is used as a test image. The original image is shown in Fig. 3(a). In this experiment, we employed the MATLAB® sym4 wavelet filter with four levels of decomposition to implement the orthogonal DWT. We use our thresholding function $\eta_b(x,t)$ with *b=0.1* throughout this work. The PSNR results, for different noise levels, are shown in Table 1. This table confirms that our proposed algorithm outperforms Zhang's method. To compare the visual quality of these two methods, the result of denoising for a noisy image with PSNR=20db is shown in Fig. 3. As we see, our method gives better visual result besides the PSNR improvement.

In the second experiment we compare the proposed method with several well-known denoising methods. These methods include: *VisuShrink*[2], MATLAB®'s spatially adaptive image filtering algorithm *Wiener2*, Donoho's *SureShrink* of soft-thresholding [9], Chang's *BayesShrink* [10], Hou's *improved Wiener-Chop* algorithm that utilizes the Wiener filtering of wavelet coefficients [11], Mihcak's method [12], Chen's *NeighShrink* thresholding [13], Bharath's *complex steerable* wavelet construction method [14]. The PSNR results of these denoising methods are shown in Table 2. For this experiment, we use 512×512 *Lena* image. As we see our methods outperforms all these established denoising approaches.

Noisy	Zhang's TNN	Proposed Method
20	26.64	**27.28**
25	29.97	**30.77**
30	33.73	**33.99**

Table 1. Comparison between the PSNRs (dB) resulted from Zhang's TNN method and our method for denoising 'cameramen'.



(a) Original image (b) Noisy image (PSNR=20db)
(c) Zhang's TNN (PSNR=26.6) (d) Our method (PSNR=27.28db)

Fig. 3. Comparison of the proposed method with Zhang's TNN method [3].

In the third experiment we compare the proposed method with another cycle-spinning based method [7] which uses basic hard-thresholding for the operator T and two different transforms, wavelet and contourlet. Fig. 4 compares our method with this cycle-spinning based method. As we see, our method is better both subjectively and objectively (PSNR).

5. CONCLUSION

In this paper, we proposed a new efficient wavelet-based image denoising method based on improved thresholding neural network (TNN) and cycle spinning methods. We presented a new class of thresholding functions as an

Noise Level	σ_n=10	σ_n=15	σ_n=20	σ_n=25	σ_n=30
VisuShrink [1]	31.37	29.58	28.51	27.74	27.19
Wiener2	33.44	30.77	28.87	27.20	25.98
SureShrink [9]	33.74	-	30.33	-	28.59
BayesShrink [10]	33.65	-	30.41	-	28.67
Improved WienerChop [11]	34.35	-	30.85	-	28.96
Mihcak [12]	34.39	30.44	28.52	26.95	-
NeighShrink [13]	33.69	31.68	30.10	28.90	27.98
Complex steerable [14]	32.81	-	31.07	-	29.69
Our Method	**34.89**	**33.87**	**32.48**	**31.43**	**30.49**

Table 2. Comparison of PSNRs (dB) for different denoising methods using *Lena* image with different noise levels.

(a) Original image (b) Noisy image (PSNR=22.18db)

(c) Wavelet+Cyclespinning (PSNR=28.87db) (d) Contourlet+Cyclespinning (PSNR=29.39db)

(e) Our method (PSNR=30.05db)

Fig. 4. Comparison of the proposed method with a cycle-spinning based method [7].

activation function of TNNs, which utilizes good properties of hard-thresholding such as edge preserving. Also we applied cycle-spinning as a post-processing filter to reduce image artifacts. Experimental results clearly showed the capability of the proposed method in image denoising and its superiority to several other established wavelet denoising techniques, in terms of PSNR and visual quality. For future work we will consider new thresholding functions and other transforms such as contourlet transform.

6. ACKNOWLEDGEMENT

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