JOINT IMAGE REGISTRATION AND SUPER-RESOLUTION USING NONLINEAR LEAST SQUARES METHOD

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ABSTRACT

This paper proposes a new algorithm to integrate image registration into image super-resolution (SR) by fusing multiple blurred low-resolution (LR) images to render a high-resolution (HR) image. Conventional super-resolution (SR) image reconstruction algorithms assume either the estimated motion (displacement) errors by existing registration methods are negligible or the displacement is known *a priori*. This assumption, however, is impractical as the performance of existing registration algorithms is still less than perfect. In view of this, we present a new estimation framework that performs joint image registration and HR reconstruction. An iterative scheme based on nonlinear least squares method is developed to estimate the motion shift (displacement) and HR image progressively. The motion model that is considered in this work includes both translation as well as rotation. Experimental results show that the proposed method is effective in performing image super-resolution.

***Index Terms*—** Image super-resolution, image registration, least squares methods

1. INTRODUCTION

Image super-resolution (SR) is the fusion of a number of low-resolution (LR) images to produce a high-resolution (HR) image. These LR images are shifted at subpixel level, and the information obtained in each LR image can be combined to obtain a HR image. The wide applications of image SR include remote sensing, military surveillance, and medical imaging, among others. Registration is a very important step to ensure the success of image SR. Many existing SR image reconstruction algorithms [1] assume the displacement is known *a priori* (i.e., an image-formation system using multiple CCD sensor arrays [2]). Other existing SR methods [3], [4] attempt to estimate the registration parameters by assuming a translational motion model or through an iterative two-phase estimation procedure. Therefore, this motivates the study of image super-resolution while taking into account the impact of registration error. This work attempts to incorporate image registration into the SR algorithm to form an iterative framework where the SR image and motion shifts can be estimated progressively with increasing accuracy.

Traditional SR techniques [1] are commonly performed in two stages, namely (i) image registration and (ii) inverse estimation that integrates fusion and deblurring into a single process. Generally, these SR algorithms assume the estimated motion parameters by existing registration methods to be error-free. Due to the presence of aliasing inherent in imaging systems, accurate registration for SR is difficult to achieve. Currently, some researchers [5] assume that the registration errors can be modeled by Gaussian noise and proceed to propose a regularized adaptive HR estimation method. However, the convergence of this method is not guaranteed. An iterative scheme based on alternating minimization (AM) has also been developed to estimate HR image and refine the estimated motion parameters in [4]. Nevertheless, it has been shown in [6] that optimization using the AM approach tends to be trapped at local minima. To the best of our knowledge, even though some SR methods consider registration errors, they impose strict restriction that the motion model contains only global translation. Hence, they are unable to handle both rotational and translational motion models.

In this paper, we propose a new framework for joint image registration and HR reconstruction. For the motion model, not only is the translational motion considered, but the rotational motion is also taken into account. The new problem is a more difficult task, as we cannot use the properties of Toeplitz structure to simplify computation. Traditional linear least squares method cannot be extended directly to image SR in this case. In view of this, a joint image registration and SR estimation framework is developed and formulated into a nonlinear parameter estimation problem. As opposed to traditional two-stage SR methods, the image registration in the proposed method is estimated from the HR image iteratively instead of LR images. An iterative scheme based on nonlinear least squares method is developed to estimate the displacement vector and HR image progressively. Experimental results show that the proposed method is effective in performing image super-resolution.

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2. PROBLEM FORMULATION

Let us consider that the k-th $(1 \le k \le N)$ acquired LR image $g_k(m,n)$ can be modeled by rotating the HR image $f(x,y)$ by θ_k, shifting it by a translational vector (s_{xk}, s_{yk}), blurring the result by a point-spread function (PSF) h_k, then down-sampling it to the resolution of the observed image by a factor of r. The process can be expressed as [7]:

$$
\begin{aligned}
& g_k(m,n) \\
& = \big(f(x\cos\theta_k - y\sin\theta_k + s_{xk}, x\sin\theta_k + y\cos\theta_k + s_{yk}) \\
& \quad \otimes h_k \otimes h_c \big) \downarrow + n_k(m,n)
\end{aligned} \tag{1}
$$

where $\otimes$ is the two dimensional convolution operator, $\downarrow$ is the down-sampling operator, and n_k denotes the additive white Gaussian noise(AWGN). h_k represents the camera lens blur in each LR image and h_c represents the effect of spatial integration of light intensity over a square surface region to simulate image acquisition by the sensors during the down-sampling process. h_c takes the form of a uniform PSF with support $(r \times r)$, where r is dependent on the desired HR resolution. In this work, it is assumed that the lens condition (h_k) is known. The SR formulation in (1) can be expressed in a vector-matrix form as:

$$
\boldsymbol{g} = \boldsymbol{W}(\boldsymbol{\alpha})\boldsymbol{f} + \boldsymbol{n} \tag{2}
$$

where $\boldsymbol{g} = \begin{bmatrix} \boldsymbol{g}_1 \\ \vdots \\ \boldsymbol{g}_N \end{bmatrix}$ and $\boldsymbol{n} = \begin{bmatrix} \boldsymbol{n}_1 \\ \vdots \\ \boldsymbol{n}_N \end{bmatrix}$ are the vectors representing the discrete, concatenated and lexicographically-ordered g_k and n_k, respectively. The matrix $\boldsymbol{W}$ is formed by nonlinear differentiable functions of unknown motion parametric vector $\boldsymbol{\alpha}$. They are given as:

$$
\boldsymbol{W}(\boldsymbol{\alpha}) = \begin{bmatrix} \boldsymbol{D}\boldsymbol{H}_1\boldsymbol{S}_1(\boldsymbol{\alpha}_1) \\ \vdots \\ \boldsymbol{D}\boldsymbol{H}_N\boldsymbol{S}_N(\boldsymbol{\alpha}_N) \end{bmatrix}; \boldsymbol{\alpha} = \begin{bmatrix} \boldsymbol{\alpha}_1 \\ \vdots \\ \boldsymbol{\alpha}_N \end{bmatrix} = \begin{bmatrix} [\cos\theta_1, \sin\theta_1, s_{x1}, s_{y1}]^T \\ \vdots \\ [\cos\theta_N, \sin\theta_N, s_{xN}, s_{yN}]^T \end{bmatrix}
$$

where $\boldsymbol{D}$ is the down-sampling operator which is taken to be the same for all LR images. $\boldsymbol{H}_k$ denotes the corresponding matrices constructed from $h_c \otimes h_k$. $\boldsymbol{S}_k(\boldsymbol{\alpha}_k)$ represents the geometric motion operator for the k-th LR image. Using the first LR image as the reference, we only need to estimate N−1 unknown motion parameters during the image SR process. Therefore, the objective of image SR is to reconstruct the HR image $\boldsymbol{f}$ from N observed LR images with unknown motion parametric vector $\boldsymbol{\alpha}$.

3. ITERATIVE MAP-BASED ALGORITHM USING NONLINEAR LEAST SQUARES METHOD

In this work, we develop an iterative maximum-a-posteriori (MAP)-based algorithm by using nonlinear least square method for joint image registration and super-resolution. Given the imaging model (1), the MAP estimate of the HR image $\boldsymbol{f}$ and the unknown motion parametric vector is a function of the observed LR images. Assuming that the noise $\boldsymbol{n}$ is AWGN and using total variation (TV) technique as the HR image *prior*, the MAP estimate of the HR image $\boldsymbol{f}$ and the unknown motion parametric vector $\boldsymbol{\alpha}$ is given as the minimum of the nonlinear least squares cost function as follows:

$$
\begin{aligned}
E(\boldsymbol{\alpha}, \boldsymbol{f}) &= (\boldsymbol{g} - \boldsymbol{W}(\boldsymbol{\alpha})\boldsymbol{f})^T(\boldsymbol{g} - \boldsymbol{W}(\boldsymbol{\alpha})\boldsymbol{f}) + \lambda \boldsymbol{f}^T\boldsymbol{T}(\varepsilon)\boldsymbol{f} \\
&= \left\| \begin{matrix} r(\boldsymbol{\alpha}, \boldsymbol{f}) \\ \sqrt{\lambda}\boldsymbol{L}(\varepsilon)\boldsymbol{f} \end{matrix} \right\|^2
\end{aligned} \tag{3}
$$

where $\|\cdot\|$ denotes L_2 norm, $r(\boldsymbol{\alpha}, \boldsymbol{f})$ is defined as the residual vector $\boldsymbol{g} - \boldsymbol{W}(\boldsymbol{\alpha})\boldsymbol{f}$. $\boldsymbol{T}(\varepsilon)$ is constructed by using half-quadratic scheme in [7]. Its pseudo-decomposition is given by $\boldsymbol{T}(\varepsilon) = \boldsymbol{L}(\varepsilon)^T\boldsymbol{L}(\varepsilon)$. In fact, we do not need to compute $\boldsymbol{L}(\varepsilon)$ explicitly during the minimization. It is noted that the minimization problem is linear with respect to $\boldsymbol{f}$ but nonlinear with respect to $\boldsymbol{\alpha}$. Extending the principle of nonlinear parametric estimation algorithm in [8], a linear approximation to $r(\boldsymbol{\alpha}, \boldsymbol{f})$ is used. Let $\Delta\boldsymbol{f}$ represent a small change in the HR image $\boldsymbol{f}$, and $\Delta\boldsymbol{\alpha}$ a small change in the motion parametric vector $\boldsymbol{\alpha}$. Then, ignoring the second order terms in $\Delta\boldsymbol{f}$ and $\Delta\boldsymbol{\alpha}$ since their values are small, we can estimate the minima of (3) iteratively by linearizing the residual vector $r(\boldsymbol{\alpha}, \boldsymbol{f})$ as follows:

$$
\begin{aligned}
& r(\boldsymbol{\alpha} + \Delta\boldsymbol{\alpha}, \boldsymbol{f} + \Delta\boldsymbol{f}) \\
& = \boldsymbol{g} - \boldsymbol{W}(\boldsymbol{\alpha} + \Delta\boldsymbol{\alpha})(\boldsymbol{f} + \Delta\boldsymbol{f}) \\
& = \boldsymbol{g} - \boldsymbol{W}(\boldsymbol{\alpha} + \Delta\boldsymbol{\alpha})\boldsymbol{f} - \boldsymbol{W}(\boldsymbol{\alpha} + \Delta\boldsymbol{\alpha})\Delta\boldsymbol{f} \\
& = \boldsymbol{g} - \boldsymbol{W}(\boldsymbol{\alpha})\boldsymbol{f} - \boldsymbol{J}(\boldsymbol{f}, \boldsymbol{\alpha})\Delta\boldsymbol{a} - \boldsymbol{W}(\boldsymbol{\alpha})\Delta\boldsymbol{f} - \boldsymbol{J}(\Delta\boldsymbol{f}, \boldsymbol{\alpha})\Delta\boldsymbol{\alpha} \\
& = r(\boldsymbol{f}, \boldsymbol{\alpha}) - \boldsymbol{J}(\boldsymbol{f}, \boldsymbol{\alpha})\Delta\boldsymbol{\alpha} - \boldsymbol{W}(\boldsymbol{\alpha})\Delta\boldsymbol{f}
\end{aligned} \tag{4}
$$

where $\boldsymbol{J}(\boldsymbol{\alpha}, \boldsymbol{f})$ is the Jacobian of $\boldsymbol{W}(\boldsymbol{\alpha})\boldsymbol{f}$ with respect to $\boldsymbol{\alpha}$. We will discuss how to construct $\boldsymbol{J}(\boldsymbol{\alpha}, \boldsymbol{f})$ in Section 4. Therefore, given the current estimate of the HR image $\boldsymbol{f}$ and the motion parametric vector $\boldsymbol{\alpha}$, the minimization problem in (3) can be solved as:

$$\min_{\Delta\boldsymbol{\alpha},\Delta\boldsymbol{f}} \left\| \begin{pmatrix} \boldsymbol{J}(\boldsymbol{\alpha},\boldsymbol{f}) & \boldsymbol{W}(\boldsymbol{\alpha}) \\ 0 & \sqrt{\lambda}\boldsymbol{L}(\varepsilon) \end{pmatrix} \begin{pmatrix} \Delta\boldsymbol{\alpha} \\ \Delta\boldsymbol{f} \end{pmatrix} + \begin{pmatrix} -r(\boldsymbol{\alpha},\boldsymbol{f}) \\ \sqrt{\lambda}\boldsymbol{L}(\varepsilon)\boldsymbol{f} \end{pmatrix} \right\|^2 \quad (5)$$

The proposed iterative MAP-based algorithm to perform joint image registration and HR reconstruction using nonlinear least squares method is summarized in Table I.

TABLE I SUMMARY OF JOINT IMAGE REGISTRATION AND SUPER-RESOLUTION ALGORITHM

Step 1.	Initialize $\boldsymbol{\alpha}^0$ by using existing image registration method, then fix $\boldsymbol{\alpha}=\boldsymbol{\alpha}^0$, compute $\boldsymbol{f}^0$ by minimizing (3).
Step 2.	Construct $\boldsymbol{W}(\boldsymbol{\alpha}^0)$, $\boldsymbol{J}(\boldsymbol{\alpha}^0,\boldsymbol{f}^0)$ and $r(\boldsymbol{\alpha}^0,\boldsymbol{f}^0)$.
Step 3.	At the (*i*+1)th iteration Solving (5) for $\Delta\boldsymbol{f}^i$ and $\Delta\boldsymbol{\alpha}^i$ is equivalent to solving the following equation:

$$\begin{pmatrix} \boldsymbol{J}^T(\boldsymbol{\alpha}^i,\boldsymbol{f}^i) & 0 \\ \boldsymbol{W}^T(\boldsymbol{\alpha}^i) & \sqrt{\lambda}\boldsymbol{L}^T(\varepsilon) \end{pmatrix} \begin{pmatrix} \boldsymbol{J}(\boldsymbol{\alpha}^i,\boldsymbol{f}^i) & \boldsymbol{W}(\boldsymbol{\alpha}^i) \\ 0 & \sqrt{\lambda}\boldsymbol{L}(\varepsilon) \end{pmatrix} \begin{pmatrix} \Delta\boldsymbol{\alpha}^i \\ \Delta\boldsymbol{f}^i \end{pmatrix} = \begin{pmatrix} \boldsymbol{J}^T(\boldsymbol{\alpha}^i,\boldsymbol{f}^i) & 0 \\ \boldsymbol{W}^T(\boldsymbol{\alpha}^i) & \sqrt{\lambda}\boldsymbol{L}^T(\varepsilon) \end{pmatrix} \begin{pmatrix} -r(\boldsymbol{\alpha}^i,\boldsymbol{f}^i) \\ \sqrt{\lambda}\boldsymbol{L}(\varepsilon)\boldsymbol{f}^i \end{pmatrix} \quad (6)$$

Step 4.	Update the (*i*+1)th iteration estimates $\boldsymbol{f}^{i+1}$ and $\boldsymbol{\alpha}^{i+1}$: $\boldsymbol{f}^{i+1}=\boldsymbol{f}^i+\Delta\boldsymbol{f}^i$, $\boldsymbol{\alpha}^{i+1}=\boldsymbol{\alpha}^i+\Delta\boldsymbol{\alpha}^i$.
Step 5.	Update $\boldsymbol{W}(\boldsymbol{\alpha}^{i+1})$, $\boldsymbol{J}(\boldsymbol{\alpha}^{i+1},\boldsymbol{f}^{i+1})$ and $r(\boldsymbol{\alpha}^{i+1},\boldsymbol{f}^{i+1})$
Step 6.	Go to Step 3 until convergence or a maximum number of iterations is reached.

We use conjugate gradient optimization to solve (6). In this work, the dimension of unknown motion vector $\boldsymbol{\alpha}$ is much smaller than the dimension of the unknown HR image $\boldsymbol{f}$. Hence, its computation cost is similar to the traditional SR algorithms where the estimated $\boldsymbol{\alpha}$ is considered to be accurate or known *a priori*.

4. CONSTUCTION OF THE JACOBIAN $\boldsymbol{J}(\boldsymbol{\alpha},\boldsymbol{f})$

$\boldsymbol{J}(\boldsymbol{\alpha},\boldsymbol{f})$ is the Jacobian of $\boldsymbol{W}(\boldsymbol{\alpha})\boldsymbol{f}$ with respect to $\boldsymbol{\alpha}$. As each LR image is independent, $\boldsymbol{J}(\boldsymbol{\alpha},\boldsymbol{f})$ can be written as:

$$\boldsymbol{J}(\boldsymbol{\alpha},\boldsymbol{f}) = \begin{bmatrix} \boldsymbol{J}(\boldsymbol{\alpha}_1,\boldsymbol{f}) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \boldsymbol{J}(\boldsymbol{\alpha}_N,\boldsymbol{f}) \end{bmatrix} \quad (7)$$

where $\boldsymbol{J}(\boldsymbol{\alpha}_k,\boldsymbol{f})$ is the Jacobian of $\boldsymbol{D}\boldsymbol{H}_k\boldsymbol{S}_k(\boldsymbol{\alpha}_k)\boldsymbol{f}$ with respect to $\boldsymbol{\alpha}_k$. It is not easy to compute $\boldsymbol{J}(\boldsymbol{\alpha}_k,\boldsymbol{f})$ directly. To simplify this problem, we use bilinear interpolation.

We define $[\Delta x_k, \Delta y_k]^T$ as the displacement vector between the *k*-th LR image and the reference LR image as shown in Fig. 1. The pixel at $(x+\Delta x_k, y+\Delta y_k)$ of the *k*-th LR image is determined by four pixel values f_{ul}, f_{ur}, f_{dl} and f_{dr}. Given the imaging model (1), the displacement vector can be written as:

$$\begin{bmatrix} \Delta\boldsymbol{x}_k \\ \Delta\boldsymbol{y}_k \end{bmatrix} = \boldsymbol{C}\boldsymbol{\alpha}_k - \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix}, \text{where } \boldsymbol{C} = \begin{bmatrix} \boldsymbol{x} & -\boldsymbol{y} & \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{y} & \boldsymbol{x} & \boldsymbol{0} & \boldsymbol{1} \end{bmatrix} \quad (8)$$

where $\Delta\boldsymbol{x}_k$, $\Delta\boldsymbol{y}_k$, $\boldsymbol{x}$ and $\boldsymbol{y}$ are the vectors representing the discrete and lexicographically-ordered displacement vectors Δx_k, Δy_k, and the coordinates of the reference LR image x and y, respectively. $\boldsymbol{1}$ and $\boldsymbol{0}$ are vectors of all ones and zeros, respectively. By using bilinear interpolation, the shifted and rotated $\boldsymbol{S}_k(\boldsymbol{\alpha}_k)\boldsymbol{f}$ can be obtained by:

$$\begin{aligned} \boldsymbol{S}_k(\boldsymbol{\alpha}_k)\boldsymbol{f} &= \Delta\boldsymbol{x}_k \odot (\boldsymbol{1}-\Delta\boldsymbol{y}_k) \odot \boldsymbol{f}_{dl} + \Delta\boldsymbol{x}_k \odot \Delta\boldsymbol{y}_k \odot \boldsymbol{f}_{dr} \\ &+ (\boldsymbol{1}-\Delta\boldsymbol{x}_k) \odot (\boldsymbol{1}-\Delta\boldsymbol{y}_k) \odot \boldsymbol{f}_{ul} + (\boldsymbol{1}-\Delta\boldsymbol{x}_k) \odot \Delta\boldsymbol{y}_k \odot \boldsymbol{f}_{ur} \end{aligned} \quad (9)$$

where $\odot$ is a corresponding entry-by-entry multiplication operator. $\boldsymbol{f}_{ul}, \boldsymbol{f}_{ur}, \boldsymbol{f}_{dl}$ and $\boldsymbol{f}_{dr}$ are the vectors representing the lexicographically-ordered f_{ul}, f_{ur}, f_{dl} and f_{dr}, respectively.

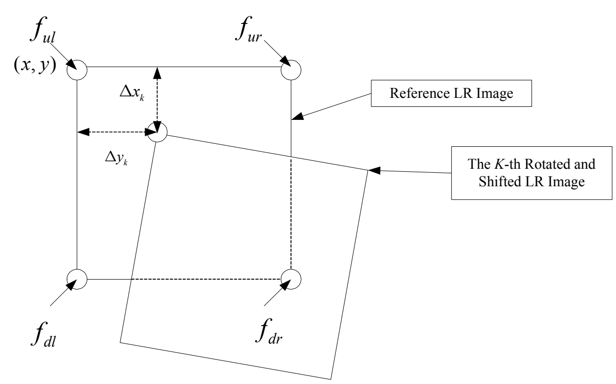


Fig. 1 Displacement between two LR images

Therefore, the Jacobian of $\boldsymbol{S}_k(\boldsymbol{\alpha}_k)\boldsymbol{f}$ with respect to the displacement $[\Delta\boldsymbol{x}_k{}^T, \Delta\boldsymbol{y}_k{}^T]^T$ can be written as:

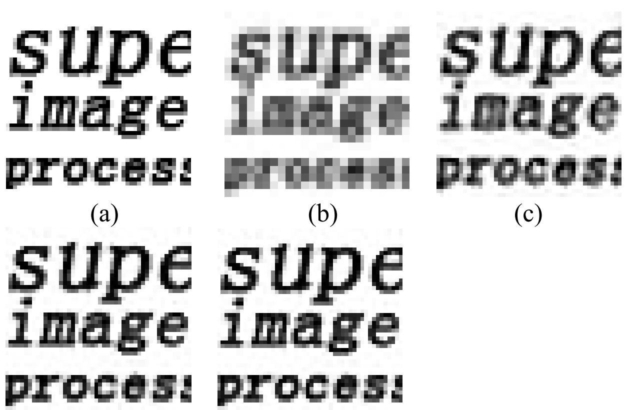
$$\begin{aligned} \boldsymbol{B} = \big[& diag\{(\boldsymbol{1}-\Delta\boldsymbol{y}_k)\odot(\boldsymbol{f}_{dl}-\boldsymbol{f}_{ul}) + \Delta\boldsymbol{y}_k \odot (\boldsymbol{f}_{dr}-\boldsymbol{f}_{ur})\}, \\ & diag\{(\boldsymbol{1}-\Delta\boldsymbol{x}_k)\odot(\boldsymbol{f}_{ur}-\boldsymbol{f}_{ul}) + \Delta\boldsymbol{x}_k \odot (\boldsymbol{f}_{dr}-\boldsymbol{f}_{dl})\}\big] \end{aligned} \quad (10)$$

Using the chain rule, we obtain $\boldsymbol{J}(\boldsymbol{\alpha}_k,\boldsymbol{f})$ as:

$$\boldsymbol{J}(\boldsymbol{\alpha}_k,\boldsymbol{f}) = \boldsymbol{D}\boldsymbol{H}_k\boldsymbol{B}\boldsymbol{C} \quad (11)$$

5. EXPERIMENTAL RESULTS

Several experiments were carried out to determine the effectiveness of the proposed algorithm. The “Text” image was selected as the test image in Fig. 2(a). To generate the LR images, the HR image was rotated by different degrees of (0, 12.5, 5, -7.5, 15), shifted by translations of (0,0), (0.15,0.75), (0.55,0.55), (0.65,0.25), (1,1) subpixels, and blurred by uniform blurs with support of 2×2. A decimation factor of 2 was chosen and the images were degraded by AWGN to produce a signal-to-noise ratio (SNR) at 30 dB. A sample of the scaled-up LR images is shown in Fig. 2(b). We first initialized the motion parameters by using image registration method in [9]. The reconstructed HR image without considering the registration errors is shown in Fig. 2(c). It is observed that there are obvious artifacts around the edge of the words. Next, we performed joint image registration and SR image reconstruction using the proposed algorithm and the result is given in Fig. 2(d). We also compared our result with the estimated image when the exact motion parameters are known in Fig. 2(e). Comparing our result in 2(d) with the reconstructed HR image in Fig. 2(e), it is noticed that even if the proposed method does not have the information of the exact displacements, it can still recover the HR image that is almost as good as that recovered using the exact displacements. Fig. 3 shows the normalized mean squared errors (NMSE) of the identified motion parameters using our proposed method. This objective performance measure further reconfirms the effectiveness of the proposed method in performing joint image registration and SR.



(a) (b) (c) (d) (e)

Fig. 2 Joint image registration and SR. (a) Original HR image, (b) A sample of the scaled-up LR images, (c) Reconstructed image without considering the impact of registration error, (d) Reconstructed image using the proposed algorithm, (e) Reconstructed image using known exact motion parameters.

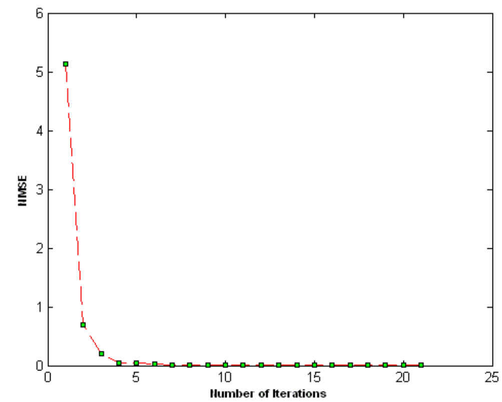


Fig. 3 NMSE of estimated motion parameters

6. CONCLUSION

This paper presents a new algorithm to address the SR image reconstruction while taking into consideration of potential registration errors. The new method performs joint image registration and SR from the observed LR images. An iterative scheme based on nonlinear least squares method is developed to estimate the displacement and HR image progressively. As opposed to other SR algorithms, the adopted motion model contains both translational and rotational motions. Experimental results show that the new method is effective in performing image super-resolution.

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