DETECTING CURVED UNDERGROUND TUNNELS USING PARTIAL RADON TRANSFORMS

Ali Cafer Gurbuz, James H. McClellan and Waymond R. Scott

Georgia Institute of Technology
Atlanta, GA 30332–0250

ABSTRACT

The Radon Transform (RT) is known to be effective in detecting lines in noisy images, but it is not capable of detecting curves unless the curve parametrization is given. In this paper, partial Radon transforms (PRT) are investigated as a tool to detect curved features such as underground tunnels in ground penetrating radar (GPR) images. The algorithm applies the Radon Transform to small batches of the total image and updates the tunnel position parameters as new batches are used. Missing data, as well as finding the ends of tunnels can be handled with the proposed algorithm. Performance analysis is given for various signal-to-noise ratios (SNR) and batch sizes. The effect of the curvature level on the performance is also analyzed.

Index Terms— Radon transforms,tunnel detection, curve estimation, partial Radon Transform

1. INTRODUCTION

The Hough Transform (HT) [1] and the Radon Transform (RT) [2] are two commonly used techniques for line detection. These transforms map an image into a parameterized domain such that the parameterized shapes (i.e., lines) correspond to peaks in the parameter domain. This property makes it easier to develop detection algorithms in parameter space, which has led to many applications in image processing [3, 4], computer vision [3, 5] and underground imaging [6, 7]. Line detection techniques can be extended to the detection of curves using the generalized Radon Transform (GRT) [8–10]. Previous work on curve estimation using Radon transforms has focused on detection of parameterized curves, such as circles in images [8], or hyperbolas and other parameterized curves [9,10]. However, none of the work on this area has addressed the problem of estimating a random non-parameterized curve.

The RT has been successfully applied to detect buried linear features [6, 7, 11], assuming that the buried object (e.g., a tunnel, or pipeline) is linear over the distance of interest. However, it is not reasonable to assume that buried objects are always linear over long distances, so the GRT fails to detect curving features when the actual curve parametrization is not known. In this paper, we propose using the RT in small batches of data where the buried object might be assumed linear, and then update the position of the buried object when new data is gathered. Thus, selection of the batch size is crucial; for small batch sizes the Radon gain from summing the signal values decreases, for large batch sizes the object might curve which decreases the Radon gain. A batch size that takes into account both of these factors must be used.

In Section 3 simulated GPR reflections from a curving tunnel are generated. The proposed method is able to detect the tunnel when the classical RT fails. Performance of the algorithm for varying levels of signal-to-noise ratio (SNR) and different batch sizes are analyzed. It is observed that the mean square error (MSE) decreases for increasing SNR. Also, it is seen that there is an optimum batch size that depends on the curvature of the buried tunnel. For a fixed batch size and SNR the tunnels having higher curvature had higher MSE values. The next section explains the proposed algorithm in detail.

2. THEORY

The RT maps an image into a parameterized domain such that the parameterized shapes (i.e., lines) correspond to peaks in the parameter domain. The Radon transform of a digital image $f(x, y)$ using the parametrization of a line can be given as follows:

$$R(m, n)[f(x, y)] = \int_{-\infty}^{\infty} f(x, mx + n)dx \qquad (1)$$

where m and n are the slope and the intercept of the line, respectively. The GRT, which maps specific parameterized curves into peaks in parameter space can be formulates as

$$R(\mathbf{j})[f(x, y)] = \sum_{l=0}^{L-1} f(\phi_x(l, \mathbf{j}), \phi_y(l, \mathbf{j})) \qquad (2)$$

where $\mathbf{j}$ is a multi-dimensional vector defining the curve parameters and $\phi_x(l, \mathbf{j})$ and $\phi_y(l, \mathbf{j})$ are functions that define the specific curve. While the GRT is effective in detecting any

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kind of specific known curve in images, it cannot detect a curve whose structure is unknown. The proposed method applies the RT with line parametrization (1) to smaller batches of an image and tries to estimate the curving structure from linear segments obtained from different batches.

Our problem can be stated as estimating a curving function $f(n)$ from partial linear approximations. An illustration of the problem and the approach taken is shown in Fig. 1 Assume we have an image of size $L \times L$ and the RT is applied on batches of size $M \times L$ where $M \ll L$. For batch i if a line is detected in the parameter domain a linear approximation is obtained as

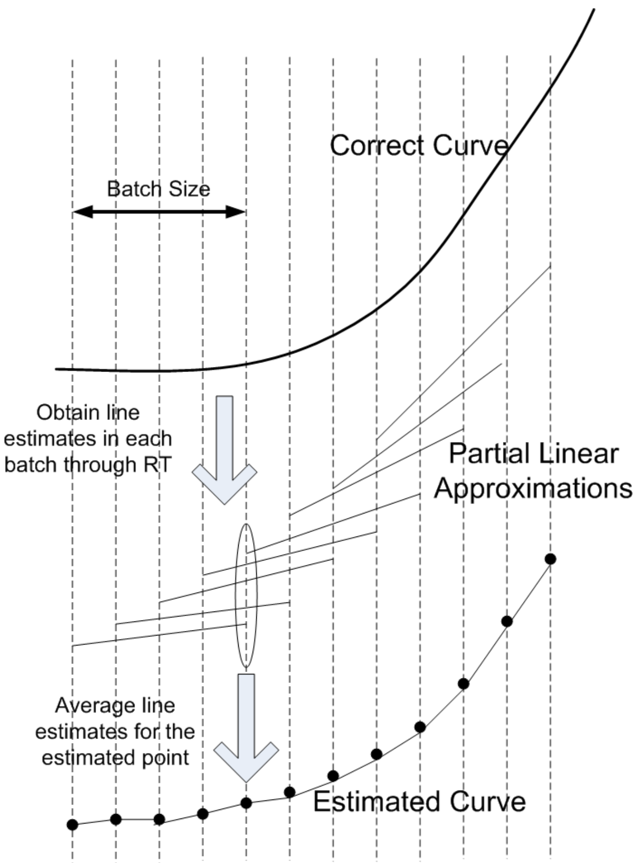

Fig. 1. Estimating a curve from partial linear approximations

$$\hat{f}_i(l) = m_i l + n_i \tag{3}$$

where $i \leq l \leq i + M - 1$ for $i = 1, 2, 3, ...L - M + 1$ and (m_i, n_i) are the slope and intercept values of the detected line. In this way $L - M + 1$ linear approximations of the curving function $f(n)$ are obtained. These partial linear approximations can be seen in Fig. 1. For each point n the estimate can be found as the average of the linear approximations for that point:

$$\hat{f}(n) = \begin{cases} \frac{1}{n} \sum\limits_{k=1}^{n} \hat{f}_k(k) & n < M \\ \frac{1}{M} \sum\limits_{k=n-M+1}^{n} \hat{f}_k(k) & M \leq n \leq L - M \\ \frac{1}{L-n+1} \sum\limits_{k=n}^{L} \hat{f}_k(k) & n > L - M \end{cases} \tag{4}$$

There may be no line detections in some batches. This is not a problem since the estimator only uses the average of detected lines for estimating the position of curve at one point. For a curve to end, or not be detected, at one point at least M consecutive batches should not have any line detections.

If multiple line detections in batches are obtained, an association of the line parameters with the curves are needed. It is possible for a high GPR clutter response (i.e., cans, rocks) to dominate the RT detection, resulting in incorrect line parameters. Line detection in dominant clutter can be solved iteratively [11]. This paper focuses on the detection of one curve in high noise; while the problem of detecting curving structures in dominant clutter will be addressed in a future work.

Selection of the batch size M is critical. On one hand, increasing M might decrease the advantage of Radon gain if the tunnel is curving, on the other hand decreasing M might cause missed detections in Radon space due to the adding up only a small number of signal values while not getting enough incoherent averaging of the noise. The optimal batch size should be selected to maximize the probability of detection (P_D) of a line in the batch [6]

$$P_D = Q\left(Q^{-1}(P_{FA}) - \frac{\mu \min(M, R_S)}{\sqrt{M\sigma^2}}\right) \tag{5}$$

where μ is average signal value, σ^2 the noise variance and R_S the maximum Radon votes from the curving structure in the batch. The Radon Transform implementation finds the line with most votes from the curve. Note that without discretization all lines are tangent to a curve at one point only. Most votes come from the lowest sloped part of the curve; in a second-order curve, $f(x) = ax^2 + bx + c$ the number of Radon votes can be approximated as $R_S \approx \sqrt{(2/a)}$. The optimal batch size M should maximize (5). Since Q is the area in the tail of a Gaussian function and $Q^{-1}(P_{FA})$ is positive for all P_{FA}, an optimal M must satisfy

$$M^* = \arg\max_{M} \frac{\mu \min(M, R_S)}{\sqrt{M\sigma^2}} \quad \Rightarrow \quad M^* = R_S \tag{6}$$

From (6) it can be seen that the optimal batch size should be equal to the maximum number of Radon votes, R_S. Since R_S depends on the curvature, a, this varies from curve to curve. However, an average value of curvature can be selected for detecting the structure in images.

3. SIMULATIONS

The proposed algorithm was tested on simulated underground tunnels. For this purpose, a GPR simulated data generation program was developed in MATLAB. System specifications such as aperture size, antenna height, the transmitter-receiver distance, target depth, and soil type, conductivity are all adjustable parameters of the program. Targets are simulated as point targets. A tunnel is represented with many point-like targets forming a defined curve.

Result 1: In this example, a tunnel described by the curve parametrization $y = 40 + 0.01(x - 100)^2$ at a depth of 20 cm is simulated. A two-layer model with dry soil type is used. The system parameters are antenna height 10 cm, transmitter-receiver distance 10 cm, antenna step size 2 cm (100 total steps are simulated). The tunnel is formed with point-like targets at each x position on the specified curve and depth. For the simulated tunnel response, a data cube is obtained. The square of the data along the depth axis is summed to obtain a surface energy image, shown in Fig. 2. Clutter is assumed to be

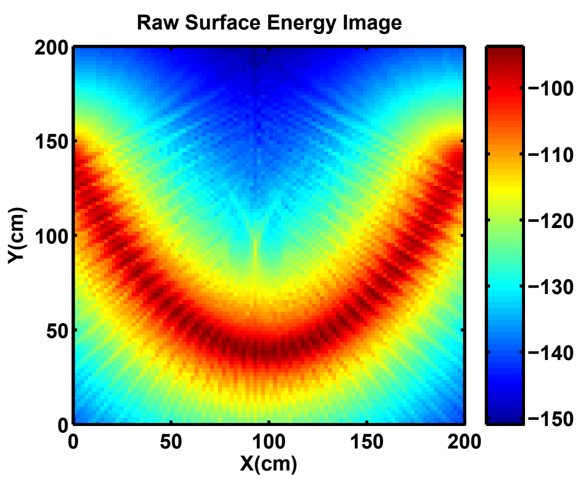

Fig. 2. Raw Surface Energy Image

zero-mean additive white gaussian noise (WGN). This clutter model is added to the raw tunnel data. The signal-to-noise ratio (SNR) is defined as the ratio of the maximum signal power in the image to the average noise variance. Figure 3 shows results from the RT (line parametrization) and the proposed PRT algorithm on a surface energy image.

As seen in Fig. 3, while the RT incorrectly finds a line nearly tangent to the curving structure, the proposed algorithm finds the curve correctly. In the PRT algorithm, $M = 20$ is used as the batch size.

To compare the performance of the algorithm for varying parameters, a mean square error metric is defined as

$$\text{MSE} = \frac{1}{N}\sqrt{\sum_{n=1}^{N}(f(n) - \hat{f}(n))^2} \qquad (7)$$

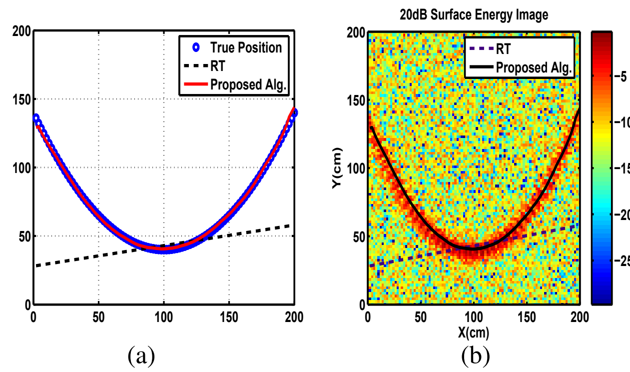

Fig. 3. (a) Processing results with RT and PRT detected for the noise-free case, (b) Detected Structures for RT and PRT on a 20-dB surface energy image.

where N is the total number of points. For the case in Fig.3, the MSE for the RT and proposed algorithm are 31 cm and 0.88 cm, respectively.

To analyze the performance of the algorithm with varying noise levels the algorithm is applied to images with SNRs of 10 dB, 0 dB −7 dB. The batch size is $M = 20$ for all noise levels. The detected lines with the correct target positions are shown in Fig. 4. Figure 4 shows how the performance of the

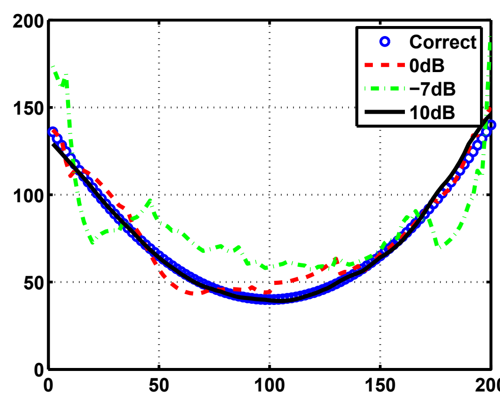

Fig. 4. Detected Lines for different noise levels

algorithm decreases with increasing noise levels, but the performance depends not only on the SNR level, but also on the batch size. A second simulation was performed to observe the effect of SNR and batch size. At each SNR and batch size the algorithm is run and the MSE is computed. This procedure is repeated 100 times and the average MSE value is obtained for each SNR and M. The figure of merit $1/\text{MSE}$ is plotted in Fig. 5 for each SNR and M.

From Fig. 5, it can be observed that there is an optimum batch size (i.e., 15–20) that gives better performance for nearly all SNR. This optimum batch size is also very close to the

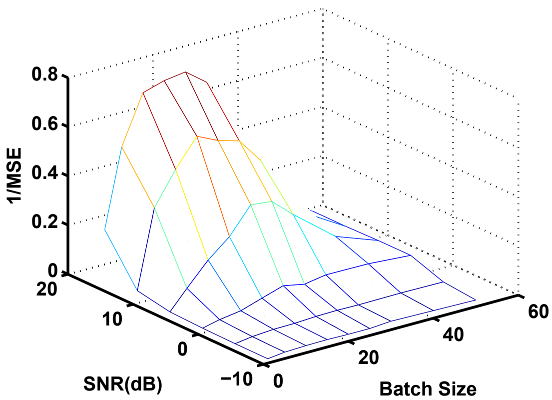

Fig. 5. 1/MSE for varying SNR and batch size values

approximate Radon vote for the curvature which is $R_S \approx \sqrt{(2/0.01)} \approx 14$. For low SNRs the MSE increases for all batch sizes and the dependence on batch size is less important.

In addition to SNR and batch size, the curvature of the structure to be detected also effects the performance of the algorithm. To observe this, simulated GPR data from three curves with varying curvatures was generated. The curves are formed by $y = 40 + a(x - 100).^2$ with the parameter a equal to 0.005, 0.01 and 0.015 for the three cases. The PRT algorithm is run on all three data sets with a 10-dB SNR level and $M = 20$. The detected curves and the correct target positions are shown in Fig. 6. The lower curvature line fits the true positions better than higher curvature ones. The MSE values for the curves (from lower to higher curvature) are 0.83 cm, 1.12 cm and 2.24 cm respectively.

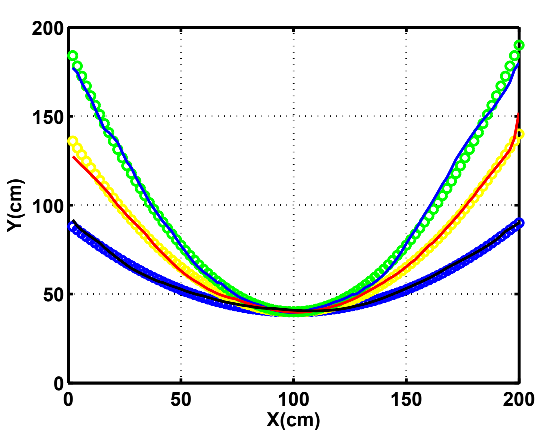

Fig. 6. Effect of curvature level on detection performance

4. CONCLUSIONS

The partial Radon Transform was used to detect curving tunnels in GPR images. Simulated data results show that the proposed algorithm successfully detects the underlying curves, where classical RT fails.

5. REFERENCES

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