ROBUST MATCHED FILTERS FOR TARGET DETECTION IN HYPERSPECTRAL IMAGING DATA

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ABSTRACT

Most detection algorithms for hyperspectral imaging applications assume a target with a perfectly *known* spectral signature. In practice, the target signature is either imperfectly measured (target mismatch) and/or it exhibits spectral variability. The objective of this paper is to introduce a robust matched filter that takes the uncertainty and/or variability of target signatures into account. It is shown that, if we describe this uncertainty with an ellipsoid in the spectral space, we can design a matched filter that provides a response of the same magnitude for all spectra within this ellipsoid. Thus, by changing the size of this ellipsoid, we can control the "spectral selectivity" of the matched filter. The ability of the robust matched filter to deal effectively with target mismatch and spectral variability is demonstrated with hyperspectral imaging data from the HYDICE sensor.

Index Terms— Infrared spectroscopy, multidimensional signal detection, array signal processing, adaptive signal detection

1. INTRODUCTION

One of the fundamental challenges for a hyperspectral imaging surveillance system is the detection of subpixel targets in background clutter [1, 2]. The background surrounding the target, which acts as interference, provides the major obstacle to successful detection. Most hyperspectral target detection algorithms are derived using the following assumptions

1. The target is characterized by a spectral signature with known shape

2. The background spectrum follows a multivariate normal distribution with known mean vector and covariance matrix

*This work was sponsored by the Department of Defense under Air Force Contract FA8721-05-C-0002. Opinions, interpretations, conclusions, and recommendations are those of the author and are not necessarily endorsed by the United States Government.

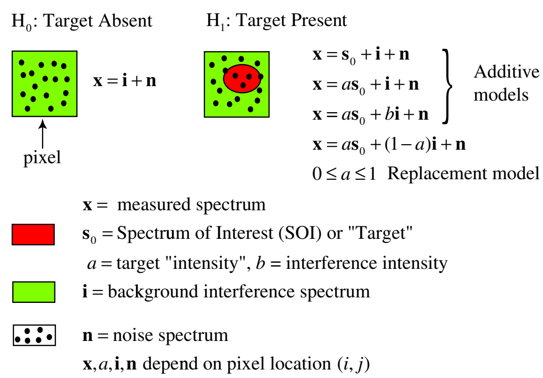

Fig. 1. Pictorial illustration of additive and replacement subpixel target models for hyperspectral imaging data.

3. For subpixel targets the background acts as an additive interference.

The additive and replacement subpixel target models are illustrated in Figure 1. In hyperspectral imaging data, the target replaces the background and we have a replacement model; however, for mathematical tractability we use an additive model [2].

The most widely used algorithm for hyperspectral target detection is the matched filter. The matched filter is a linear processor whose weights can be obtained using different optimality criteria. Indeed, for a target with known spectral signature in the presence of additive zero mean normal interference with known covariance matrix, the likelihood ratio processor, the maximum signal-to-noise-plus-interference ratio filter, and the minimum variance linear signal estimator have identical weights. Another criterion of optimality, which we shall explore in this work, is that the matched filter minimizes the output noise-plus-interference variance subject to the constraint that the response to the target is equal to one. This implies that the matched filter has maximum response in the direction of the target in the spectral space and the smaller possible response in any other direction. Unfortunately, in practical applications, the target signature is not accurately measured or the target object exhibits intrinsic spectral variability. In both cases, there is a performance degradation that increases with the measurement error or the spectral variability of the target signature. In practice, these assumptions are violated to various degrees, leading to suboptimum performance. Furthermore, in most practical applications, the background mean and covariance are estimated from the available spectral observations. The quality of these estimates depend on the available number of pixels, their spectral homogeneity, and whether there are contaminated by target pixels.

2. THE OPTIMUM MATCHED FILTER

The spectral measurements obtained by a K-band hyperspectral imaging sensor can be arranged in vector form as

$$\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & \dots & x_K \end{bmatrix}^T \tag{1}$$

where T denotes matrix transpose. Let $\boldsymbol{v}$ be a $K \times 1$ random vector from a normal distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ representing the background clutter. Finally, let $\boldsymbol{s}_0$ be a $K \times 1$ vector representing the spectral signature of the target of interest. To simplify notation, we assume that $\boldsymbol{\mu}$ is removed from all spectra, that is, we deal with zero mean clutter and a "clutter-centered" target signature.

The optimum linear matched filter [3]

$$y = \boldsymbol{h}^T \boldsymbol{x} \tag{2}$$

can be determined by minimizing the output clutter power $\text{Var}(y^2) = \boldsymbol{h}^T \boldsymbol{\Sigma} \boldsymbol{h}$ subject to a unity gain constraint in the direction of the target spectral signature

$$\min_{\boldsymbol{h}} \boldsymbol{h}^T \boldsymbol{\Sigma} \boldsymbol{h} \quad \text{subject to} \quad \boldsymbol{h}^T \boldsymbol{s}_0 = 1 \tag{3}$$

The solution to (3) is given by

$$\boldsymbol{h} = \frac{\boldsymbol{\Sigma}^{-1} \boldsymbol{s}_0}{\boldsymbol{s}_0^T \boldsymbol{\Sigma}^{-1} \boldsymbol{s}_0} \tag{4}$$

which is the formula for the widely used matched filter.

In the array processing area, where the data and filter vectors are complex, the matched filter (4) is known as the standard Capon beamformer (SCB) [4].

In practice, the clutter covariance matrix $\boldsymbol{\Sigma}$ and the target signature $\boldsymbol{s}_0$ have to be estimated from the available data. It turns out that the matched filter (4) is sensitive to signature errors and the quality of the clutter covariance matrix. Therefore, the development of matched filters that are robust to signature and clutter covariance errors is highly desirable. This problem has been traditionally dealt with using a diagonal loading approach or an eigenspace-based approach. However, in both case the selection of diagonal loading or the subspace dimension is ad-hoc [4].

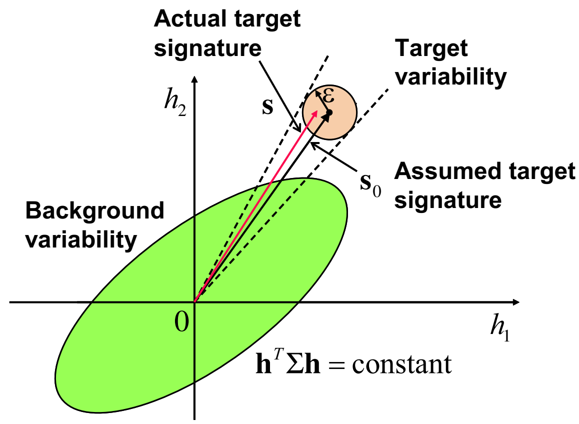

Fig. 2. Illustration of robust matched filter design principle using two spectral bands.

3. THE ROBUST MATCHED FILTER

In this section, we shall use the theory of robust Capon beamformer (RCB) [5] to develop a robust matched filter that takes measurement errors and the spectral variability of hyperspectral target signatures into consideration. The robust matched filter (RMF) addresses robustness to target signature errors by introducing an uncertainty region constraint into the optimization process. To this end, assume that the only knowledge we have about the signature $\boldsymbol{s}$ is that it belongs to an uncertainty ellipsoid

$$(\boldsymbol{s} - \boldsymbol{s}_0)^T \boldsymbol{C}^{-1} (\boldsymbol{s} - \boldsymbol{s}_0) \le 1 \tag{5}$$

where the vector $\boldsymbol{s}_0$ and the positive definite matrix $\boldsymbol{C}$ are given. In most hyperspectral target detection applications, it is difficult to get sufficient data to reliably estimate the full matrix $\boldsymbol{C}$. Therefore, we usually set $\boldsymbol{C} = \varepsilon \boldsymbol{I}$, so that (5) becomes

$$||\boldsymbol{s} - \boldsymbol{s}_0||^2 \le \varepsilon \tag{6}$$

where ε is a positive number. These ideas are illustrated in Figure 2. It has been shown in [5] that the RMF can be obtained as the solution to the following optimization problem

$$\min_{\boldsymbol{s}} \boldsymbol{s}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{s} \quad \text{subject to} \quad ||\boldsymbol{s} - \boldsymbol{s}_0||^2 \le \varepsilon \tag{7}$$

It turns out that the solution of (7) occurs on the boundary of the constraint set; therefore, we can reformulate (7) as a quadratic optimization problem with a quadratic equality constraint

$$\min_{\boldsymbol{s}} \boldsymbol{s}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{s} \quad \text{subject to} \quad ||\boldsymbol{s} - \boldsymbol{s}_0||^2 = \varepsilon \tag{8}$$

This problem can be efficiently solved using the method of Lagrange multipliers [6]. The solution involves an estimated target signature

$$\hat{s} = \beta(\boldsymbol{\Sigma}^{-1} + \beta \boldsymbol{I})^{-1} \boldsymbol{s}_0 \tag{9}$$

which is subsequently used to determine the RMF by

$$\boldsymbol{h}_\beta = \frac{\boldsymbol{\Sigma}^{-1}\hat{\boldsymbol{s}}}{\hat{\boldsymbol{s}}^T\boldsymbol{\Sigma}^{-1}\hat{\boldsymbol{s}}} \tag{10}$$

The Lagrange multiplier $\beta \geq 0$ can be obtained by solving the nonlinear equation

$$\boldsymbol{s}_0^T(\boldsymbol{I}+\beta\boldsymbol{\Sigma})^{-2}\boldsymbol{s}_0 = \sum_{k=1}^{L}\frac{|\tilde{s}_k|^2}{(1+\beta\lambda_k)^2} = \varepsilon \tag{11}$$

where λ_k and $\tilde{s}_k$ are obtained from the eigen-decomposition

$$\boldsymbol{\Sigma} = \boldsymbol{Q}\boldsymbol{\Lambda}\boldsymbol{Q}^T = \sum_{k=1}^{K}\lambda_k\boldsymbol{q}_k\boldsymbol{q}_k^T \tag{12}$$

and the orthogonal transformation

$$\tilde{\boldsymbol{s}} = \boldsymbol{Q}^T\boldsymbol{s}_0 \tag{13}$$

The solution of (11) can be easily done using some nonlinear optimization algorithm, for example, Newton's method.

Finally, we note that the RMF (10) can be expressed in diagonal loading form as follows

$$\boldsymbol{h}_\beta = \frac{(\boldsymbol{\Sigma}+\beta^{-1}\boldsymbol{I})^{-1}\boldsymbol{s}_0}{\boldsymbol{s}_0^T(\boldsymbol{\Sigma}+\beta^{-1}\boldsymbol{I})^{-1}\boldsymbol{\Sigma}(\boldsymbol{\Sigma}+\beta^{-1}\boldsymbol{I})^{-1}\boldsymbol{s}_0} \tag{14}$$

where β^{-1} is the loading factor [5] is computed from (11).

Figure 3 illustrates the validity of the optimization approach leading to the RMF. We note that the RMF is obtained as a standard MF for a modified target signature. As expected the "assumed" target signature specifies the center of the uncertainty region, whereas the modified signature "touches" the boundary of the uncertainty region.

4. EXPERIMENTAL INVESTIGATION

To illustrate the validity of the RMF approach, we use airborne hyperspectral imagery data collected by the HYDICE sensor at the U.S. Army Aberdeen Proving Grounds, Maryland, on 24 August 1995. HYDICE collects calibrated (post-processed) spectral radiance data in 210 wavelengths spanning 0.4 to 2.5 μm in nominally 10-nm wide bands. After removing the atmospheric opaque bands, we end up with 155 usable spectral bands. Figure 4 shows an image of the ground surface and the available target signatures obtained by ground measurements. The target signature $\boldsymbol{s}_0$ is determined by the mean of the target spectra shown in Figure 4. Clearly, due to environmental and target variability there is a mismatch between the spectra of the in-scene target pixels and the spectral signature used by the matched filter. Figure 5 shows the detection statistics in the neighborhood of the four target objects for a standard MF and a RMF with $\varepsilon = 0.01$. We note that the responses of the two filters are almost identical. Using smaller values for

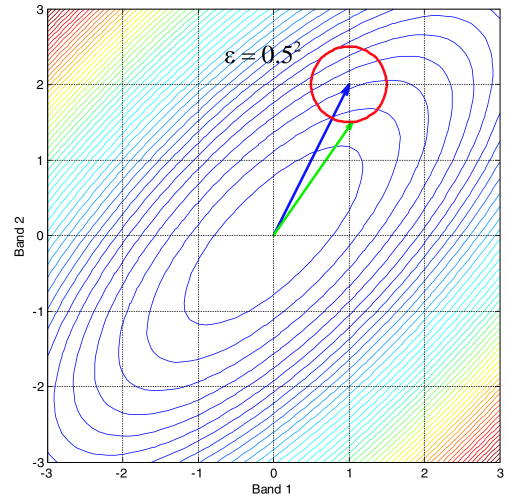

Fig. 3. Illustration of robust matched filter when there is a target signature mismatch. The algorithm uses the available signature specifying the center of the uncertainty region, to produce a "robust" signature that is subsequently used to determine the RMF coefficients.

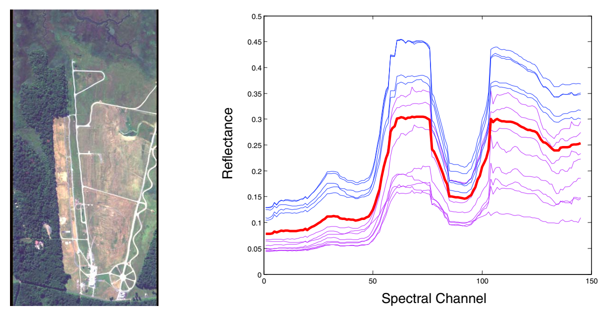

Fig. 4. Color image representing the hyperspectral data from the HYDICE sensor data cube and a plot of the available spectral signatures for the target of interest. The red curve represents the mean target signature.

ε results to identical detection statistics. In Figure 6, where we have increased the uncertainty to $\varepsilon = 0.1$, we note that the RMF "picks-up" more target pixels around the third target object. This illustrates the trade-off between "selectivity" (small ε) and "collectivity" (large ε) of the algorithm. Clearly, a "less selective" algorithm may "pick-up" more targets and false alarms. Figure 7 shows the signature used by the MF and the "robust" signature created by the RMF. Clearly, the robust signature $\hat{s}$ can be used to "robustify" other detection algorithms, like ACE, GLRT, and AMF [2]. Due to the high dimensionality of hyperspectral imaging data, covariance regularization is another practical necessity, which can be meaningfully addressed using the robust matched filter approach.

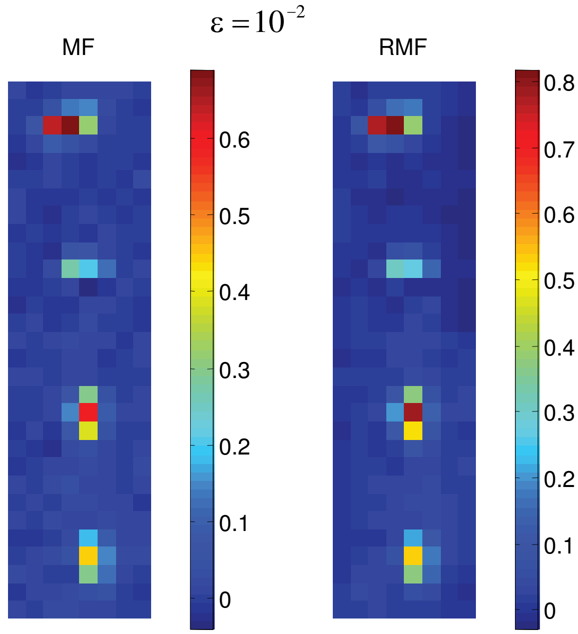

Fig. 5. Detection statistics of a standard MF and a RMF with small target uncertainty.

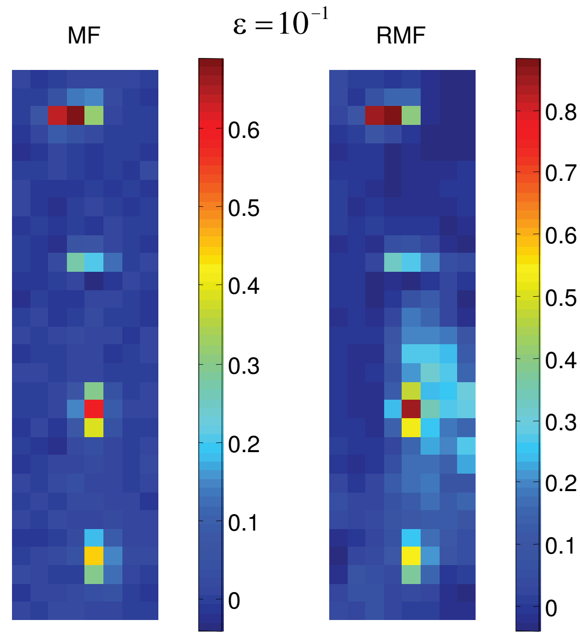

Fig. 6. Detection statistics of a standard MF and a RMF with large target uncertainty.

5. SUMMARY

Classical matched filter is sensitive to target signature missmatch and background covariance invertibility. Covariance matrix regularization, through diagonal loading, provides an ad-hoc method to address both problems [4]. In this paper, we have described a robust matched filtering approach that relates systematically target signature uncertainty to covariance matrix regularization and provides the tools to (a) study robustness of target detection algorithms and (b) develop robust and effective detection algorithms for targets with spectral variability or uncertainty. This technique ia particularly useful in hyperspectral imaging applications where target variability can be bounded by angular and sensor noise considerations. More experimental results will be presented at the conference.

6. REFERENCES

[1] D. Manolakis and G. Shaw, "Detection algorithms for hyperspectral imaging application," *IEEE Signal Processing Magazine*, pp. 29–43, January 2002.

[2] D. Manolakis, D. Marden, and G. Shaw, "Target detection algorithms for hyperspectral imaging applications," *Lincoln Laboratory Journal*, vol. 14, no. 1, pp. 79–116, 2003.

[3] S. M. Kay, *Fundamentals of Statistical Signal Processing*, Prentice Hall, New Jersey, 1998.

[4] H. L. Van Trees, *Optimum Array Processing*, Wiley, New York, 2002.

[5] J. Li, P. Stoica, and Z. Wang, "On robust Capon beamforming and diagonal loading," *IEEE Trans. on Signal Processing*, vol. 51, no. 7, pp. 1702–1715, July 2003.

[6] P. Gill, W. Murray, and M. Wright, *Practical Optimization*, Academic Press, London, UK, 1981.

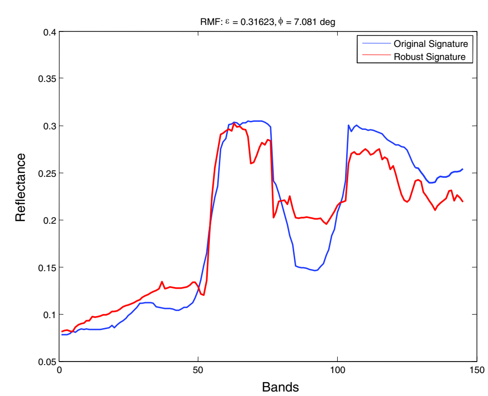

Fig. 7. The RMF algorithm modifies the target signature to produce a new signature that is used to generate the RMF coefficient vector.