ADAPTIVE BLOOD VELOCITY ESTIMATION IN MEDICAL ULTRASOUND

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ABSTRACT

This paper investigates the use of data-adaptive spectral estimation techniques for blood velocity estimation in medical ultrasound. Current commercial systems are based on the averaged periodogram, which requires a large observation window to give sufficient spectral resolution. Herein, we propose a novel data-adaptive method to form the blood velocity spectral estimate. The method is evaluated using realistic Field II simulations for both steady and unsteady flow. The latter representing the femoral artery with strong tissue interference. The method is compared to the averaged periodogram and a Capon-based estimator. The simulations indicate that the proposed method offer a significant performance gain, suggesting that the frame-rate may be increased dramatically by using adaptive spectral estimation techniques.

Index Terms— Blood velocity estimation, medical ultrasound, adaptive spectral estimation.

1. INTRODUCTION

A typical B-mode ultrasound image consists of about 100 image lines, with each line being created by focusing the ultrasound transducer array on a single point on the current line. The backscattered waves are processed with dynamic receive focusing, and, ideally, only objects along the image line are interrogated. The velocity of moving blood can be estimated by imaging the same image line repeatedly. The slow-time signal (sampled with the pulse repetition frequency), for a specific depth, exhibits a modulation in its center frequency which is proportional to the axial velocity [1]. A common way of estimating the blood velocity is to estimate the power spectral density (psd) of the slow-time signal. Displaying the psd as a function of time (the so-called sonogram or spectrogram) not only visualizes the blood velocity but also allows the operator to track the time-variations of the blood.

In ultrasound imaging, the psd is normally estimated using an averaged periodogram approach, also known as Welch's method [2]. However, as is well-known, the method suffers from either low resolution or high leakage, or both, and to achieve sufficient spectral resolution, the duration of the observation window has to be long. This implies that a large number of transmissions (about 100) has to be carried out to obtain a sufficiently accurate psd estimate. This lowers the frame-rate of the system significantly and makes the temporal resolution poor. This limits the usability of the system as it is desirable to update the B-mode image frequently to allow the physician to examine the surrounding tissue. As a result, there is a need for algorithms that yield sufficient spectral resolution from fewer image lines, thus allowing the frame-rate and the temporal resolution of the system to be increased.

In this paper, we formulate a data-adaptive blood velocity spectral estimator based on the matched filterbank (MAFI) framework [2,3]. The performance of the method is evaluated using realistic Field II [5] blood flow data, for both steady and unsteady flow. The method is compared to a Capon-based estimator and Welch's method. The latter is used extensively in commercial ultrasound systems. The simulations indicate that the proposed method offer a pronounced performance gain as compared to Welch's method, suggesting that the frame-rate may be increased significantly.

2. THEORY AND METHODS

To estimate the blood velocity at a given location, a number of transmissions are carried out in the same direction. After focusing, the resulting image lines represent a time series over depth. Let $y_k(l)$ denote the available (stationary) data sample at slow-time (transmission-number) l and fast-time (proportional to depth) k, of which the blood velocity spectrum is to be estimated. For a generic (axial) velocity v_z, $y_k(l)$ can be expressed as [1] (assuming that In-phase and Quadrature channels have been created)

$$y_k(l) = \alpha_{v_z} e^{j\left\{k\frac{\omega_c}{f_s} - l\frac{2\omega_c v_z}{c f_{prf}}\right\}} + w_k(l), \qquad (1)$$

for $k = 0, \ldots, K-1$ and $l = 0, \ldots, \widetilde{L}-1$, where α_{v_z} denotes the (complex-valued) amplitude of the sinusoidal signal at velocity v_z, assumed to be constant over the slow-time observation window, and $w_k(l)$ is the residual (or noise) term containing the signal components at all velocities different from v_z. Furthermore, $\omega_c = 2\pi f_c$, with f_c denoting the center frequency of the transducer, f_s the sampling frequency of the

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system, f_{prf} the pulse repetition frequency, and c the speed of sound. We note that the second term in the exponential in (1) is due to the time shift occurring when the blood particles move between transmissions. The problem of estimating the blood velocity spectral density can thus be expressed as forming an estimate of $|\alpha_{v_z}|^2$, for each velocity of interest. For a given depth (fast-time sample) k, let

$$\mathbf{y}_k(\ell) \triangleq \left[\begin{array}{cccc} y_k(\ell) & \dots & y_k(\ell+N-1) \end{array} \right]^T, \qquad (2)$$

for $\ell = 0, \dots, L-1$, with $L = \widetilde{L} - N + 1$ representing the total number of slow-time vectors in the time-series. By introducing $\phi = \omega_c / f_s$ and $\psi = -2\omega_c v_z / c f_{prf}$, (2) can be written as

$$\mathbf{y}_k(\ell) = \alpha_{v_z} \mathbf{a}_\psi e^{jk\phi + j\ell\psi} + \mathbf{w}_k(\ell), \qquad (3)$$

where $\mathbf{w}_k(\ell)$ is formed similar to $\mathbf{y}_k(\ell)$, and

$$\mathbf{a}_\psi \triangleq \left[\begin{array}{cccc} 1 & e^{j\psi} & \dots & e^{j\psi(N-1)} \end{array} \right]^T. \qquad (4)$$

Using the MAFI framework, we proceed to form a bank of matched filters, $\mathbf{h}_\psi$, for each ψ of interest, such that

$$\mathbf{h}_\psi = \arg\min_{\mathbf{h}_\psi} \mathbf{h}_\psi^* \mathbf{Q}_\psi \mathbf{h}_\psi \quad \text{subject to} \quad \mathbf{h}_\psi^* \mathbf{a}_\psi = 1, \qquad (5)$$

where $\mathbf{Q}_\psi = E\{\mathbf{w}_k(\ell)\mathbf{w}_k^*(\ell)\}$ denotes the covariance matrix of the noise term, where, by notation, we stress that $\mathbf{Q}_\psi$ will depend on ψ. Here, $E\{\cdot\}$ and $(\cdot)^*$ denote the expectation and the conjugate transpose, respectively. Thus, the filter centered at a given velocity represented by ψ, $\mathbf{h}_\psi$, will minimize the noise power at the filter output, while being constraint to pass the sinusoidal component at ψ undistorted. As is well known, the solution to (5) is given as [2]

$$\mathbf{h}_\psi = \frac{\mathbf{Q}_\psi^{-1} \mathbf{a}_\psi}{\mathbf{a}_\psi^* \mathbf{Q}_\psi^{-1} \mathbf{a}_\psi}. \qquad (6)$$

We proceed to note that

$$\mathbf{h}_\psi^* \mathbf{y}_k(\ell) = \alpha_{v_z} e^{jk\phi + j\ell\psi} + \mathbf{h}_\psi^* \mathbf{w}_k(\ell), \qquad (7)$$

suggesting that α_{v_z} can be estimated by averaging the L slow-time vectors at depth k

$$\hat{\alpha}_{v_z}^{(k)} = \mathbf{h}_\psi^* \mathbf{Y}_\psi^{(k)}, \qquad (8)$$

where

$$\mathbf{Y}_\psi^{(k)} \triangleq \frac{1}{L} \sum_{l=0}^{L-1} \mathbf{y}_k(l) e^{-jk\phi - jl\psi}. \qquad (9)$$

Typically, one can assume that the velocity of the blood is slowly varying over depth [1]. An improved estimate can be formed by averaging over depth (fast-time)

$$\hat{\Phi}_\psi = \frac{1}{K} \sum_{k=0}^{K-1} \left| \hat{\alpha}_{v_z}^{(k)} \right|^2. \qquad (10)$$

To evaluate (10), it remains to form an estimate of $\mathbf{Q}_\psi$. From (3), we note that

$$\mathbf{R} \triangleq E\{\mathbf{y}_k(\ell)\mathbf{y}_k^*(\ell)\} = |\alpha_{v_z}|^2 \mathbf{a}_\psi \mathbf{a}_\psi^* + \mathbf{Q}_\psi, \qquad (11)$$

suggesting the estimate

$$\hat{\mathbf{Q}}_\psi = \hat{\mathbf{R}} - |\hat{\alpha}_{v_z}|^2 \mathbf{a}_\psi \mathbf{a}_\psi^* \qquad (12)$$

$$= \hat{\mathbf{R}} - \frac{1}{K} \sum_{k=0}^{K-1} \mathbf{Y}_\psi^{(k)} \mathbf{Y}_\psi^{(k)*} \qquad (13)$$

where the second equality follows from (8), and where

$$\hat{\mathbf{R}} = \frac{1}{LK} \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} \mathbf{y}_k(l) \mathbf{y}_k^*(l) \qquad (14)$$

Given the close resemblance to the APES[1] power spectral estimator, proposed in [4], we term the estimator in (10), using (13), the *B*lood spectral *APES* (BAPES) estimator.

For performance analysis, we compare the proposed method with both a Capon- and a Welch-based blood spectral estimator. To form the Capon-based blood spectral estimate, write (see, e.g., [2] for further details on this estimator)

$$\tilde{\Phi}_\psi = \frac{1}{\mathbf{a}_\psi^* \hat{\mathbf{R}}^{-1} \mathbf{a}_\psi}. \qquad (15)$$

We term this estimator the *B*lood spectral *P*ower *C*apon (BPC) estimator. Finally, we note that the Welch estimate can be formed as

$$\breve{\Phi}_\psi = \mathbf{a}_\psi^* \hat{\mathbf{R}} \mathbf{a}_\psi. \qquad (16)$$

3. RESULTS

In this section, we will evaluate the performance of the discussed estimators using both simplistic artificial data and realistic Field II blood flow data with both steady and unsteady flow. Initially, we evaluate performance on simplistic data with a single velocity component at $v_z = 0.5$ m/s, observed in white zero mean circularly symmetric Gaussian distributed noise with a signal-to-noise-ratio of 57 dB. Further, $f_{prf} = 10$ kHz, $f_c = 7$ MHz, $c = 1540$ m/s and $f_s = 100$ MHz. The slow-time and fast-time observation windows were chosen to be $K = 40$ and $L = 9$, respectively. Fig. 1 shows the resulting blood spectral estimates using a filter of length $N = 8$. As is clear from the figure, the adaptive estimators (BAPES and BPC) clearly outperform the Welch-based estimator, with the BAPES estimator showing the best performance.

We proceed to examine a realistic steady flow, simulated using the simulation tool Field II [5]. A cylindric vessel with a parabolic flow profile was created, where the number of blood scatterers was chosen so that one resolution cell[2] contained at

[1] The acronym APES stands for *A*mplitude and *P*hase *ES*timation.

[2] One resolution cell was $\lambda \times \lambda \times \lambda$, where λ is the wavelength of the ultrasound.

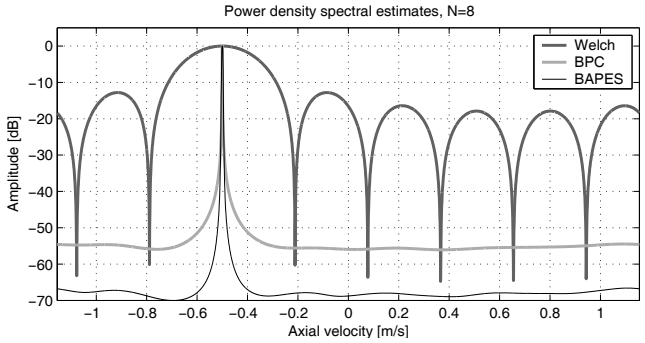

Fig. 1. The blood spectral estimates for the simplistic model with velocity $v_z = 0.5$ m/s. The BAPES estimate displays better sidelobe levels as compared to both BPC and especially Welch's method.

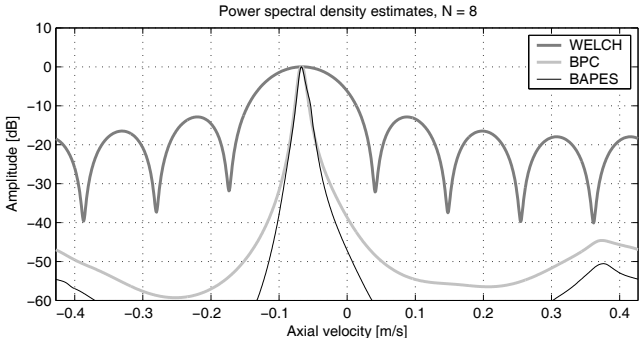

Fig. 2. The blood spectral estimates for a cylinder flow vessel with axial velocity $v_z = 0.071$ m/s. The ultrasound transducer has a certain spatial resolution and therefore, a distribution of velocities will be measured.

least ten particles to assure fully developed speckle. The vessel had a radius of 5 mm and was positioned at a depth of 25 mm. The maximum velocity in the vessel was 0.1 m/s, with a beam to flow angle of 45°, implying that the measured axial velocity will be about 0.071 m/s. A 12 MHz linear array transducer with 128 elements was simulated. The excitation waveform was an eight cycle sinusoid at the center frequency of the transducer. Data was acquired with f_{prf}= 10 kHz. The transmitting aperture was the central 30 elements, focused at a depth of 25.7 mm. The backscattered signals were recorded by all 128 elements and beamformed with dynamic focusing using a Hanning window as aperture weighting. The velocity of the moving blood particles was estimated in the middle of the vessel using the discussed blood spectral estimation techniques, using $N = 8$, $K = 40$ and $L = 9$. Fig. 2 shows the resulting blood spectral estimates. The figure indicates a similar performance as was observed for the simplistic data model. It should be noted that since the ultrasound system has a finite spatial resolution, a distribution of blood particles moving with different velocities will be measured in every transmission. Therefore, the velocity spectral estimates will consist of a distribution of velocities. The results in Fig. 2 indicate that by applying adaptive spectral estimation techniques, much can be gained in terms of velocity resolution.

We proceed to examine the velocity resolution as a function of N. Using the cylinder flow data, with $K = 40$ and $L = N+1$, we evaluate the full width half maximum (FWHM) of the blood spectral estimates for varying filter lengths. Fig. 3 clearly indicates the superior performance of the adaptive estimators as compared to Welch's method, with the BAPES estimator yielding better resolution than BPC for large values of N.

Finally, we examine the performance for realistic blood flow data with unsteady flow being affected by strong tissue interference. A 5 MHz 128 element linear array transducer was simulated. The excitation waveform was a four cycle sinusoid at the center frequency of the transducer and the f_{prf}

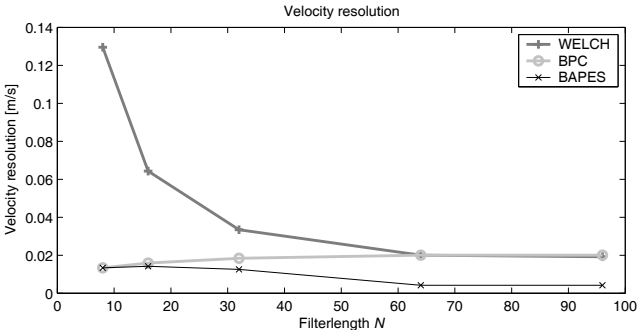

Fig. 3. The velocity resolution (FWHM) for the discussed methods for varying filter lengths, N. The BPC and BAPES display significant improvements in spectral resolution for small N as compared to Welch. As N increases, BAPES gives slightly better resolution as compared to both BPC and Welch.

was 15 kHz. The target was a vessel situated at a depth of 38 mm with a radius of 4.2 mm. The flow profile was parabolic over the vessel and varying with time. The flow was chosen to resemble the flow profile of the femoral artery in the leg. The Womersley model [6] for pulsating flow in a vessel was used to generate realistic flow data. A tissue signal was superimposed on the flow model being 40 dB stronger than the blood signal.

Prior to velocity estimation, the tissue signal was removed by a mean subtraction filter [1] of length N. The velocity of the moving blood particles was thereafter estimated in the middle of the vessel. Here, $N = 96$, $K = 40$ and $L = 97$. The resulting spectrogram as a function of time is shown in Fig. 4 where the dynamic range is 40 dB. The three methods all display sufficient spectral resolution to represent the velocity distribution as a function of time. The analysis was repeated for $N = 8$, $K = 40$ and $L = 9$. As seen in Fig. 5 the adaptive methods still manage to represent the blood velocity

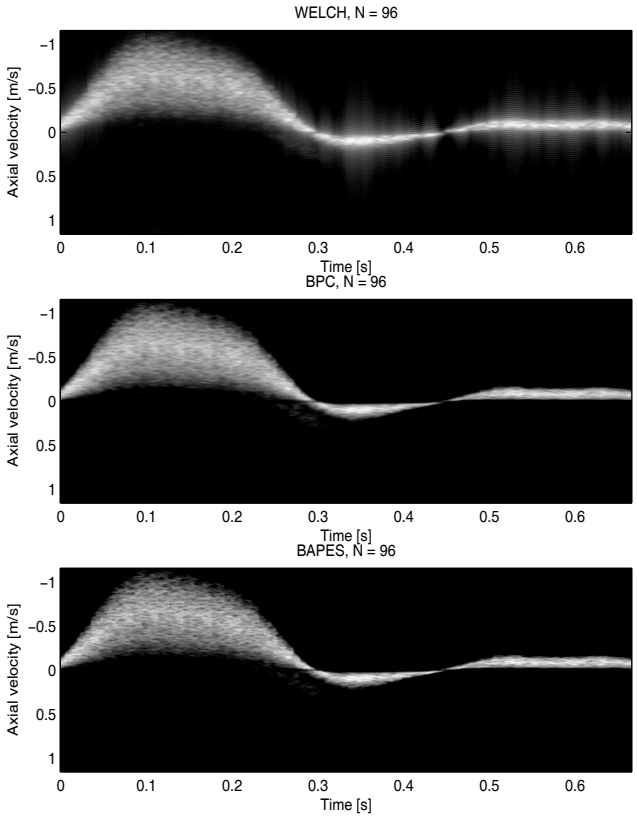

Fig. 4. The blood spectral estimates for an unsteady flow, using $N = 96$. The dynamic range is 40 dB. All three methods display similar performance and it is easy to track the changes in the blood flow. However the Welch estimate shows more spectral leakage as compared to BPC and BAPES.

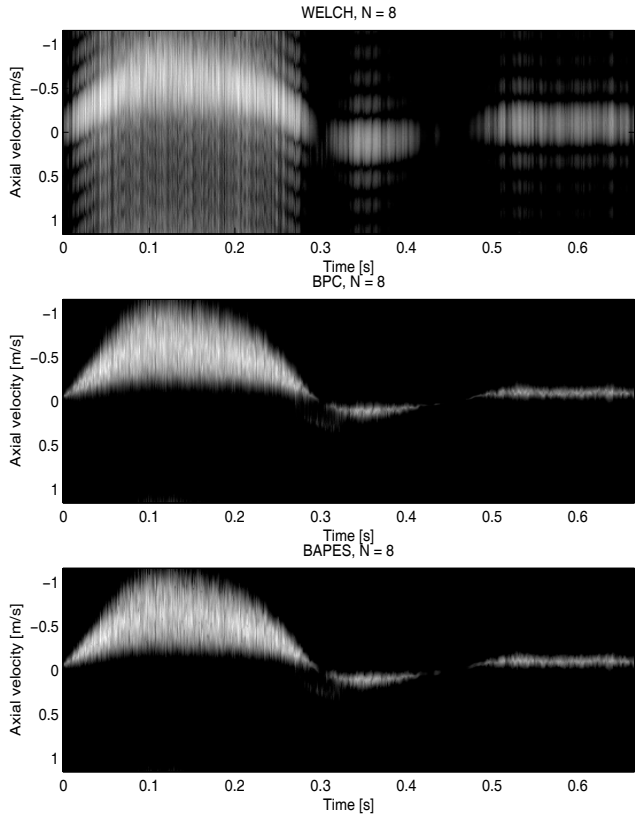

Fig. 5. The blood spectral estimates for an unsteady flow, using $N = 8$. The dynamic range is 40 dB. It is no longer possible to rely on the Welch estimate due to significant spectral leakage. However, the BPC and BAPES still manage to represent the unsteady blood velocity.

distribution satisfactory, while the Welch-based estimate has much too poor resolution to give any information about the properties of the moving blood.

4. CONCLUSION

In this paper, we have proposed a novel data-adaptive blood velocity spectral estimator based on the matched filterbank framework. The proposed method was compared to the averaged periodogram (Welch's method) and to a Capon-based approach. Via extensive simulations using both simplistic and realistic steady and unsteady blood flow data, we have concluded that the adaptive (both BPC and BAPES) estimators offer a significant performance gain as compared to the Welch-based blood velocity estimator used in commercial ultrasound scanners. We conclude that adaptive spectral estimation techniques can potentially be used to improve the temporal resolution and frame rate of blood velocity estimation systems in medical ultrasound.

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