AN ANALOGUE COMPENSATION SCHEME IN ADAPTIVE ACTIVE CONTROL SYSTEM WITH SIGNIFICANT SECONDARY PATH VARIATION

Ying Song, Wendong Xiao and Xiang Chen

Institute for Infocomm Research, 21 Heng Mui Keng Terrace, Singapore
{sying, wxiao, xchen}@i2r.a-star.edu.sg

ABSTRACT

Online secondary path modeling is used to deal with varying secondary path in active control systems. However, instability may happen when there is fast "significant" changing due to the tracking speed limitation of online secondary path modeling. In this paper a scheme involving analogue compensator is proposed to be used in the secondary path with fast "significant" changing. The function of the analogue compensator is analyzed and the guideline for design is given. Simulations have been carried out to compare the performance of the proposed scheme and the existing online secondary path modeling approach. The result shows that the proposed scheme is more effective for fast "significant" secondary path variation.

Index Terms— active control, FxLMS, online secondary path modeling, adaptive filter.

1. INTRODUCTION

Fig.1. illustrates the block diagram of the feed-forward filtered-x least mean square (FxLMS) algorithm. Where $G(z)$ is transfer function of the secondary path; $\hat{G}(z)$, $W(z)$, $x(n)$, $d(n)$ and $e(n)$ denote the estimation of $G(z)$, the transfer function of adaptive filter, reference signal, desired signal and residual error signal respectively.

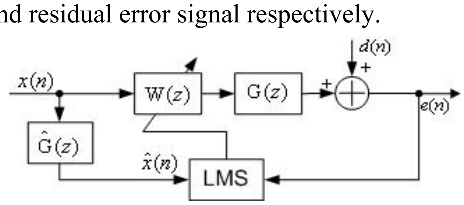

Fig.1. Block diagram of feed-forward FxLMS

The estimation of the secondary path is obtained by offline method [1] in conventional FxLMS algorithm. However, in most realistic application, the secondary path is always varying, and it causes unavoidable error between secondary path and its estimation. This leads to the poor performance or system instability [2-6]. For that reason, the approaches of online secondary path modeling [7-9] are proposed.

Online secondary path modeling performs secondary path identification during operation of the active control system. It consists of an additional adaptive filter to track the varying secondary path and injection of an identification noise into the system [1]. Fig.2 shows a typical block diagram [7], where $v(n)$ is the injected noise which is uncorrelated with the reference signal $x(n)$.

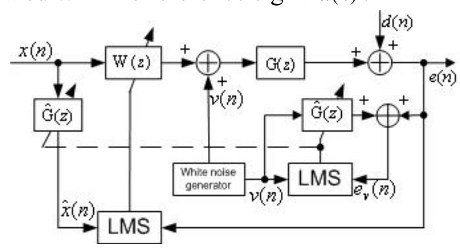

Fig.2. Block diagram of an online secondary path modeling

Online secondary path modeling works well for the secondary path with slow varying. However, the stability and performance are restricted by the updating speed of the secondary path tracking when the secondary path is with fast and "significant" changing. In this paper, an analogue compensator scheme specifically designed for secondary path exhibiting "significant" variations is presented. By adding a properly designed analogue compensator, the offline method can still be used in the secondary path with fast and "significant" variation cases.

The paper is organized as follows. In section 2, we proposed the analogue compensation scheme; the functionality of the analogue compensator is analyzed and design methodology is presented. In Section 3 the computer simulations are conducted to show the effectiveness of the proposed scheme. Finally we draw the conclusion in Section 4.

2. PROPOSED SCHEME

2.1. Analogue Compensator Scheme

Active control function was realized firstly with analogue controller. In [2, 10] the analogue controller was used in the digital and analogue active noise control system. In [10] we presented a robust hybrid analogue/digital active noise control headset system by adding an analogue feedback filter loop. The current work is an extension of the previous study. It is found that the added analogue feedback filter actually works as a compensator to compensate the secondary path changing. That is, a new secondary path with smaller variation degree can be formed with a properly designed analogue compensator.

The block diagram of the proposed analogue compensator scheme is illustrated in Fig. 3, where $H(s)$ and $G_i(s)$ denote the transfer functions of analogue compensator and the secondary path at status i ($i \in 0,...N-1$), respectively. When the dashed block is replaced by an equivalent secondary path transfer function $C_i(z)$, the estimation of the secondary path is replaced by $\hat{C}_i(z)$. In this case, the system is actually identical to a conventional feed-forward FxLMS shown in Fig.1.

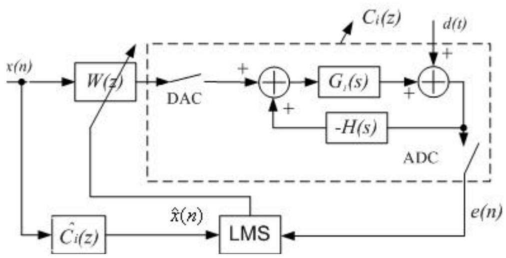

Fig.3. Block diagram of analogue compensator scheme

With the analogue compensator inserted, the new secondary path transfer function can be obtained:

$$C_i(s) = \frac{G_i(s)}{1+G_i(s)H(s)}; w < w0 \tag{1}$$

The Laplace transform rather than the z-transform is used to avoid confusion since compensator is analog. To ease the formulation we let $s = jw$, and define $H(jw)$ and $G_i(jw)$ as:

$$H(jw) = H(w)\exp(j\theta_H(w)); G_i(jw) = G_i(w)\exp(j\theta_i(w)); \tag{2}$$

It also can be presented as:

$$C_i(jw) = (G_i(w)/A_i(w))\exp(j\beta_i(w)); \tag{3}$$

Where:

$$A_i(w) = \sqrt{1+2G_i(w)H(w)\cos(\gamma_i(w)) + (G_i(w)H(w))^2};$$
$$\beta_i(w) = \theta_i(w) - \gamma_i(w) - \varphi_i(w);$$
$$\gamma_i(s) = \theta_H(s) + \theta_i(s);$$
$$\varphi_i(w) = arctg\frac{G_i(w)H(w)\sin(\gamma_i(w))}{1+G_i(w)H(w)\cos(\gamma_i(w))} - \gamma_i(w);$$

When original secondary path changes from $G_l(jw)$ to $G_i(jw)$, where $l \neq i$, the magnitude ratio of the compensated secondary path between two statuses is:

$$\left|\frac{C_l(jw)}{C_i(jw)}\right| = \frac{G_l(w)}{G_i(w)}\frac{A_i(w)}{A_l(w)}; \tag{4}$$

The phase difference for compensated secondary path between two statuses is:

$$\cos(\beta_i(w)-\beta_l(w)) = \frac{1}{A_l(w)A_i(w)}\Big[\cos((\theta_i(w)-\theta_l(w)) + G_l(w)H(w)\cos(\gamma_i(w)) + G_i(w)H(w)\cos(\gamma_l(w)) + G_l(w)G_i(w)(H(w))^2\Big] \tag{5}$$

2.2. Analogue Compensator Design Method

The system stability and the control performance are the key concerns to design analogue compensator. Two factors affect the overall system stability. The first one is the stability of the analogue feedback loop which consists of the original secondary path and the analogue compensator. To ensure a stable analogue feedback loop, all poles of the closed-loop transfer function of $C_i(s)$ must be in the open left-hand part of s plane [2]. Yu *et al.* presented detailed design method [11] to achieve this property. The second factor is the 90º phase error condition between real and estimated secondary path for FxLMS algorithm [3-4]. Suppose the estimation of compensated secondary path is obtained at status $i = 0$ then $\cos(\beta_i(w) - \beta_0(w)) > 0$ is required to keep FxLMS algorithm stable.

$$\cos(\theta_i(w)-\theta_0(w)) + G_0(w)H(w)\cos(\gamma_i(w)) + G_i(w)H(w)\cos(\gamma_0(w)) + G_i(w)G_0(w)(H(w))^2 > 0; \tag{6}$$

Beside to meet the above stability conditions, it is also required:

$$\mathrm{var}(|C_i(jw)|) \le \mathrm{var}(|G_i(jw)|); i \in (0,...N-1); \tag{7}$$

where var presents the variance, which is the constraint for compensating magnitude changing.

The following equation is the constraint for compensating the large phase difference.

$$\Big[K(w) + G_l(w)H(w)\cos(\gamma_i(w)) + G_i(w)H(w)\cos(\gamma_l(w)) + G_l(w)G_i(w)(H(w))^2\Big] \frac{1}{A_l(w)A_i(w)} > K0; i \in (0,...N-1); l \in (0,...N-1); i \neq l; \tag{8}$$

where $K(w) = \cos(\theta_i(w) - \theta_l(w))$, $K0$ is constant which may be chosen as $K0 = \cos(50^\circ)$.

For easy to observe the compensator function in the proposed scheme, we rewrite the equation (4) as:

$$\frac{|C_l(jw)|}{|C_i(jw)|} = \frac{G_l(w)}{G_i(w)}\sqrt{\frac{(G_iH+\cos\gamma_i)^2+\sin^2\gamma_i}{(G_lH+\cos\gamma_l)^2+\sin^2\gamma_l}}; \tag{9}$$

Similarly, the equation (5) can be rewritten as:

$$\cos(\beta_i-\beta_l)=\frac{(G_lH+\cos\gamma_l)(G_iH+\cos\gamma_i)+\cos(\theta_i-\theta_l)-\cos\gamma_i\cos\gamma_l}{\sqrt{\left((G_iH+\cos\gamma_i)^2+\sin^2\gamma_i\right)\left((G_lH+\cos\gamma_l)^2+\sin^2\gamma_l\right)}} \tag{10}$$

It is clear from equations (9) and (10), when the increase of $|H(jw)|$, the magnitude compensation factor $\sqrt{(1+2G_iH\cos\gamma_i+(G_iH)^2)/(1+2G_lH\cos\gamma_l+(G_lH)^2)}$ will approach to G_i/G_l, this leads to $|C_i(jw)/C_l(jw)|$ approaching to 1. Similar to the phase compensation from equation (10), we have as the increase of $|H(jw)|$, $\cos(\beta_i-\beta_l)$ will approach to 1. It means when meeting the requirement of the stability condition, higher gain compensator can improve the compensation performance. The phase compensation effect is not as simple as magnitude compensation. However, suppose $G_i(w)/G_l(w)>1$, the modification effect of γ_l is more obvious than γ_i according to equations (10). This is the second interesting finding. Since the secondary path changing in phase is different for different frequencies, it is impossible to get the optimum compensation for every frequency. Therefore we notice those large phase difference compensation and balance the other part to design the phase.

In summary, the design of the analogue compensator requires detailed knowledge of secondary path variation. We need to meet those analogue feedback loop and FxLMS algorithm 90° stability conditions first. Large magnitude of the compensator will reduce the phase determination degree, therefore ease the compensator design. The compensator phase needs to be chosen carefully to obtain an overall optimum compensation.

3. SIMULATIONS AND DISCUSSIONS

To evaluate the effectiveness of the proposed scheme computer simulations were conducted. The Eriksson's online secondary path modeling method and conventional feed-forward FxLMS were used for the comparison.

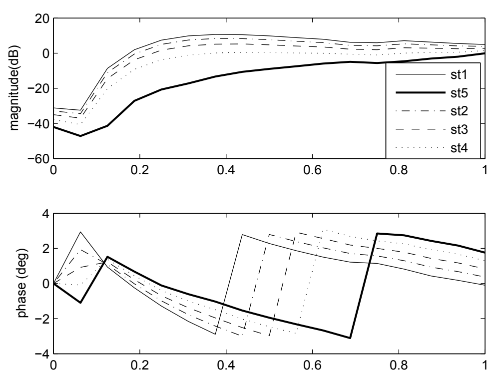

Fig.4. 5 groups of secondary path frequency response

The "significant" secondary path changing can be classified into 1) slight and slow per occurrence but large accumulated changing with time; 2) large changing at each occurrence. We will not explicitly consider the mixture of above two types in our simulations.

In the simulation, the frequency response of the secondary path transfer function without compensation varying in unit frequency is shown in Fig.4. There are 5 groups of secondary path transfer functions corresponding to 5 statuses changing which label as st1, st2, st3, st4 and st5. The st1 and st5 are obtained by measuring headsets plant in [10]. The st2, st3 and st4 are generated by magnitude and phase interpolation separately using st1 and st5. The frequency response of analogue compensator is shown in Fig.5. It also employs the analogue filter in [10]. The estimation of the secondary path used in the conventional FxLMS and proposed schemes are st1 and compensated st1. The reference signal $x(n)$ is a tone noise with 0.5 normalized frequency buried in the wide-band noise. The injected noise $v(n)$ is white Gaussian noise with variance equal to 0.0001. The $d(n)$ signal is obtained by $x(n)$ passes all-pass filter.

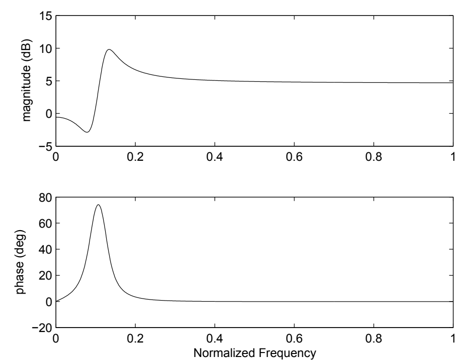

Figure 5 – Frequency response of the analogue compensator

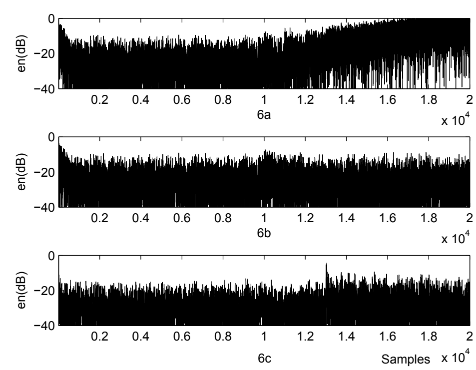

Fig.6. Residual noise for "significant" variation type1 6a).Conventional feed-forward FxLMS; 6b).Online secondary path modeling; 6c).Proposed analogue compensation scheme;

Fig.6 shows the result where the secondary path with "significant" varying type1. The changing is occurs at samples 10000, 11000, 12000 and 14000 which correspond to the secondary path status changing from st1 to st2, st2 to st3, st3 to st4 and st4 to st5 respectively. When every secondary path varying occurs, the changing in the amplitude and the phase are small. However, the accumulated changing, such as from st1 to st5, is “significant”.

Fig.6a shows the result of the conventional feed-forward FxLMS. Until the sample 10000, the algorithm converges. Even in transit from st1 to st2, the system is still in convergence. However, starting in the transition from st2 to st3, it gradually diverges. It shows that conventional FxLMS algorithm has certain degrees of adaptation ability to small secondary path changing. Fig.6b shows the result of online secondary path modeling. It works well for this kind of secondary path variation. The residual noise level keeps consistent. Fig.6c shows the result of the proposed scheme. Although the residual noise level suddenly increases at the sample 13000, it gradually converges to a lower level.

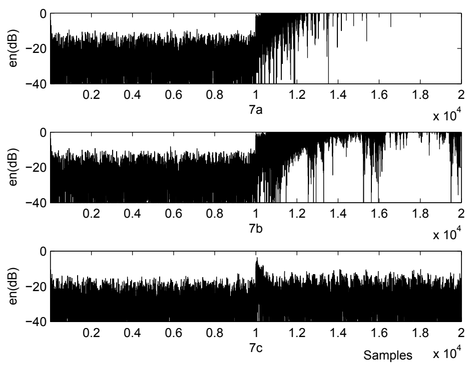

Fig.7.Residual noise for “significant” variation type2 7a).Conventional feed-forward FxLMS; 7b).Online secondary path modeling; 7c).Proposed analogue compensation scheme

Fig.7 shows the simulation results where the fast “significant” varying of the secondary path occurs at sample 10000. From sample 1 to 9999, the secondary path is in the st1 status. Staring from sample 10,000, the secondary path is changed from st1 to st5.

Fig.7a shows that the conventional FxLMS starts divergence at the sample 10,000. Fig.7b shows a similar result using the online secondary path modeling method. Fig.7c shows the result of the proposed scheme, where we can find it still manages to converge although there is a transient surge of the residual noise level at sample 10,000. It is observed that the noise level in the period of samples 10000 to 20000 is higher than that in the period of samples 1 to 9999. The reason is the proposed scheme actually works as the conventional FxLMS except the change of the extent of the secondary path variation.

Simulation result shows that the proposed scheme is effective in the secondary path with both “significant” changing types, particularly in the fast “significant” variation secondary path cases.

4. CONCLUSION

In this paper an analogue compensation scheme has been proposed for active control system with “significant” secondary path variations. Using a properly designed analogue compensator, the fast “significant” variation path can be changed to one with smaller variations which can increase the overall system stability and performance. Comparing with the online secondary path modeling approach, the proposed scheme is more suitable for the system with fast “significant” changing secondary path. Simulations verify the theoretical analysis.

REFERENCES

[1] S. M. Kuo and D. R. Morgan, *Active noise control systems: algorithms and DSP implementations*, New York, John Wiley, 1996.

[2] S. J. Elliott, *Signal processing for active noise control*, London: Academic Press, 2001.

[3] D. R. Morgan, “An analysis of multiple correlation cancellation loops with a filter in the auxiliary path,” *IEEE Trans. on ASSP.,* vol. ASSP-28, pp. 454-467, 1980.

[4] S. D. Snyder and C. H. Hansen, “The effect of transfer function estimation errors on the filtered-X LMS algorithm,” *IEEE Trans. on SP.,* vol. 42, pp. 950-953, 1994.

[5] A. K. Wang and W. Ren, “Convergence analysis of the multi-variable filtered-x LMS algorithm with application to active noise control,” *IEEE Trans. on SP.,* vol. 47, pp. 1166-1169, 1999.

[6] S. J. Elliott and P. A. Nelson, “Algorithms for the active control of periodic sound and vibration,” in ISVR Technical Memorandum, No. 679, University of Southampton, 1986.

[7] L. J. Eriksson and M. C. Allie, “Use of random noise for on-line transducer modelling in an adaptive attenuation system,” *JASA,* vol. 85, pp. 797-802, 1989.

[8] S. M. Kuo and D. Vijayan, “A secondary path estimated techniques for active noise control systems,” *IEEE Trans. SAP.,* vol. 5, pp. 374-377, 1997.

[9] M. Zhang, H. Lan and W. Ser, “A robust secondary path modelling method with auxiliary noise power scheduling strategy and norm constraint manipulation,” *IEEE Trans. on SAP.*, vol.11, pp. 45-53, Jan, 2003.

[10] Y. Song, Y. Gong and S. M. Kuo, “A robust hybrid feedback active noise cancellation headset,” *IEEE Trans. on SAP.,* vol.13, pp. 607-617, July, 2005.

[11] S. H. Yu and J. S. Hu, “Controller design for active noise cancellation active noise control headphones using experimental raw data,” *IEEE/ASME Trans. Mechatronics.,* vol.6, pp.483-490, 2001.