OPTIMAL POLARIZED WAVEFORM DESIGN FOR ACTIVE TARGET PARAMETER ESTIMATION USING ELECTROMAGNETIC VECTOR SENSORS

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ABSTRACT

We develop optimal design methods of polarimetric radar signals that improve the estimation accuracy of the target parameters. A weighted sum of the Cramér-Rao bound (CRB) of the parameters of interest is used as the cost function of the optimization. We employ an array of electromagnetic vector sensors to fully recover the polarization information from the target returns. Simulation examples illustrate the improved system performance.

1. INTRODUCTION

In active sensing systems, the polarimetric aspects of the reflected signals can be exploited to improve the parameter estimation accuracy and the resolution of targets. In practice, the target scattering matrix is usually unknown and also changes as the target moves with respect to the radar system. Hence, in order to select the optimal polarimetric signal, it must be estimated from the target echoes jointly with other target parameters of interest.

The problem of selecting the transmit polarized signal has been receiving increasing attention in recent years. For example, Wang and Nehorai [1] have developed an adaptive waveform design algorithm for a target in the presence of compound-Gaussian clutter intended to improve the estimation of the scattering matrix. Hochwald and Nehorai [2] have briefly discussed the optimal transmit signal for estimating the polarimetric response of a distributed target where the entries of its scattering matrix are modeled as complex Gaussian random variables. However, the active models and algorithms presented in [1] and [2] were developed assuming a static target located at a known position.

In this paper, we consider a radar system that supports waveform diversity, including polarization agility, i.e. the polarimetric characteristics of the transmit signal can be changed in time. We assume the receiver is able to fully exploit the polarization information using vector sensors to measure the six components of the electromagnetic (EM) field [3]. It has been shown that employing vector sensors improves the estimation of the signal direction of arrival and resolution of closely spaced signal arrivals [3].

We address the problem of selecting the radar polarized waveform that minimizes the estimation error of the target parameters, i.e. bearing angles, range, Doppler shift, and scattering matrix. The optimal polarized waveform is selected in order to minimize a weighted sum of the Cramér-Rao bound (CRB) of these parameters. By computer simulations, we illustrate the improved performance on the parameter estimation for different scenarios.

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2. MEASUREMENT MODEL

We consider a target characterized by azimuth ϕ, elevation ϑ, range r, Doppler shift ω_{D}, and scattering matrix S_{t}. These parameters are assumed to be deterministic and unknown. To uniquely identify the polarimetric aspects of the target, polarization diversity is required and the complete EM information of the signal reflected from the target has to be processed [2]. To provide these measurements, we assume the receiver of the active sensing system is an array of electromagnetic vector sensors [3], where each sensor measures the six components of the EM field. These 6D vector sensors provide several advantages, such as ability to resolve coherent and closely spaced signals, improving the estimation accuracy of the signal direction-of-arrivals, and avoiding spatial undersampling ambiguities [3].

In the following sections, we extend the active model presented in [2] for different scenarios of interest.

2.1. High-Elevation Target

Consider an array of M vector sensors receiving the signal returns from a target at high elevation, free of interferences and clutter returns. The complex envelope of the measurements can be expressed as

$$\boldsymbol{y}(t) = A(\boldsymbol{\theta}_{\mathrm{t}}) S_{\mathrm{t}} \boldsymbol{\xi}(t-\tau) e^{j\omega_{\mathrm{D}} t} + \boldsymbol{e}(t), \;\; t = 1, \ldots, N, \quad (1)$$

where $\boldsymbol{\theta}_{\mathrm{t}} = [\phi, \vartheta]$ is the bearing angles vector. The matrix $A(\boldsymbol{\theta}_{\mathrm{t}}) = \boldsymbol{q}(\boldsymbol{\theta}_{\mathrm{t}}) \otimes V(\boldsymbol{\theta}_{\mathrm{t}})$ is the array response, where $\otimes$ is the Kronecker product, $\boldsymbol{q}(\boldsymbol{\theta}_{\mathrm{t}}) = \left[e^{j2\pi \boldsymbol{k}^T \boldsymbol{r}_{1/\lambda}}, \ldots, e^{j2\pi \boldsymbol{k}^T \boldsymbol{r}_{M/\lambda}}\right]^T$ represents the phase of the planewave arriving in the direction given by the vector $\boldsymbol{k}$ at the position $\boldsymbol{r}_m$ of the m^{th} sensor ($m = 1, \ldots, M$), λ is the signal wavelength, and $V(\boldsymbol{\theta}_{\mathrm{t}})$ is the response of a single vector sensor given by [3]

$$V(\boldsymbol{\theta}_{\mathrm{t}}) = \begin{bmatrix} -\sin\phi & -\cos\phi\sin\vartheta \\ \cos\phi, & -\sin\phi\sin\vartheta \\ 0 & \cos\vartheta \\ -\cos\phi\sin\vartheta & \sin\phi \\ -\sin\phi\sin\vartheta & -\cos\phi \\ \cos\vartheta & 0 \end{bmatrix}. \quad (2)$$

The complex scattering matrix S_{t} represents the polarization change of the transmit signal upon its reflection on the target:

$$S_{\mathrm{t}} = \begin{bmatrix} s_{\mathrm{hh}} & s_{\mathrm{hv}} \\ s_{\mathrm{vh}} & s_{\mathrm{vv}} \end{bmatrix}. \quad (3)$$

The variables s_{hh} and s_{vv} are the co-polar scattering coefficients, whereas s_{hv} and s_{vh} are the cross-polar coefficients. For the monostatic radar case, $s_{\rm hv} = s_{\rm vh}$. The polarized transmit wave is a narrowband signal which can be represented by a complex vector [2], [3]

$$\boldsymbol{\xi}(t) = \begin{bmatrix} \xi_{\rm h}(t) \\ \xi_{\rm v}(t) \end{bmatrix} = s(t)Q(\alpha)\boldsymbol{w}(\beta), \tag{4}$$

where

$$Q(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}, \quad \boldsymbol{w}(\beta) = \begin{bmatrix} \cos\beta \\ j\sin\beta \end{bmatrix}.$$

The angles α and β are the orientation and ellipticity angles of the polarization ellipse depicted by the electric field vector. The function of time $s(t)$ represents the scalar complex envelope of the transmitted signal. The time delay is $\tau = 2r/c$, where c is the wave propagation speed. The vector $\boldsymbol{e}(t)$ is the additive noise corrupting the radar measurements. The number of recorded snapshots is N.

We note that the waveform design problem consists of selecting the envelope $s(t)$ and polarization angles α and β in equation (4). A different approach is selecting different waveforms for each component of the vector $\boldsymbol{\xi}(t)$, known as dual signal transmission [3]. We will discuss both methods in Section 3.

2.2. Low-Grazing-Angle Target

In this section, we discuss the measurement model for a vector-sensor array receiving the signal from a target located at low elevation angle, over a reflecting surface. Due to the presence of strong coherent interferences, this scenario represents a challenging problem that can be found in different defense applications, such as tracking sea-skimming missiles. Vector sensors become a valuable tool given their ability to resolve coherent and closely spaced signals.

Under the above conditions, the received signal is a superposition of the direct field and multipath components generated by surface reflections. The multiple signals reflected from the surface and impinging on the receiver can be modeled as coherent and incoherent components [4]-[6], which are also known as specular and diffuse multipath components. If the surface were flat and smooth, a four-path model would represent the total coherent target returns [7], [8], as sketched in Fig. 1. However, more multipath signals exist due to the irregularities of the surface. It is assumed that these additional components are the result of the reflection from many independent scatterer points. Because of the random nature of these points (and the surface irregularities), these reflections are added incoherently. Hence, the total field produced by the incoherent components behaves like noise. As a consequence of the central limit theorem, this total incoherent signal can be modeled as a random Gaussian process with zero mean [4], increasing the additive sensor noise [6].

In the presence of a reflecting surface, the measurement model for a vector-sensor array becomes

$$\begin{aligned} \boldsymbol{y}(t) \;&=\; \left[A(\boldsymbol{\theta}_{\rm t}) + e^{j\delta}A(\boldsymbol{\theta}_{\rm i})S_{\rm s}\right] S_{\rm t} \left[I_2 + e^{j\delta}S_{\rm s}\right] \\ &\quad \cdot\boldsymbol{\xi}(t-\tau)e^{j\omega_{\rm D}t} + \boldsymbol{e}(t), \;\; t = 1,\ldots,N, \end{aligned} \tag{5}$$

where $\boldsymbol{\theta}_{\rm i} = [\phi, \psi]$ is the vector of the interference bearing angles, ψ is the elevation angle of the interference, also known as the grazing angle. Note that both the target and the interference have the same azimuth ϕ. The angle δ is the phase shift due to the length difference between the direct radar-target path and the radar-surface-target path. For the low-grazing-angle case, the phase shift is approximated by $\delta \approx 4\pi h_{\rm r}h_{\rm t}/r_{\rm g}$, where $h_{\rm r}$ and $h_{\rm t}$ are the radar and target height, and $r_{\rm g} = r\cos\vartheta$ is the ground range. The surface scattering matrix is $S_{\rm s} = \rho_{\rm s}\cdot{\rm diag}(\gamma_{\rm h}, \gamma_{\rm v})$. The coefficients $\gamma_{\rm h}$ and $\gamma_{\rm v}$ are the Fresnel reflection coefficients; their expressions can be found in [6]. For the low-angle propagation problem over a sea-surface, they can be approximated by $\gamma_{\rm h} = \gamma_{\rm v} \approx -1$. The coefficient $\rho_{\rm s}$ represents a reduction in the magnitude of the reflected field due to the surface roughness [6]. I_2 is the 2×2 identity matrix. We assume that $h_{\rm r}$ and $\rho_{\rm s}$ are known.

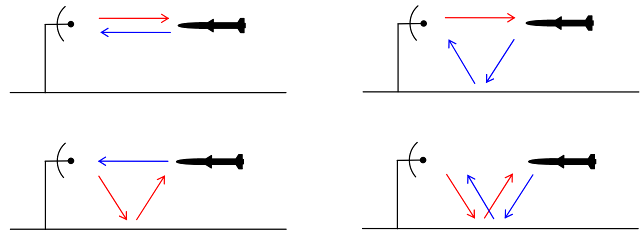

Fig. 1. Four-component signal multipath model.

3. OPTIMAL POLARIZED WAVEFORM DESIGN

The design of the waveform involves selecting the signal envelope and its polarization, see equation (4). Herein, we consider linear frequency modulated (LFM) pulses with Gaussian envelope, which are defined as

$$s(t) = \left(\pi\eta^2\right)^{-1/4}\exp\left(-\left(\frac{1}{2\eta^2} - jb\right)t^2\right), \tag{6}$$

where η is the pulse length and b is the frequency sweep rate. In addition, it is considered that in order to uniquely identify $S_{\rm t}$, the polarization must vary in time (see [1], [2]). Bellow, we present two different approaches for designing the polarized waveform.

Scheme A. Two consecutive polarized pulses with fixed duration η and frequency sweep rate b are transmitted. The polarization parameters α and β change for each pulse:

$$\boldsymbol{\xi}(t) = s(t)Q(\alpha_1)\boldsymbol{w}(\beta_1) + s(t-T_s)Q(\alpha_2)\boldsymbol{w}(\beta_2),$$

where $T_s = 7.4\eta$ is the effective pulse length, chosen to be the time interval over which the signal amplitude is greater than 0.1% of its maximum value.

Scheme B. Two simultaneous pulses are transmitted using the components of the vector signal $\boldsymbol{\xi}(t)$; method known as dual signal transmission [3]. The parameters η and b of each vector-signal component must be different in order to generate time-variant polarization:

$$\boldsymbol{\xi}(t) = \begin{bmatrix} s(t;\eta_{\rm h}, b_{\rm h}) \\ s(t;\eta_{\rm v}, b_{\rm v}) \end{bmatrix}.$$

For a fair comparison, the signal to noise ratio, which we define as SNR$= \int \|\boldsymbol{\xi}(t)\|^2 dt/\sigma^2$, is assumed to be constant for all the cases and waveform configurations.

Note that the current research can be extended by using waveform libraries instead of a single form of pulse, see for example [9].

3.1. Cost Function

To calculate the optimal transmitted signal parameters, it is necessary to define a cost function or a performance measure. The CRB is a universal lower bound on the variance of all unbiased estimators of a set of parameters and provides a measure of potential performance attainable by the system. This bound is defined as [10]

$$\mathrm{CRB}(\boldsymbol{\nu}) = \left\{ -\mathrm{E}\left[\frac{\partial \ln p(\boldsymbol{y})}{\partial \boldsymbol{\nu}} \frac{\partial \ln p(\boldsymbol{y})}{\partial \boldsymbol{\nu}^T} \right] \right\}^{-1}, \quad (7)$$

where $p(\boldsymbol{y})$ is the joint probability density function of the N measurements described in Section 2, $\boldsymbol{\nu} = [\phi, \vartheta, r, \omega_{\mathrm{D}}, \boldsymbol{s}]^T$ is the vector of target parameters, and the scattering coefficients are represented by $\boldsymbol{s} = [\mathrm{Re}\{s_{\mathrm{hh}}, s_{\mathrm{hv}}, s_{\mathrm{vv}}\}, \mathrm{Im}\{s_{\mathrm{hh}}, s_{\mathrm{hv}}, s_{\mathrm{vv}}\}]$. Assuming that the noise $\boldsymbol{e}(t)$ is independently and identically complex Gaussian distributed with zero mean and known covariance $\sigma^2 I_{6M}$, then the measurements $\boldsymbol{y}(t)$ are also Gaussian distributed. The derivation of a CRB expression can be considered arduous, but straightforward for Gaussian models; due to space constrains, it is not shown in this paper.

The aim of our optimal design is to minimize the estimation error of the target parameters. Hence, the cost function should be an operator summarizing the CRB matrix in a scalar value. Different optimization measures have been proposed based on this concept. D-optimality uses the determinant of the CRB [11] and A-optimality employs the trace [12]. However, these measures cannot be applied for our design problem. Due to the different physical nature of the target parameters, the variance of their estimators may differ in several orders of magnitude and units.

We apply a criterion that consists of a weighted sum of the CRB of each target parameter:

$$\begin{aligned} J \;=\; & c_\phi \mathrm{CRB}_\phi + c_\vartheta \mathrm{CRB}_\vartheta + c_r \mathrm{CRB}_r \\ & + c_{\omega_{\mathrm{D}}} \mathrm{CRB}_{\omega_{\mathrm{D}}} + c_{\boldsymbol{s}} \mathrm{tr}(\mathrm{CRB}_{\boldsymbol{s}}), \end{aligned} \quad (8)$$

where "tr" is the trace operator and the c variables are the weighting coefficients. In our problem, the c coefficients are intended to weight the CRB of the different target parameters.

In order order to evaluate the cost function J and the CRB, it is required to know the value of the target parameters. We assume that those parameters are estimated and provided by a tracking filter of the radar system as a prediction of the next target state.

4. NUMERICAL EXAMPLES

The simulation setup to test the proposed waveform design schemes consists of a single target characterized by scattering coefficients $s_{\mathrm{hh}} = 0.8 - j0.1$, $s_{\mathrm{hv}} = j0.2$ and $s_{\mathrm{vv}} = 0.5$, Doppler shift $\omega_{\mathrm{D}} = 1\mathrm{KHz}$, range $r = 10\mathrm{km}$, and azimuth $\phi = 45^\circ$. The elevation is $\vartheta = 15^\circ$ and $\vartheta = 0^\circ$ for each of the following examples. The receiver is an array of two vector sensors located along the y-axis, separated by 0.5λ ($\lambda = 0.3\mathrm{m}$). For the LFM signals applied on Scheme A, the duration parameter is $\eta = 1\mu\mathrm{s}$, the frequency rate is $b = 135 \cdot 10^9 \mathrm{s}^{-2}$ (bandwidth $B = 1\mathrm{MHz}$), and the polarization angles belong to the following intervals $\alpha \in [-90^\circ, 90^\circ]$ and $\beta \in [-45^\circ, 45^\circ]$. For Scheme B, the signals are constrained to $\eta \in [0.5, 2]\mu\mathrm{s}$ and $b \in [13.5, 1350] \cdot 10^9 \mathrm{s}^{-2}$ (maximum bandwidth $B = 20\mathrm{MHz}$).

Example 1. We first test the proposed waveform schemes for the case of a high-elevation target.

Scheme A is a generalization of the method used by conventional radars, which transmit pulses with fixed orthogonal polarization. Usually, they transmit linearly polarized signals ($\beta_1 = \beta_2 = 0^\circ$) with horizontal ($\alpha_1 = 0^\circ$) and vertical ($\alpha_2 = 90^\circ$) polarization orientation. However, this arbitrary selection of the signal orientations may not be optimal. Fig. 2 shows the optimization cost

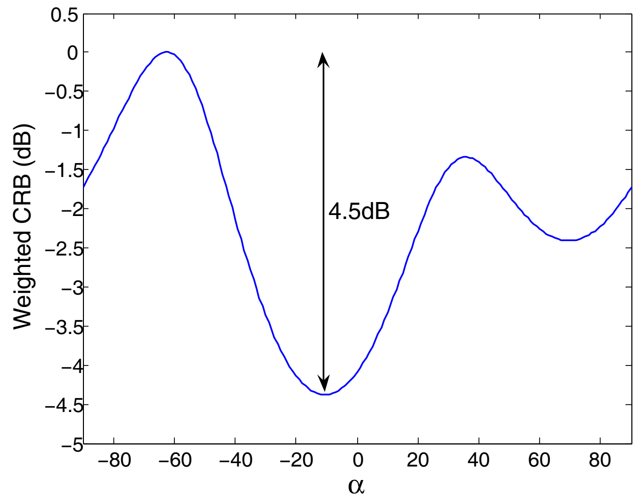

Fig. 2. Design cost as a function of the orientation angle of two orthogonally and linearly polarized pulses for the high-elevation target, in Example 1.

function J for two orthogonally and linearly polarized pulses as a function of their orientation angle, i.e. $\alpha_1 = \alpha$ and $\alpha_2 = \alpha + 90^\circ$. There is 4.5dB of performance difference between the worst and best signal orientation. More significant improvement can be achieved by removing the linear and orthogonal constrains. We have found an improvement of 10dB with respect to the worst case shown in Fig. 2 when the polarization of the pulses is given by $\alpha_1 = -1^\circ$, $\alpha_2 = 46^\circ$, $\beta_1 = 26^\circ$, and $\beta_2 = -42^\circ$.

For Scheme B, the optimal parameters that minimizes the cost function are $\eta_{\mathrm{h}} = 2\mu\mathrm{s}$, $\eta_{\mathrm{v}} = 2\mu\mathrm{s}$, $b_{\mathrm{h}} = 13.5 \cdot 10^9 \mathrm{s}^{-2}$ and $b_{\mathrm{v}} = 1350 \cdot 10^9 \mathrm{s}^{-2}$. The cost function for this set of parameters is 6dB higher than the optimal waveform provided by Scheme A.

Example 2. The former simulations are repeated for the case of a low-grazing-angle target. It assumed that the radar and target heights are $h_{\mathrm{r}} = h_{\mathrm{t}} = 50\mathrm{m}$, and the sea state is such that $\rho_s = 0.7$.

Fig. 3 shows that the performance difference between the worst and best polarization orientation for linearly polarized signals is 2.3dB. The optimal parameters provided by Scheme A are $\alpha_1 = 0^\circ$, $\alpha_2 = -4^\circ$, $\beta_1 = 5^\circ$, and $\beta_2 = -12^\circ$, leading to a reduction of 6.5dB of the cost function with respect the worst case shown in Fig. 3. For Scheme B, the best set of parameters is $\eta_{\mathrm{h}} = 2\mu\mathrm{s}$, $\eta_{\mathrm{v}} = 2\mu\mathrm{s}$, $b_{\mathrm{h}} = 13.5 \cdot 10^9 \mathrm{s}^{-2}$ and $b_{\mathrm{v}} = 1350 \cdot 10^9 \mathrm{s}^{-2}$; the cost function is 4.5dB higher than the optimal signal resulted from Scheme A.

Discussion. Studying the CRB of the target parameters separately, we found that each scheme is capable of improving the estimation accuracy of the various parameters in different magnitude. Hence, each scheme may be used for different applications, depending on which parameter is considered more important. Table 1 shows the CRB of the target parameters for the optimal waveforms which result from Schemes A and B. The values on the table are relative to the worst case of linearly polarized waveforms shown in Fig. 2 and Fig. 3 for Examples 1 and 2, respectively. Scheme B yields an excellent performance on the estimation of the target range by choosing the optimal time-frequency parameters of the signal. On the other hand, Scheme A has the freedom to select the polarization matching the target aspects; hence it is able to improve the estimation of the scattering coefficients. In addition, it provides better estimation

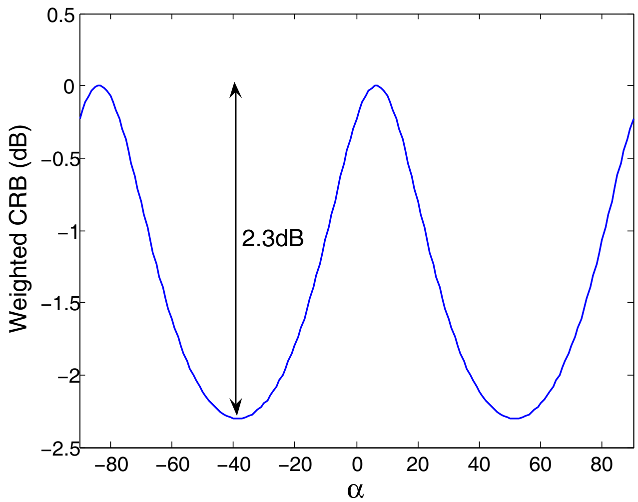

Fig. 3. Design cost as a function of the orientation angle of two orthogonally and linearly polarized pulses for the low-grazing-angle target, in Example 2.

Example 1

	CRB_ϕ	CRB_ϑ	CRB_r	$\text{CRB}_{\omega_\text{D}}$	$\text{CRB}_{\boldsymbol{s}}$
Scheme A	+0.5	+0.5	+2.3	+13.3	+10.8
Scheme B	-5.3	-0.2	+19.7	+3.0	+4.2

Example 2

	CRB_ϕ	CRB_ϑ	CRB_r	$\text{CRB}_{\omega_\text{D}}$	$\text{CRB}_{\boldsymbol{s}}$
Scheme A	+2.3	+2.5	+3.7	+14.2	+2.4
Scheme B	-0.2	-0.2	+17.2	+2.7	-0.2

Table 1. Cramér-Rao bound on the target parameters for the optimal wavefrom resulting from Schemes A and B, for Examples 1 and 2, with respect to the worst case shown in Fig. 2 and Fig. 3, respectively (units:dB).

of the Doppler shift than Scheme B, due to the latter is forced to reduce the duration of one of its pulses to generate the polarization diversity. It is also noted that Scheme A gives 2.5dB of performance improvement on the estimation of the bearing angles for the case of low-grazing-angle target.

5. CONCLUSIONS

We have considered the problem of selecting radar polarized waveforms to decrease the estimation error of the target parameters, i.e. bearing angles, range, Doppler shift, and scattering matrix. We studied two different problems; first, we addressed the case of a high-elevation target, free of interferences and clutter returns, and second, the problem of a low-grazing-angle target in the presence of multipath interference. We proposed two different approaches for designing the polarized waveforms. One was shown to yield high accuracy in estimating the target range, whereas the other proved to be more effective in improving the accuracy of the scattering coefficient and Doppler shift estimates.

In our future work, we will develop adaptive algorithms for joint estimation of the target parameters and dynamic optimal design of the transmit polarized signals.

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