MAXIMUM A POSTERIORI (MAP)-BASED ALGORITHM FOR DISTRIBUTED SOURCE LOCALIZATION USING QUANTIZED ACOUSTIC SENSOR READINGS

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ABSTRACT

In this paper, we propose a distributed source localization algorithm based on the Maximum A Posteriori (MAP) criterion, where the observations generated by each of the distributed sensors are quantized before being transmitted to a fusion node for localization. If the source signal energy is known, each quantized sensor reading corresponds to a region in which the source can be located. Aggregating the information obtained from multiple sensors corresponds to generating intersections between the regions. In our previous work we developed quantizer design techniques aimed at optimizing localization accuracy for a given aggregate rate. In this paper we develop localization algorithms based on estimating the likelihood of each of the intersection regions. This likelihood can incorporate uncertainty about the source signal energy as well as measurement noise. We show that the computational complexity of the algorithm can be significantly reduced by taking into account the correlation of the received quantized data. We also propose a technique, based on a weighted average of estimators, to address the case when the signal energy is unknown. Our simulation results show that our localization algorithm achieves good performance with reasonable complexity as compared with Minimum Mean Square Error (MMSE) estimation.

1. INTRODUCTION

Source localization algorithms have been proposed for distributed sensor networks operating with different sensor types, e.g., acoustic, seismic or thermal. Because the sensors tend to have limited power and the fusion node can be physically separate from the sensors, the efficiency of a localization algorithm should be assessed in terms of both its accuracy and the amount of bandwidth it requires. Thus, practical systems will normally have to operate using quantized sensor readings in order to reduce bandwidth. In our work we have focused on (i) quantization techniques that are optimized for localization and (ii) localization algorithms that take into account the fact that only quantized measurements are available.

Source localization based on acoustic energy measured at individual sensors is proposed in [5], where each sensor transmits unquantized acoustic energy readings to a fusion node. The localization problem has been solved mostly through nonlinear least squares estimation (see [5], [6]), which is sensitive to local optima and saddle points. To overcome this drawback, the authors in [2] formulated this problem as a convex feasibility problem. None of these approaches take explicitly into account the effect of sensor reading quantization.

In this paper, we extend our work in [4, 3], which focused on quantizer optimization for source localization, and propose a distributed source localization algorithm that uses the Maximum A Posteriori (MAP) criterion. First, consider the localization problem under the assumptions of no measurement noise and known source signal energy. In this scenario, the only source location uncertainty is due to quantization. Because the source signal energy is known, each quantized reading can be mapped to a region in the sensor field, whose shape depends on the characteristics of the sensor. For the case of an acoustic sensor that provides no directional information, the region corresponding to one quantized reading is a "ring" centered at the sensor location (see Figure 1). Since the measurements are assumed to be noiseless we expect the source to be located in the intersection of all the ring regions, one generated by each sensor. A vector of readings (one per sensor) will correspond to a unique location and, after quantization, there will be a unique (non-empty) intersection region (as shown in Figure 1.) Here, efficient quantizer designs seek to minimize the average area of all admissible intersections [4, 3]. Our previous work assumed low measurement noise and known source signal energy and used heuristic techniques for estimation in cases when quantized sensor readings were "inconsistent" (i.e., when intersections were empty).

Clearly, localization becomes more difficult when measurement noise is not negligible and/or the source signal energy is not known; these situations make it more likely that vector readings will lead to regions whose intersection is empty. Also, quantized sensor readings may change over time, even if the source does not move. To address these problems we use a probabilistic formulation, where we model the likelihood that a given candidate source location would produce a given vector reading. In this context, we first formulate the source localization problem as a Minimum Mean Square Error (MMSE) estimation problem; this approach generally has significant computational complexity, especially for the non-Gaussian case considered in this paper. We show that the complexity can be significantly reduced by taking into account the quantization effect and the distributed property of the quantized data, without significant impact on localization accuracy. Based on this, under the assumption of known source signal energy we propose a distributed algorithm based on the MAP criterion. We then show that for the unknown source signal energy case, a good estimator of the source location can be found by computing a weighted average of the estimates obtained by our MAP-based algorithm under different source energy assumptions. Simulation results show that our distributed localization algorithm achieves good localization performance with reasonable complexity, as compared with MMSE estimation.

This paper is organized as follows. The problem formulation is

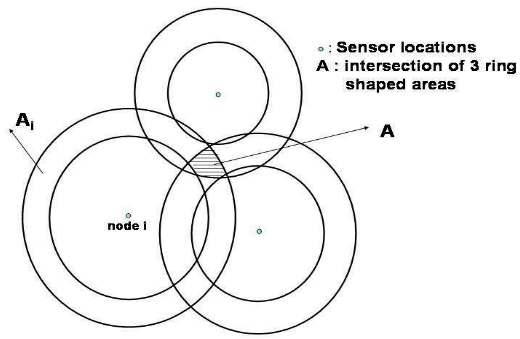

Fig. 1. Source Localization based on quantized energy readings

presented in Section 2. Our distributed source localization algorithm is presented in Section 3 and its implementation is discussed in Section 4. The case of unknown source signal energy is treated in Section 5. Simulation results are given in Section 6.

2. PROBLEM FORMULATION

Consider a sensor field $S \in \mathbf{R}^2$ containing M nodes that measure an acoustic signal emitted from a source assumed to be static during the localization process. By adopting the energy decay sensor model proposed in [5], we assume that the signal energy measured at node i over a time interval k can be expressed as follows:

$$z_i(x,k) = g_i \frac{a}{\|x - x_i\|^\alpha} + w_i(k), \quad i = 1, ..., M \tag{1}$$

where z_i is the acoustic energy reading at node i, x the source location, and x_i the position of node i. Note that $x, x_i \in \mathbf{R}^2$. The model parameters consist of the gain factor of the i-th sensor $g_i(\approx 1)$, an energy delay factor $\alpha(\approx 2)$, and the source signal energy a. We assume that the measurement noise $w_i(k)$ is independent of x and can be approximated by a normal distribution $N(0, \sigma_i^2)$. The signal energy a in (1) is assumed to remain constant during the localization process and to be in the range $[a_{min} \quad a_{max}]$. We also assume that the positions of all nodes are known and each node collects its noise-corrupted energy reading, $z_i(x,k)$ at time interval k, quantizes it with a given quantization level L_i, and sends it to a fusion node. Source localization is performed at the fusion node, based on the received noisy M-tuple, $\mathbf{Q_r} = (Q_1, ..., Q_M)$, where Q_i is the quantization index sent from i-th node. In what follows Q_i will be used to denote both a quantization index $Q_i \in \{1, ..., L_i\}$ and the quantization interval, $[Q_{i,l} \quad Q_{i,h}]$, corresponding to noisy measurement z_i at node i. Throughout the paper we assume that there is only one-way communication from sensors to fusion node, i.e., there is no feedback channel and the sensors do not communicate with each other; we also assume that the fusion node receives $\mathbf{Q_r}$ without transmission error.

Consider first the case of a known source signal energy, a and no measurement noise ($w_i = 0$ in (1)). Localization based on quantized energy readings is illustrated by Figure 1, where at least three energy readings are required to achieve a connected intersection. More formally:

$$A = \bigcap_{i=i}^{M} A_i, \quad A_i = \{x : g_i \frac{a}{\|x - x_i\|^\alpha} \in Q_i\}, \tag{2}$$

where A_i is the ring-shaped area corresponding to Q_i obtained from z_i (Figure 1). Let $p(x)$ be the pdf associated to our a priori knowledge about the source location. Then the MMSE estimate of the source location in the noiseless case would be $\hat{x} = E(x|x \in A)$. If $p(x)$ is uniform $\hat{x}$ would simply be the sample mean in A.

Consider now the case where there is measurement noise and/or the source signal energy is unknown. There is no guarantee that obtaining the intersection of the quantized readings will lead to the true intersection A. Assuming the statistics of the measurement noise, w_i, are known, we can formulate the source localization problem as an MMSE estimation problem as follows:

$$\begin{aligned} \hat{x} &= \int_{x \in S} x p(x|\mathbf{Q_r}) dx = \int_{x \in S} x p(\mathbf{Q_r}|x) p(x) / p(\mathbf{Q_r}) dx \\ &= \frac{\int_{x \in S} x [\prod_{i=1}^{M} \int_{z_i \in Q_i} p(z_i|x) dz_i] p(x) dx}{p(\mathbf{Q_r})}, \end{aligned} \tag{3}$$

where $p(\mathbf{Q_r}) = \int_{x \in S} [\prod_{i=1}^{M} \int_{z_i \in Q_i} p(z_i|x) dz_i] p(x) dx$. The conditional probability $p(z_i|x), i = 1, ..., M$ in (3) can be obtained using the sensor model in (1) along with a probabilistic characterization of a (e.g., its pdf): that is, $p(z_i|x) = \int_a p(z_i|x,a) p(a) da$ where $p(z_i|x,a)$ is normal distribution, $N(g_i \frac{a}{\|x - x_i\|^\alpha}, \sigma_i^2)$. When $p(z_i|x)$ and $p(x)$ are available (3) provides the MMSE estimator, which allows us to take into consideration optimally the unknown signal energy and the measurement noise. However, estimating $\hat{x}$ as in (3) is highly complex and results may be sensitive to accuracy of the a priori models $p(x)$ and $p(a)$. In this paper, we use an uninformative or uniform prior distribution whenever there is ignorance about the parameters to be estimated, since this allows us to obtain the posteriori distribution which will be approximately proportional to the likelihood. In addition, the uninformative prior has another advantage of keeping subsequent computations relatively simple. In the following sections, we focus on developing our localization algorithm.

3. PROPOSED LOCALIZATION ALGORITHM BASED ON MAXIMUM A POSTERIORI (MAP) CRITERION

We first consider the case where the signal energy, a, is known to the fusion node. The case of unknown signal energy will be treated in Section 5. With no measurement noise, only certain M-tuples can be produced by the sensors, those corresponding to non-empty intersection regions in S. Denote S_Q^f the set of M-tuples that can be generated in a noise-free environment, which can be written as:

$$S_Q^f = \{(Q_1, ..., Q_M) | g_i \frac{a}{\|x - x_i\|^\alpha} \in Q_i, i = 1, ..., M \quad x \in S\} \tag{4}$$

Denote A^j the region corresponding to the j-th M-tuple, $\mathbf{Q^j}$ in S_Q^f. Then, $A^j \cap A^i = \emptyset$, if $j \neq i$, so that we can partition S into $|S_Q^f|$ regions[1].

We now consider the noisy case. Our approach is to first identify the most likely region A^j corresponding to an M-tuple in S_Q^f, given the noisy observation $\mathbf{Q_r}$ and then compute the estimated source location within the chosen region. Note that now there is no guarantee that $\mathbf{Q_r}$ will belong to S_Q^f. Denote H_j the hypothesis corresponding

[1]Note that our proposed algorithm can be applied to other sensor types; the only assumption we make is that the sensors lead to a partition of the sensor field and given the specific partition for our chosen sensors, the same algorithm can be applied.

to region A^j and vector reading $\mathbf{Q^j} \in S_Q^f$. Our goal is to find H^* based on the received noisy M-tuple $\mathbf{Q_r}$ as follows:

$$\begin{aligned} H^* &= \arg\max_j p(H_j|\mathbf{Q_r}) = \arg\max_j p(\mathbf{Q_r}|x \in A^j)p_j \\ &= \arg\max_j \prod_{i=1}^{M} p(Q_i|x \in A^j)p_j, \quad j = 1, ..., |S_Q^f| \end{aligned} \quad (5)$$

where $p_j = p(x \in A^j)$ and $p(Q_i|x \in A^j)$ is computed as

$$\begin{aligned} & p(Q_i|x \in A^j) = p(\mu_i(x) + w_i \in Q_i|x \in A^j) \\ = & \int_{x \in A^j} [\Phi(\frac{Q_{i,h} - \mu_i(x)}{\sigma_i}) - \Phi(\frac{Q_{i,l} - \mu_i(x)}{\sigma_i})]p_j(x)dx \end{aligned} \quad (6)$$

where $\mu_i(x) = g_i \frac{a}{\|x-x_i\|^\alpha}$ and $p_j(x) = p(x|x \in A^j)$. Here, $\Phi(.)$ is the cdf for the normal distribution, $N(0, 1)$. Once H^* is obtained, the source estimate $\hat{x}$ is computed by $E(x|H^*) = E(x|x \in A^*)$.

4. IMPLEMENTATION OF PROPOSED ALGORITHM

The most complex step in our proposed algorithm is the integration in (6), which is required for each hypothesis. To simplify the process, we propose to reduce the number of hypotheses to evaluate, based on $\mathbf{Q_r}$. We note that, even in the noisy case, if the source is in A^j corresponding to $\mathbf{Q^j} \in S_Q^f$ the corresponding quantized vector reading is likely to be $\mathbf{Q^j}$. Based on this observation, we construct the set, $A_s(\subset S)$ such that $p(x \in A_s|\mathbf{Q_r}) \approx 1$, i.e., the set containing the most likely hypotheses. By considering only the hypotheses in A_s, we reduce the complexity of localization. Construction of A_s can be accomplished by noting that given a source location, x and the corresponding noisy M-tuple $\mathbf{Q_r}$, it would be very likely that $p(x \in A^i|\mathbf{Q_r}) > p(x \in A^j|\mathbf{Q_r})$ as long as $p(\hat{x}_i|\mathbf{Q_r}) \gg p(\hat{x}_j|\mathbf{Q_r})$, where $\hat{x}_j$ is the centroid of the set A^j. Thus, to select A_s, we first find a coarse estimate of the source location, $\hat{x}_c$, by determining which of the centroids of all A^j is most likely, i.e., we compute

$$\begin{aligned} \hat{x}_c &= \arg\max_{\hat{x}_j} p(\hat{x}_j|\mathbf{Q_r}) = \arg\max_{\hat{x}_j} p(\mathbf{Q_r}|\hat{x}_j)p_j \\ &= \arg\max_{\hat{x}_j} \prod_{i=1}^{M} [\Phi(\frac{Q_{i,h} - \mu_i(\hat{x}_j)}{\sigma_i}) - \Phi(\frac{Q_{i,l} - \mu_i(\hat{x}_j)}{\sigma_i})]p_j, \end{aligned}$$

and then we define $A_s(\delta) = \bigcup_{k=1}^{K} A^k$ $(A_s \subset S)$ such that $|\hat{x}_c - \hat{x}_k| < \delta, \quad \forall k = 1, ..., K$ where $\hat{x}_k = E(x|x \in A^k)$, i.e., we include all hypothesis that are within a certain distance of our coarse estimate. We select the most likely hypothesis following (5) but among only those candidates in $A_s(\delta)$. Thus, by choosing δ we can control the trade-off between search complexity and localization accuracy (a smaller δ will lead to a faster search, since fewer hypotheses will be tested.)

5. UNKNOWN SOURCE SIGNAL ENERGY CASE

The source signal energy, a, is generally unknown and should also be estimated along with the source location. To eliminate this energy term in our estimation problem, we could adopt the energy ratios-based source localization method proposed in [5]. However, this has drawbacks in our problem because with quantized readings we can only estimate a range of possible values for the signal energy, and these ranges could result in significant uncertainty, especially when quantization is coarse. In what follows we show that good source localization can be achieved with a weighted average of estimated source locations under different signal energy assumptions.

Considering a to be a nuisance parameter [1], we can formulate the MMSE estimation as in Section 2.

$$\begin{aligned} \hat{x} &= \int_{x \in S} xp(x|\mathbf{Q_r})dx = \int_x x[\int_a p(x, a|\mathbf{Q_r})da]dx \\ &= \int_x x[\int_a p(x|\mathbf{Q_r}, a)p(a|\mathbf{Q_r})da]dx \\ &= \int_a \hat{x}(a)p(a|\mathbf{Q_r})da \approx \int_a \hat{x}_{prop}(a)p(a|\mathbf{Q_r})da \end{aligned} \quad (7)$$

In (7), the MMSE estimate, $\hat{x}(a)$ given by (3) is approximated by $\hat{x}_{prop}(a)$ obtained by the proposed localization algorithm developed for the case of known source signal energy in Section 3. Note that significant computational complexity is required to compute $p(a|\mathbf{Q_r}) \propto p(a) \int_{x \in S} p(\mathbf{Q_r}|x, a)p(x)dx$, which leads us to make some further approximations.

First, while the source signal energy can take continuous values in a predetermined interval $[a_{min} \quad a_{max}]$, we consider only discrete energy values, since small variations in signal energy have a small impact on localization accuracy (see Figure 2). That is,

$$\hat{x} \approx \sum_{k=1}^{N} \hat{x}_{prop}(a_k)p(a_k|\mathbf{Q_r}) = \sum_{k=1}^{N} \frac{\hat{x}_{prop}(a_k)W_k}{\sum_i^N W_i} \quad (8)$$

where N is the number of discrete energy values used and W_k is the k-th weight given by $W_k = p(a_k) \int_{x \in S} p(\mathbf{Q_r}|x, a_k)p(x)dx$.

Secondly, some signal energy values are bound to be less likely than others (for example, a particular energy value can lead to a non empty intersection of quantization regions, while under other energy values the intersections may be empty). Thus, there will be some dominant weights in (8) and if we compute the weights first, we only need to perform the localization algorithm for those weights that are sufficiently large. That is,

$$\hat{x} \approx \sum_{l=1}^{L} \frac{\hat{x}_{prop}(a_l)W_l}{\sum_i^L W_i} \quad (9)$$

where L is the number of chosen weights and the set $\{W_k\}_{k=1}^N$ is arranged such that $W_i \geq W_j$ if $i < j$. Note that W_l can be also approximated by $W_l \approx p(a_l) \int_{x \in A_s(\delta_w, a_l)} p(\mathbf{Q_r}|x, a_l)p(x)dx$ where the set $A_s(\delta_w, a_l)$ can be constructed using the approach in Section 4. Clearly, there will be some trade-offs between the computational complexity and the localization performance and this can be controlled by adjusting the parameters such as N, L, and δ_w.

6. SIMULATION RESULTS

In our experiments, we consider a sensor network with $M = 5$ nodes deployed randomly in a $10 \times 10m$ sensor field. Each sensor measures an acoustic source energy based on the energy decay model in (1), quantizes it using a quantizer designed by the algorithm in [4]. Note that the measurement noise is assumed to be normal distributed, $N(0, \sigma^2)$ and SNR is computed by $10 \log_{10} \frac{a^2}{\sigma^2}$. Note that SNR is measured at 1 meter from the source. For example, SNR=40dB corresponds to SNR≈5.5dB measured at each sensor on the average.

First, with a known, the proposed algorithm in Section 4 is tested using a test set of 2000 source locations generated with a uniform

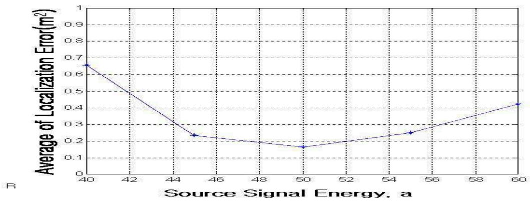

Fig. 2. Localization Accuracy (m^2) of Proposed Algorithm in Section 4 under Signal Energy Mismatch. A test set of 2000 source locations generated with $\sigma = 0.05, M = 5$ for each signal energy (=40,...,60). The algorithm is performed using $a = 50$ and $\delta = 1m$.

distribution of $p(x)$ for each SNR. In Figure 3 the proposed algorithm is compared with MMSE estimation since the latter gives us a good lower bound. It can be said that our algorithm provides good localization accuracy as compared with MMSE estimation.

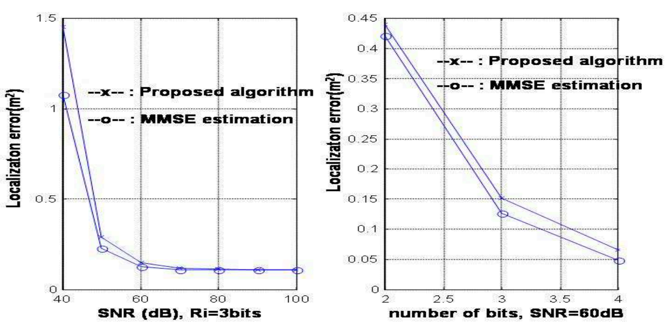

Fig. 3. Comparison of Proposed Algorithm in Section 4 with MMSE estimation. SNR =40,...,100 dB with $R_i = 3$ (left) and $R_i = 2, 3$ and 4bits with SNR=60 dB (right). The parameter δ is set as 1m.

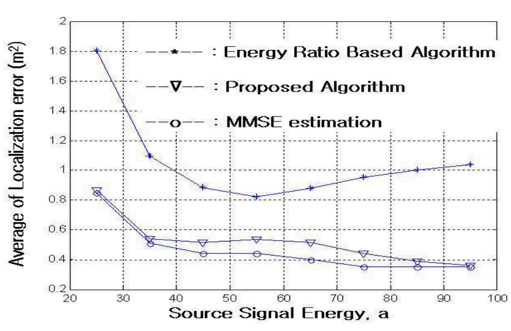

Fig. 4. Comparison of Proposed Algorithm in Section 5 with MMSE estimation and Energy Ratios-based Algorithm (ERA).

The proposed algorithm for unknown signal energy of Section 5 is tested and compared with the MMSE estimator and Energy Ratios-based Algorithm (ERA) and also evaluated under various types of mismatches. In applying the algorithm, prior distributions for $p(x)$ and $p(a)$ are assumed to be uniform over $x \in S$ and $a \in [a_{min} \quad a_{max}] = [0 \quad 100]$, respectively and the parameters are set as $N = 10, a_k \in \{10, 20, ..., 100\}, L = 3, \delta_w = 1m, \alpha = 2, g_i = 1$. In Figure 4, the interval $[a_{min} \quad a_{max}]$ is divided into 8 subintervals such as [20 30],...,[90 100] and for each subinterval, a test set of 2000 source locations with the signal energy randomly drawn from the subinterval is generated with $\sigma = 0.05$. Figure 4 shows that ERA provides worse localization accuracy than our proposed algorithm. In Table 1, we give random perturbation to one of the sensor model parameters for each test set of 2000 source locations: that is, the actual value of α is randomly drawn from $[2 - \Delta\alpha, \quad 2 + \Delta\alpha]$ for each $\Delta\alpha$ and the actual gain is also drawn randomly from $[1 - \Delta g \quad 1 + \Delta g]$. Similarly, each sensor location (x, y) is randomly generated from $[x - \Delta x \quad x + \Delta x]$ and $[y - \Delta y \quad y + \Delta y]$, respectively. In addition, a test set of 2000 source locations with normal distribution of mean $(5, 5)$ and variance (σ_x^2, σ_y^2) is generated for each (σ_x, σ_y). From the results in Table 1, it can be said that small perturbation can be allowed to maintain good localization accuracy. Finally, our proposed algorithm is also tested in a large sensor field $20 \times 20m$ and shows similar results.

$\Delta\alpha$	0	0.1	0.2	0.3	0.4
LE(Proposed)	0.5319	0.7360	1.4643	2.2653	3.6998
LE(ERA)	0.8886	1.1402	1.8658	2.7042	3.6696
Δg_i	0	0.1	0.2	0.3	0.4
LE(Proposed)	0.5414	0.6293	0.8201	1.1606	1.6215
LE(ERA)	0.8980	0.9695	1.2012	1.6407	2.0873
$(\Delta x, \Delta y)$	0	0.1	0.2	0.3	0.4
LE(Proposed)	0.5414	0.5380	0.5836	0.6242	0.7176
LE(ERA)	0.8980	0.8900	0.9167	1.0074	1.0760
(σ_x, σ_y)	1	1.5	2	2.5	3
LE(Proposed)	0.2710	0.3554	0.4806	0.8992	1.7617
LE(ERA)	0.8879	0.9233	0.9732	1.4556	2.2024

Table 1. Localization Error (LE) (m^2) of Proposed Algorithm in Section 5 (Proposed) vs. ERA under various mismatches.

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