CONVEX TRANSMIT BEAMFORMING FOR DOWNLINK MULTICASTING TO MULTIPLE CO-CHANNEL GROUPS

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ABSTRACT

We consider the problem of transmit beamforming to multiple co-channel multicast groups. Since the direct minimization of transmit power while guaranteeing a prescribed minimum signal to interference plus noise ratio (SINR) at each receiver is nonconvex and NP-hard, we present convex SDP relaxations of this problem and study when such relaxations are tight. Our results show that when the steering vectors for all receivers are of Vandermonde type (such as in the case of a uniform linear array and line-of-sight propagation), a globally optimum solution to the corresponding transmit beamforming problem can be obtained via an equivalent SDP reformulation. We also present various robust formulations for the problem of single-group multicasting, when the steering vectors are only approximately known. Simulation results are presented to illustrate the effectiveness of our SDP relaxations and reformulations.

1. INTRODUCTION

Consider a downlink transmission scenario where the transmitter is equipped with N antennas and there are M receivers. Let $\mathbf{h}_i$ denote the $N \times 1$ complex channel vector from each transmit antenna to the single receive antenna of user $i \in \{1, \ldots, M\}$. Let there be a total of $1 \leq G \leq M$ multicast groups, $\{\mathcal{G}_1, \ldots, \mathcal{G}_G\}$, where $\mathcal{G}_k$ is the index set for receivers participating in multicast group k, and $k \in \{1, \ldots, G\}$. Assume that $\mathcal{G}_k \cap \mathcal{G}_l = \emptyset$, $l \neq k$, $\cup_k \mathcal{G}_k = \{1, \ldots, M\}$, and, denoting $G_k := |\mathcal{G}_k|$, $\sum_{k=1}^{G} G_k = M$.

Let $\mathbf{w}_k^H$ denote the beamforming weight vector applied to the N transmitting antenna elements to transmit multicast stream k. The signal transmitted by the antenna array is equal to $\sum_{k=1}^{G} \mathbf{w}_k^H s_k(t)$, where $s_k(t)$ is the temporal information-bearing signal directed to receivers in multicast group k. This setup includes the case of *broadcasting* ($G = 1$) [6], and the case of individual user transmissions ($G = M$) [2]) as special cases. If each $s_k(t)$ is zero-mean white with unit variance, and the waveforms $\{s_k(t)\}_{k=1}^{G}$ are mutually uncorrelated, then the total power radiated is equal to $\sum_{k=1}^{G} ||\mathbf{w}_k||_2^2$.

The joint design of transmit beamformers subject to received SINR constraints can then be posed as follows:

$\mathcal{P}$:

$$\min_{\left\{\mathbf{w}_k \in \mathbb{C}^N\right\}_{k=1}^{G}} \sum_{k=1}^{G} \|\mathbf{w}_k\|_2^2$$

$$\text{s.t.}: \quad \frac{|\mathbf{w}_k^H \mathbf{h}_i|^2}{\sum_{l \neq k} |\mathbf{w}_l^H \mathbf{h}_i|^2 + \sigma_i^2} \geq c_i, \ \forall i \in \mathcal{G}_k, \ \forall k \in \{1, \ldots, G\}.$$

Problem $\mathcal{P}$ was considered in [5] and it was found to be NP-hard, in the case of general steering vectors, based on arguments proved in earlier work [6]. Therefore, a two step approach was proposed and shown to yield high-quality approximate solutions at manageable complexity cost. Specifically, in the first step, the original non-convex quadratically constrained quadratic programming (QCQP) problem $\mathcal{P}$ is relaxed to a semidefinite program (SDP) (denoted as $\mathcal{R}$), by changing the optimization variables to $\mathbf{X}_k := \mathbf{w}_k \mathbf{w}_k^H$ and dropping the associated non-convex constraints $\{\text{rank}(\mathbf{X}_k) = 1\}_{k=1}^{G}$. In the second step, a randomization procedure is employed to generate candidate beamforming vectors from the solution of $\mathcal{R}$. For each candidate set of vectors, a multi-group power control ($\mathcal{MGPC}$) linear programming (LP) problem is solved to ensure that the constraints of the original problem $\mathcal{P}$ are met. The final solution of this algorithm is the set of beamforming vectors yielding the smallest $\mathcal{MGPC}$ objective. The overall complexity of the algorithm is manageable, since the SDP and LP problems can be solved efficiently using interior point methods and the randomization procedure is designed so that its computational cost is negligible compared to the aforementioned problems.

2. EXACT GLOBALLY OPTIMAL SOLUTION IN THE VANDERMONDE CASE

When a uniform linear array (ULA) is used for far-field transmit beamforming, the $N \times 1$ complex vectors which model the phase shift from each transmit antenna to the receive antenna of user $i \in \{1, \ldots, M\}$ are Vandermonde $\mathbf{h}_i = [1 \ e^{j\theta_i} \ e^{j2\theta_i} \ \cdots \ e^{j(N-1)\theta_i}]^T$. In this scenario, we observed that when the relaxed SDP problem $\mathcal{R}$ in [5] is feasible, its optimal solution, i.e., the blocks $\{\mathbf{X}_k^{\text{opt}}\}_{k=1}^{G}$, are all consistently rank-one. This means that problem $\mathcal{R}$ is then equivalent to, and not a relaxation of, the original problem $\mathcal{P}$. Thus, the second step of the proposed algorithm, comprising the randomization - multicast power control loop, turns out being redundant and the set of the optimum beamforming vectors $\{\mathbf{w}_k^{\text{opt}}\}_{k=1}^{G}$ can be formed simply using the principal components of the blocks $\{\mathbf{X}_k^{\text{opt}}\}_{k=1}^{G}$. This observation suggests that, in the case of Vandermonde channel vectors, the original problem $\mathcal{P}$ is no longer NP-hard and can be equivalently posed as a convex optimization problem.

Towards this end, note that for the special case of Vandermonde steering vectors, the signal power received at each user can be rewrit-

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ten as

$$\left|\mathbf{w}_k^H \mathbf{h}_i\right|^2 = \sum_{\ell=-(N-1)}^{N-1} r_k(\ell) e^{j\theta_i \ell}, \tag{1}$$

where $\ell := n - m$ and $r_k(\ell) := \sum_{m=\max(1-\ell,1)}^{\min(N-\ell,N)} w_k(m) w_k^*(m+\ell)$. Let us consider $r_k(\ell)$ for $0 < \ell \leq N-1$, i.e., $r_k(\ell) = \sum_{m=1}^{N-\ell} w_k(m) w_k^*(m+\ell)$. Then $r_k^*(-\ell) = r_k(\ell)$, i.e., $r_k(\ell)$ is conjugate-symmetric about the origin. Define the $(2N-1) \times 1$ vector

$$\mathbf{r}_k := [r_k(-N+1), \cdots, r_k(-1), r_k(0), r_k(1), \cdots, r_k(N+1)]^T, \tag{2}$$

and the associated $(2N-1) \times 1$ "extended" steering vector

$$\mathbf{f}_i := [e^{-j\theta_i(N-1)}, \cdots, e^{-j\theta_i}, 1, e^{j\theta_i}, \cdots, e^{j\theta_i(N-1)}]^T. \tag{3}$$

Then $\left|\mathbf{w}_k^H \mathbf{h}_i\right|^2 = \mathbf{f}_i^T \mathbf{r}_k$. Furthermore, note that $r_k(0) = \mathbf{r}_k(N) = \sum_{m=1}^{N} w_k(m) w_k^*(m) = ||\mathbf{w}_k||_2^2$. It therefore follows that the original problem $\mathcal{P}$ can be equivalently written as follows

$$\min_{\{\mathbf{r}_k\}_{k=1}^G} \sum_{k=1}^{G} \mathbf{r}_k(N)$$

$$\textit{s.t.}: \quad \mathbf{f}_i^T \mathbf{r}_k \geq c_i \sum_{\ell \neq k} \mathbf{f}_i^T \mathbf{r}_\ell + c_i \sigma_i^2, \ \forall i \in \mathcal{G}_k, \ \forall k \in \{1, \ldots, G\},$$

$$\mathbf{r}_k : \text{autocorrelation vector}, \ \forall k \in \{1, \ldots, G\},$$

where the fact that the terms in the denominator are all non-negative has also been taken into account.

This is a problem comprising a linear cost, M linear inequality constraints, and autocorrelation constraints. Each of the latter is equivalent to a linear matrix inequality (LMI) constraint [1]. Specifically, $r_k(m)$, $\forall m \in \{-N+1, \ldots, N-1\}$ belongs to the set of finite autocorrelation sequences if and only if $r_k(m) = \text{trace}(\mathbf{E}^m \mathbf{Y}_k)$, $\forall m \in \{-N+1, \ldots, N-1\}$, for some positive semidefinite matrix $\mathbf{Y}_k \in \mathbb{C}^{N \times N}$, where $\mathbf{E}$ is the $N \times N$ unit-shift matrix with ones in the first lower sub-diagonal and zeros elsewhere.

Thus, introducing G positive semidefinite $N \times N$ "slack" matrices, one for each autocorrelation vector $\mathbf{r}_k$, the autocorrelation constraints are equivalently converted to linear equality constraints plus positive semidefinite constraints as follows

$\mathcal{V}$:

$$\min_{\{\mathbf{r}_k\}_{k=1}^G, \{\mathbf{Y}_k\}_{k=1}^G} \sum_{k=1}^{G} \mathbf{r}_k(N)$$

$$\begin{aligned} \textit{s.t.}: \quad & \mathbf{f}_i^T \mathbf{r}_k - c_i \textstyle\sum_{\ell \neq k} \mathbf{f}_i^T \mathbf{r}_\ell \geq c_i \sigma_i^2, \\ & \forall i \in \mathcal{G}_k, \ \forall k \in \{1, \ldots, G\}, \\ & \mathbf{r}_k(m) = \text{trace}(\mathbf{E}^m \mathbf{Y}_k), \\ & \forall m \in \{-N+1, \ldots, N-1\}, \ \forall k \in \{1, \ldots, G\} \\ & \mathbf{Y}_k \succeq \mathbf{0}, \ \forall k \in \{1, \ldots, G\}. \end{aligned}$$

Problem $\mathcal{V}$ is an SDP problem which can be efficiently solved by any standard SDP solver, such as SeDuMi [7], by means of interior point methods. Once the optimum autocorrelation sequences $\left\{\mathbf{r}_k^{\text{opt}}\right\}_{k=1}^G$ are found, they can be factored to obtain the respective optimum beamforming vectors $\left\{\mathbf{w}_k^{\text{opt}}\right\}_{k=1}^G$, using spectral factorization techniques [9].

A simple simulation experiment illustrates the equivalence of the aforementioned algorithm to the one proposed in [5]. Figures 1 and 2 show the optimized transmit beam patterns generated by algorithm 1 (SDP relaxation problem $\mathcal{R}$ and randomization - multicast power control problem $\mathcal{MGPC}$) and algorithm 2 (SDP problem $\mathcal{V}$ and spectral factorization), respectively. The ULA consists of $N = 4$ transmit antenna elements spaced $\lambda/2$ apart. The $M = 24$ users are considered evenly clustered in $G = 2$ groups, at an angle of 0.5 degrees to their neighboring ones. The angular cluster separation (defined as the minimum angle between any 2 users belonging to different groups) is set to 10 degrees. The received SINR constraints are set to 10dB for all users and the noise variance to $\sigma^2 = 1$ for all channels.

3. ROBUST RELAXATION OF SINGLE-GROUP MULTICAST BEAMFORMING

In this section we provide a robust relaxation to the problem of downlink transmit beamforming towards a single multicast group, which was considered in [6]. The key difference here is that full channel state information (CSI) is no longer available; instead, the channel vectors are assumed to lie in a ball with known center and radius. Specifically, letting $\tilde{\mathbf{h}}_i := \mathbf{h}_i / \sqrt{c_i \sigma_i^2}$ denote the normalized channel vectors, we assume that $\tilde{\mathbf{h}}_i \in \mathrm{B}_\epsilon(\bar{\mathbf{h}}_i) := \{\tilde{\mathbf{h}}_i | \tilde{\mathbf{h}}_i = \bar{\mathbf{h}}_i + \mathbf{e}, \|\mathbf{e}\| \leq \epsilon\}$. The robust design of the beamformer that minimizes the transmitted power, subject to constraints on the received SNR can be written as

$\mathcal{RB}$:

$$\min_{\mathbf{w} \in \mathbb{C}^N} \|\mathbf{w}\|_2^2$$

$$\textit{s.t.}: \quad |\mathbf{w}^H \tilde{\mathbf{h}}_i|^2 \geq 1, \ \forall \ \tilde{\mathbf{h}}_i \in \mathrm{B}_\epsilon(\bar{\mathbf{h}}_i), \ \forall \ i \in \{1, \ldots, M\}.$$

The constraints in problem $\mathcal{RB}$ guarantee that the received signal power in all M users will be larger than unity in the *worst case*, i.e. for the particular channel vector $\tilde{\mathbf{h}}_i$ that corresponds to the smallest value of $|\mathbf{w}^H \tilde{\mathbf{h}}_i|^2$. Each one of these constraints is equivalent to the semi-infinite nonconvex constraint

$$|\mathbf{w}^H \tilde{\mathbf{h}}_i| \geq 1, \ \forall \ \tilde{\mathbf{h}}_i \in \mathrm{B}_\epsilon(\bar{\mathbf{h}}_i), \tag{4}$$

which admits a convex (SOC) reformulation, as it was shown in [8]. First note that equation (4) can be equivalently written as

$$\min_{\tilde{\mathbf{h}}_i \in \mathrm{B}_\epsilon(\bar{\mathbf{h}}_i)} |\mathbf{w}^H \tilde{\mathbf{h}}_i| \geq 1. \tag{5}$$

Under the natural constraint $|\mathbf{w}^H \bar{\mathbf{h}}_i| \geq \epsilon \|\mathbf{w}\|_2$, it can be shown [8] that

$$\min_{\tilde{\mathbf{h}}_i \in \mathrm{B}_\epsilon(\bar{\mathbf{h}}_i)} |\mathbf{w}^H \tilde{\mathbf{h}}_i| = |\mathbf{w}^H \bar{\mathbf{h}}_i| - \epsilon \|\mathbf{w}\|_2, \tag{6}$$

and we can recast equation (5) as

$$|\mathbf{w}^H \bar{\mathbf{h}}_i| - \epsilon \|\mathbf{w}\|_2 \geq 1 \Leftrightarrow |\mathbf{w}^H \bar{\mathbf{h}}_i| \geq 1 + \epsilon \|\mathbf{w}\|_2. \tag{7}$$

The robust beamforming problem $\mathcal{RB}$ is thus equivalently written as

$\mathcal{RB}'$:

$$\min_{\mathbf{w} \in \mathbb{C}^N} \|\mathbf{w}\|_2^2$$

$$\textit{s.t.}: \quad |\mathbf{w}^H \bar{\mathbf{h}}_i| \geq 1 + \epsilon \|\mathbf{w}\|_2, \ \forall \ i \in \{1, \ldots, M\}.$$

Let us also consider the corresponding original non-robust beamforming (ONRB) problem:

$$\min_{\mathbf{w}\in\mathbb{C}^N} \|\mathbf{w}\|_2^2$$

$$\textit{s.t.}: \quad |\mathbf{w}^H\tilde{\mathbf{h}}_i| \geq 1, \quad \forall i \in \{1,\ldots,M\}.$$

Our main result in this section is the following:

Claim 1 *Let $\mathbf{w}'$ be an exact solution of $\mathcal{RB}'$. Then $\mathbf{w}'/(1+\epsilon\|\mathbf{w}'\|)$ is an exact solution of ONRB. Conversely, if $\mathbf{w}_o$ is an exact solution of ONRB, then $\mathbf{w}_o/(1-\epsilon\|\mathbf{w}_o\|)$ is an exact solution of $\mathcal{RB}'$.*

Proof: Forward: The proof is based on two Lemmas. The first is the following *Scaling Lemma*:

Lemma 1 *$\mathbf{w}_o$ is an exact solution of ONRB if and only if $t\mathbf{w}_o$ is an exact solution of*

$$\min_{\mathbf{w}\in\mathbb{C}^N} \|\mathbf{w}\|_2^2$$

$$\textit{s.t.}: \quad |\mathbf{w}^H\tilde{\mathbf{h}}_i| \geq t, \quad \forall i \in \{1,\ldots,M\}.$$

Proof: $|\mathbf{w}_o^H\tilde{\mathbf{h}}_i| \geq 1 \implies |t\mathbf{w}_o^H\tilde{\mathbf{h}}_i| \geq t$. Suppose there exists $\mathbf{w}_1$ with $|\mathbf{w}_1^H\tilde{\mathbf{h}}_i| \geq t$, $\forall i$, and $\|\mathbf{w}_1\|_2^2 < t^2\|\mathbf{w}_o\|_2^2$. Consider $\mathbf{w}_2 := \mathbf{w}_1/t$. It satisfies $|\mathbf{w}_2^H\tilde{\mathbf{h}}_i| \geq 1$, and

$$\|\mathbf{w}_2\|_2^2 = \frac{1}{t^2}\|\mathbf{w}_1\|_2^2 < \frac{1}{t^2}t^2\|\mathbf{w}_o\|_2^2 = \|\mathbf{w}_o\|_2^2, \tag{8}$$

which contradicts optimality of $\mathbf{w}_o$ for ONRB. The converse is obvious. □

Lemma 2 *Let $\mathbf{w}'$ be an exact solution of $\mathcal{RB}'$. Then, $\mathbf{w}'$ is an exact solution of the following non-robust beamforming problem (NRB)*

$$\min_{\mathbf{w}\in\mathbb{C}^N} \|\mathbf{w}\|_2^2$$

$$\textit{s.t.}: \quad |\mathbf{w}^H\tilde{\mathbf{h}}_i| \geq 1+\epsilon\|\mathbf{w}'\|_2, \quad \forall i \in \{1,\ldots,M\}.$$

Proof: Clearly, $\mathbf{w}'$ is a feasible solution of NRB, since it satisfies the constraints. Suppose there exists $\mathbf{w}''$ that also satisfies the constraints of NRB, but with $\|\mathbf{w}''\|_2^2 < \|\mathbf{w}'\|_2^2$. Then $1+\epsilon\|\mathbf{w}'\|_2 > 1+\epsilon\|\mathbf{w}''\|_2^2$, and thus $\mathbf{w}''$ also satisfies the constraints of problem $\mathcal{RB}'$, with $\|\mathbf{w}''\|_2^2 < \|\mathbf{w}'\|_2^2$. This contradicts optimality of $\mathbf{w}'$ for $\mathcal{RB}'$. □

Now suppose that $\mathbf{w}'$ is an exact solution of $\mathcal{RB}'$. It follows from the last Lemma that it is also an exact solution of NRB. Then, from the *Scaling Lemma*, it follows that $\mathbf{w}'/(1+\epsilon\|\mathbf{w}'\|)$ is an exact solution of ONRB. This completes the forward part of the proof of Claim 1. □

Converse: Let $\mathbf{w}_o$ be a solution of ONRB. Then, according to the Scaling Lemma

$$\mathbf{w}' = \frac{\mathbf{w}_o}{1-\epsilon\|\mathbf{w}_o\|_2} \tag{9}$$

is a solution of the modified NRB (MNRB) problem

$$\min_{\mathbf{w}\in\mathbb{C}^N} \|\mathbf{w}\|_2^2$$

$$\textit{s.t.}: \quad |\mathbf{w}^H\tilde{\mathbf{h}}_i| \geq \frac{1}{1-\epsilon\|\mathbf{w}_o\|_2}, \quad \forall i \in \{1,\ldots,M\}.$$

We will show that $\mathbf{w}'$ is also a solution of $\mathcal{RB}'$. Since $\mathbf{w}'$ is a solution of MNRB, it follows that

$$|\mathbf{w}'^H\tilde{\mathbf{h}}_i| \geq \frac{1}{1-\epsilon\|\mathbf{w}_o\|_2}. \tag{10}$$

However, from (9), it follows (provided that $1-\epsilon\|\mathbf{w}_o\|_2 \geq 0$, i.e., $\epsilon \leq \frac{1}{\|\mathbf{w}_o\|_2}$) that

$$\|\mathbf{w}'\|_2 = \frac{\|\mathbf{w}_o\|_2}{1-\epsilon\|\mathbf{w}_o\|_2} \Leftrightarrow \|\mathbf{w}_o\|_2 = \frac{\|\mathbf{w}'\|_2}{1+\epsilon\|\mathbf{w}'\|_2}.$$

Hence

$$\frac{1}{1-\epsilon\|\mathbf{w}_o\|_2} = \frac{1}{1-\frac{\epsilon\|\mathbf{w}'\|_2}{1+\epsilon\|\mathbf{w}'\|_2}} = 1+\epsilon\|\mathbf{w}'\|_2, \tag{11}$$

so $\mathbf{w}'$ indeed satisfies the constraints of $\mathcal{RB}'$. Suppose there exists $\mathbf{w}''$, such that $\|\mathbf{w}''\|_2 < \|\mathbf{w}'\|_2$ which also satisfies the constraints of $\mathcal{RB}'$. From the forward proof it follows that $\frac{\mathbf{w}''}{1+\epsilon\|\mathbf{w}''\|_2}$ satisfies the constraints of ONRB, with norm $\frac{\|\mathbf{w}''\|_2}{1+\epsilon\|\mathbf{w}''\|_2}$. On the other hand, $\mathbf{w}_o$ in (9) is an exact solution of ONRB, and $\|\mathbf{w}'\|_2 = \frac{\|\mathbf{w}_o\|_2}{1-\epsilon\|\mathbf{w}_o\|_2}$ yielding $\|\mathbf{w}_o\|_2 = \frac{\|\mathbf{w}'\|_2}{1+\epsilon\|\mathbf{w}'\|_2}$. But $\frac{x}{1+x}$ is monotone increasing in $x>0$. Therefore, $\|\mathbf{w}''\| < \|\mathbf{w}'\|$ implies that

$$\frac{\|\mathbf{w}''\|_2}{1+\epsilon\|\mathbf{w}''\|_2} < \frac{\|\mathbf{w}'\|_2}{1+\epsilon\|\mathbf{w}'\|_2} = \|\mathbf{w}_o\|_2, \tag{12}$$

which contradicts optimality of $\mathbf{w}_o$ for ONRB. Thus, the proof of Claim 1 is complete. □

Claim 1 implies that we can derive an exact solution of the robust beamforming problem $\mathcal{RB}'$ by a simple scaling of a solution to ONRB. Since both problems are NP-hard in general, in practice this translates to the following algorithm:

1. Compute a good feasible solution $\mathbf{w}_o$ for ONRB using the SDP relaxation approach in [6].
2. A good feasible solution of $\mathcal{RB}'$ is then $\mathbf{w}_o/(1-\epsilon\|\mathbf{w}_o\|_2)$.

Letting c_o and c' denote the norms of the optimal solutions of ONRB and $\mathcal{RB}'$, respectively, we also have

$$c_o = \frac{c'}{1+\epsilon c'} \Leftrightarrow c' = \frac{c_o}{1-\epsilon c_o}. \tag{13}$$

Claim 1 further suggests that if we set $\epsilon > 1/\|\mathbf{w}_o\|_2$, then the robust problem would be infeasible.

4. EXACT ROBUST SOLUTION IN THE SINGLE-GROUP VANDERMONDE CASE

Let us consider again the case when the steering vectors are Vandermonde. Then, the single-group ($G=1$) version of problem $\mathcal{V}$ can be written as

$$\boxed{\begin{aligned}&\mathcal{V}1:\\ &\qquad\qquad \min_{\mathbf{r}\in\mathbb{R}\times\mathbb{C}^{N-1}} \mathbf{e}_1^T\mathbf{r}\\ &\textit{s.t.}: \quad \mathrm{Re}[\mathbf{h}_i^H\tilde{\mathbf{I}}\mathbf{r}] \geq c_i\sigma_i^2,\ \forall\, i\in\{1,\ldots,M\},\\ &\qquad\quad r_\ell = \mathrm{trace}(\mathbf{E}^\ell\mathbf{Y}),\ \forall \ell\in\{0,\ldots,N-1\},\\ &\qquad\quad \mathbf{Y}\succeq\mathbf{0}.\end{aligned}}$$

where $\mathbf{e}_1$ is the first column of the $N \times N$ identity matrix,

$$r_\ell = \sum_{m=1}^{N-\ell} w_m^* w_{m+\ell}, \quad \forall \ell \in \{0, \dots, N-1\}, \tag{14}$$

$$\mathbf{r} = [r_0 \; r_1 \; \cdots r_{N-1}]^T \in \mathbb{R} \times \mathbb{C}^{N-1}, \tag{15}$$

and

$$\tilde{\mathbf{I}} = \begin{bmatrix} 1 & \mathbf{0} \\ \mathbf{0} & 2\mathbf{I}_{N-1} \end{bmatrix} \in \mathbb{R}^N. \tag{16}$$

A robust extension of the problem $\mathcal{V}1$ would be to ask that the SNR constraints are still met, when the angles $\{\theta_i\}_{i=1}^M$ are not known exactly, but allowing an estimation error up to Δ, i.e., they are assumed to lie within the intervals $\theta_i \in [\bar{\theta}_i - \Delta, \bar{\theta}_i + \Delta]$. In such scenario, the SNR constraints are defined as

$$\mathrm{Re}[\mathbf{h}_i^H \tilde{\mathbf{I}} \mathbf{r}] \geq c_i \sigma_i^2, \; \forall \, i \in \{1, \dots, M\}, \; \forall \theta_i \in [\bar{\theta}_i - \Delta, \bar{\theta}_i + \Delta]. \tag{17}$$

An interpretation of these constraints is that they require (the real part of) certain trigonometric polynomials to be nonnegative over a segment of the unit circle. As it is shown in [4], constraints of this form can be equivalently reformulated to the LMI constraints

$$\tilde{\mathbf{I}}\mathbf{r} - (c_i \sigma_i^2 + j\xi_i)\mathbf{e}_1 = \mathbf{L}^*(\mathbf{X}_i) + \mathbf{\Lambda}^*(\mathbf{Z}_i; \bar{\theta}_i - \Delta, \bar{\theta}_i + \Delta), \tag{18}$$

$\forall \, i \in \{1, \dots, M\}$, where $\mathbf{X}_i \in \mathbb{C}^{N \times N} \succeq \mathbf{0}$, $\mathbf{Z}_i \in \mathbb{C}^{(N-1)\times(N-1)} \succeq \mathbf{0}$, $\xi_i \in \mathbb{R}$ is unconstrained, and the linear operators $\mathbf{L}^*$ and $\mathbf{\Lambda}^*$ are defined by equations (35) and (36)(along with (16)) in [4], respectively. Hence, the problem encountered in this section is an SDP problem, since it consists of a linear cost, MN linear equality constraints and $2M$ positive semidefinite constraints.

5. CONCLUSIONS

Whereas multi-group multicast transmit beamforming under SINR constraints is NP-hard in general [5, 6], we have shown that, in the special case of Vandermonde steering vectors it is in fact a semidefinite problem, which can be efficiently solved. We have also considered robust beamforming solutions under channel uncertainty for the case of a single multicast group. For general steering vectors, we have shown that exact solutions of the robust and non-robust versions of the problem are related via a simple one-to-one scaling transformation. Since both problems are NP-hard, this suggests an algorithm to generate a quasi-optimal solution for one given a quasi-optimal solution for the other. In the important special case of Vandermonde steering vectors, we have shown that the robust version of the problem is convex as well. This robust solution can be extended to the multi-group Vandermonde case.

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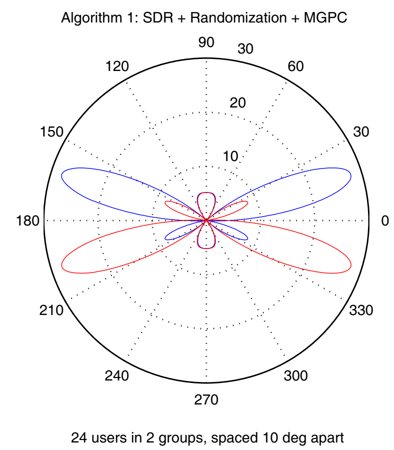

Fig. 1. SDP Relaxation + Randomization result for ULA, $N = 4$, $M = 2 \times 12$, $SINR = 10$dB

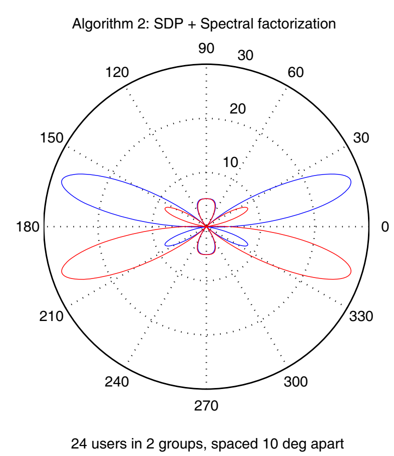

Fig. 2. Exact SDP + Spectral Factorization result for ULA, $N = 4$, $M = 2 \times 12$, $SINR = 10$dB