BER ANALYSIS OF A COOPERATIVE RANDOM ACCESS PROTOCOL IN RAYLEIGH FADING CHANNELS[†]

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ABSTRACT

A random access protocol for wireless networks was recently proposed that via cooperation of network nodes can resolve collisions and thus achieve high throughput. In this paper, we provide analytical expressions for the Bit-Error-Rate (BER) performance of that scheme in a Rayleigh flat fading scenario. Our analysis indicates that the spatial diversity introduced by user cooperation enables lower BER than non-cooperative protocols that avoid collisions, such as ALOHA or TDMA. The BER performance shows that the cooperative random access protocol is best suited for variable rate traffic. The analytical results are validated via simulations.

I. INTRODUCTION

Recently a new cooperative media access control (MAC) protocol of random access wireless network was proposed in [7], [8]. Due to that scheme, when there is a collision, the destination node (base station) does not discard the collided packets but rather saves them in a buffer. In the slots following the collision, a set of nodes designated as relays, form an alliance and forward the signal that they received during the collision slot. Based on these transmissions, the base station formulates a multiple-input multiple-output (MIMO) problem, the solution of which yields the collided packets. The method of [7], [8], referred to here as ALLIANCES [9], maintains the benefits of ALOHA systems [1] in the sense that all nodes share access to media resources efficiently and with minimal scheduling overhead, and enables efficient use of network power. The ability to resolve collisions avoids time slot waste, furthermore, the spatial diversity introduced via the cooperative relaying enables one to effectively deal with the wireless channel without any bandwidth expansion or additional antenna hardware. The NDMA approach of [6] also uses retransmissions to resolve collisions, however, it only exploits time diversity which might not exist in slowly varying channel cases.

Most MAC protocols in random access network are designed to "avoid" collisions. Examples include CSMA/CA used in IEEE 802.11g standard for wireless LANs [2], competition based protocols [5], tree-splitting algorithm [4], etc. However, due to the random arrival nature of traffic, collisions cannot be completely avoided without perfect scheduling, which comes at the expense of control overhead. As opposed to these protocols, ALLIANCES focuses on "resolving" collisions. In [7] the attainable throughput was evaluated via simulations and it was shown that the throughput approaches 1 for high Signal-to-Noise Ratio (SNR) cases.

In this paper we provide analytical expressions for the BER performance of ALLIANCES in a Rayleigh flat fading scenario. A bound on the pair-wise error probability (PEP) of ALLIANCES under high SNR was given in [8]. We here propose a tighter bound that is valid under any SNR, and is also easier to compute than that of [8]. Our analysis suggests that the spatial diversity introduced by user cooperation can provide the wireless network with lower BER than non-cooperative protocols that avoid collisions, such as ALOHA or TDMA. The analytical results are validated via simulations.

II. THE ALLIANCES MODEL

Consider a small-scale slotted multi-access system with J users, where each node can hear from a base station or access point (BS/AP) on a control channel. Link delay and online processing (packet decoding) time are ignored and all transmitters are assumed synchronized. Each user operates in a half-duplex mode. Every user and the BS/AP are equipped with only one antenna. All transmitted packets have the same length and each packet requires one time unit/slot for transmission.

Suppose that K packets have collided in the *n*-th slot. All nodes not involved in the collision enter a waiting mode and remain there until the collision is resolved. We define this period as a cooperative transmission epoch (CTE), beginning with the n-th slot. In each of the subsequent $\hat{K} - 1$ slots ($\hat{K} \ge K$), one randomly chosen node will retransmit or forward the signal involved in the collision. For example, during each of the n + k $(1 \le k \le \hat{K} - 1)$ slots, the r-th node, taken as r = mod(n + k, J) + 1 will be chosen as a relay. If this chosen node happens to be a source node, it will simply retransmit its own packet. Otherwise, it will relay the packets mixture received in the slot n. In summary, the nodes in the network form an alliance and cooperate during the retransmissions. We should emphasize here that we consider non-regenerative relays. Once the BS/AP collects at least K independent mixtures of the original transmitted packets, the collision can be resolved. ALLIANCES requires minimal overhead. 1 bit control information indicating the beginning or ending of a CTE is needed, which is the same as NDMA approach in [6].

We consider a flat fading channel. Let *n* denote the collision slot, and let the packet transmitted by the *i*-th node at slot *n* consist of *N* symbols, i.e., $\mathbf{x}_i(n) \stackrel{\triangle}{=} [x_{i,0}(n), \cdots, x_{i,N-1}(n)].$

If N symbols, i.e., $x_i(n) = [x_{i,0}(n), \cdots, x_{i,N-1}(n)]$. Let $S(n) = \{i_1, \cdots, i_K\}$ be the set of sources, and $\mathcal{R}(n) =$

 $\{r_1, \cdots, r_{\hat{K}-1}\}$ the set of nodes that will serve as relays during the CTE. During *n*-th slot, the signal heard by the BS and also by all non-source nodes is:

$$\boldsymbol{y}_{r}(n) = \sum_{i \in \mathcal{S}(n)} a_{ir}(n) \boldsymbol{x}_{i}(n) + \boldsymbol{w}_{r}(n)$$
(1)

where $r \in \{d\} \bigcup \mathcal{R}(n), r \notin \mathcal{S}(n)$, with $a_{ir}(n)$ denoting the channel coefficient between the *i*-th source node and the receiving node; $w_r(n)$ representing noise; and $\{d\}$ denoting the destination

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node. During the (n + k)-th slot, the BS/AP receives:

$$\boldsymbol{z}_{d}(n+k) = \begin{cases} a_{rd}(n+k)\boldsymbol{x}_{r}(n) + \boldsymbol{w}_{d}(n+k), \\ r \in \mathcal{R}(n) \bigcap \mathcal{S}(n) \\ a_{rd}(n+k)c(n+k)\boldsymbol{y}_{r}(n) + \boldsymbol{w}_{d}(n+k) \\ r \in \mathcal{R}(n), \quad r \notin \mathcal{S}(n) \end{cases}$$
(2)

where $z_d(n + k)$ is a $1 \times N$ vector; $w_d(n + k)$ denotes the noise vector at the access point; c(n + k) represent the scaling constant, which is selected so that the transmit power is maintained within the constraints of the relay's transmitter.

Let us define matrices X, whose rows are the signals sent by source nodes i.e., $X = [x_{i_1}^T(n), \dots, x_{i_K}^T(n)]^T$, and Z, whose rows are the signals heard by the destination node during slots $n, n + 1, \dots, n + \hat{K} - 1$, i.e., $Z = [z_d^T(n), z_d^T(n + 1), \dots, z_d^T(n + \hat{K} - 1)]^T$ with $z_d(n) = y_d(n)$. Let us assume that among the $\hat{K} - 1$ nodes, the first l nodes are relay nodes with $0 \le l \le \hat{K} - 1$ and the remaining are source nodes, i.e. $(r_{l+1}, \dots, r_{\hat{K}-1}) \subseteq S(n)$.

The received signal at the destination can then be written in matrix form as:

$$Z = HX + W \tag{3}$$

where the matrix H and W contain respectively channel coefficients and noise, and their exact structure can be easily inferred from (1) and (2).

The channel estimation and active user detection is done through the orthogonal ID sequence that are attached to each packet as in [6]. At the BS, the correlation of the received signal and the ID sequences is performed. The collision order K, can be detected by comparing the result of the correlation to a pre-defined threshold. The ID sequences are also used as pilots for channel estimation.

The CTE extends over $\hat{K} - 1$ slots with $\hat{K} \ge K$. Once the $\hat{K} \times K$ mixing matrix H is estimated, the transmitted packets, i.e., **X** can be obtained via a maximum likelihood (ML) decoder or zero forcing decoder based on (3).

III. BER ANALYSIS OF ALLIANCES

III-A. Pairwise error probability (PEP)

Let us first find the pairwise error probability for the MIMO system in (3). PEP is a measure of the diversity [11] introduced by the relays in Rayleigh fading channels, and also an intermediate step to computing BER. A PEP bound for the case of high signalto-noise ratio (SNR) was given in [8]. Here we derive a tighter bound for PEP that applies for all SNRs and is also simpler to compute.

We will make the following assumptions, which were also made in [8]: (A1) The channel coefficients $a_{ij}(.)$, i, j = ... are independent and identically distributed (i.i.d.) zero-mean, circularly symmetric complex Gaussian random variables with variance σ_a^2 ; (A2) The power of transmitted symbols is σ_x^2 ; (A3) the noise variables $w_i(m)$ are uncorrelated, complex, zero-mean white Gaussian with variance σ_w^2 ; (A4) During the CTE, the gain $c = \sqrt{\frac{\sigma_x^2}{K\sigma_a^2\sigma_x^2 + \sigma_w^2}}$ is applied at relay nodes, so that the average energy for each relay transmitter is kept equal to σ_x^2 for a K fold transmission.

To best highlight the spatial diversity advantage of the proposed method we will assume that the channel coefficients $a_{ij}(n) = a_{ij}(n+m)$ for $m = 1, ..., \hat{K} - 1$ (static channel).

Let us assume that the collision order, K and the number of non-source relays, $l (0 \le l \le K)$, are fixed. The conditional probability of the ML receiver deciding erroneously in favor of $\hat{\mathbf{X}}$ while \mathbf{X} was transmitted equals:

$$\bar{P}_{e}(\boldsymbol{X} \to \hat{\boldsymbol{X}}|K, l) \leq \prod_{j=1}^{\bar{r}} (1 + \tilde{\lambda}_{j} \frac{\sigma_{x}^{2}}{4} \sigma_{a}^{2} \rho_{1}^{2})^{-1} \\
\cdot \prod_{j=1}^{r} [-\frac{1}{\gamma_{j}} exp(\frac{1}{\gamma_{j} \sigma_{a}^{2}}) Ei(-\frac{1}{\gamma_{j} \sigma_{a}^{2}})]^{l}$$
(4)

where $\gamma_j = \frac{\sigma_x^2}{4} \lambda_j \sigma_a^2 \rho_2^2 c^2$; $\rho_1^2 = \frac{1}{\sigma_w^2}; \rho_2^2 = \frac{1}{\sigma_w^2(1+\sigma_a^2 c^2)}; Ei(.)$ is the exponential integral function; λ_j and $\tilde{\lambda}_j$ are the eigenvalues of \mathbf{R}_{Δ} and $\tilde{\mathbf{R}}_{\Delta}$, where $\mathbf{R}_{\Delta} = \frac{1}{\sigma_x^2} (\mathbf{X} - \hat{\mathbf{X}}) (\mathbf{X} - \hat{\mathbf{X}})^H$ and $\tilde{\mathbf{R}}_{\Delta} = \mathbf{R}_{\Delta} + \sum_{i=1}^{\hat{K}-(l+1)} \Phi_{l+i} \mathbf{R}_{\Delta} \Phi_{l+i}$ with Φ_{l+i} a matrix with all elements equal to zero except the element at (l+i, l+i) that equals one; r is the rank of \mathbf{R}_{Δ} and \tilde{r} is the rank of $\tilde{\mathbf{R}}_{\Delta}$.

The proof of (4) is given in the Appendix.

Let us define $SNR \stackrel{\triangle}{=} \sigma_x^2/\sigma_w^2$. For high SNR, $\gamma_j \approx \frac{\lambda_j SNR}{4(K+1)}$. Since in that case $\frac{1}{\gamma_j}$ is the dominant factor in the second product of the RHS of (4), the diversity order is determined by the number of non-source relays.

III-B. BER conditioned on K and l

To compute the average BER we need to average the PEP over all error events (sequence pairs) corresponding to a given transmitted sequence, weighted by the number of information bit errors associated with that event [3]. We statistically average this sum over all transmitted sequences and divide by the number of input bits per transmission. The BER conditioned on K and l is:

$$P_{b}(K,l) = (1/n_{c}) \sum_{\boldsymbol{X}} P(\boldsymbol{X}) \sum_{\boldsymbol{X} \neq \hat{\boldsymbol{X}}} n(\boldsymbol{X}, \hat{\boldsymbol{X}})$$
$$\cdot \bar{P}_{e}(\boldsymbol{X} \rightarrow \hat{\boldsymbol{X}} | K, l)$$
(5)

where $P(\mathbf{X})$: probability that \mathbf{X} is transmitted; $n(\mathbf{X}, \hat{\mathbf{X}})$: number of information bit errors committed by choosing $\hat{\mathbf{X}}$ instead of \mathbf{X} ; n_c : number of info bits per transmission.

By inserting (4) into (5) for all possible combinations of X and \hat{X} , one can get an exact expression for all modulation types in X and all SNRs.

In Fig. 1 we plot the theoretical result, obtained by numerically evaluating (4) and (5), for 2 specific cases. *Case 1*: No collision occurs, i.e, K = 1. In this case there is no user cooperation, nor relays. It represents a non-cooperative transmission scheme, e.g. the Slotted ALOHA with no collision, or the TDMA scheme; *Case 2*: The collision order is K = 4, thus the number of non-source relays in the CTE can be l = 0, 1, 2, 3. The transmitted signals were modulated as Binary Phase Shift Keying (BPSK). For simplicity we took each packet to contain only 1 information bit.

To validate the theoretical expressions we also plot the BER obtained as follows. A Monte Carlo simulation was run $M = 10^6$ times for each case. Each point on the curve is an average of the Moutcomes. The network population was J = 8, i.e, 8 users transmitted in a random access fashion described in section II. The channel coefficients between users and user - base station were simulated according to the sum-of-sinusoids simulation model for Rayleigh fading channels [10], according to which, each channel multi-path was a zero mean complex Gaussian random variable, with variance $\sigma_a^2 = 1$. The nodes' ID sequences were selected based on the rows of a *J*-th order Hadamard matrix. Following the ID sequence header N = 1 information bit was included in each packet. The packets were modulated as Binary Phase Shift Keying (BPSK) signal. Maximum likelihood decoding was used at the receiver to recover the packet.



Fig. 1. BER vs SNR, variable K and I: Simulations (solid lines), vs. analytical results (dashed lines). figure

Based on Fig. 1 one can see that there is good matching between theoretical expressions and simulations taking into account that the theoretical values serve as an upper bound. Also, both theory and simulations indicate that the more non-source relays are involved in the collision resolution, the lower the BER is (compare (K =4, l = 3) to (K = 4, l = 0)). It is interesting to note that the BER of ALLIANCES is actually lower than that of the non-cooperative scheme (K = 1), suggesting that collision resolution with user cooperation is actually better than preventing collisions.

We should note that in the simulations results included in Fig. 1 we used the true channel matrix H. The end effect of channel estimation errors would be a shift of the BER to a SNR region approximately 1-2 db lower.

III-C. BER conditioned on traffic load

In section III-B the BER is conditioned on K and l. In real applications both the K and l are random variables. In this section we evaluate the BER conditioned as a function of incoming traffic load of the entire network. For simplicity we assume $\hat{K} = K$. It holds:

$$P_b = \sum_{K=1}^{J} \sum_{l=0}^{\min(J-K,K-1)} P(K)P(l|K)P_b(K,l)$$
(6)

where P(K) is the probability distribution function of K, and P(l|K) is the probability distribution of l conditioned on K users transmitting simultaneously.

P(K) is a function of the traffic characteristics, e.g. traffic load, traffic distribution (Poisson or bursty), transmission control scheme as well as the buffering conditions.

In the simple case in which each user transmits with identical probability P_t , P(K) is a binomial random variable, i.e.,

$$P(K) = {}_{K}^{J} P_{t}^{K} (1 - P_{t})^{J - K}$$
(7)

If user *i* transmits with probability P_i , and $P_i \neq P_j$ for $i \neq j$, P(K) has a complex form as follows:

$$P(K) = \sum_{\mathcal{S}} (\prod_{i \in \mathcal{S}} P_i \cdot \prod_{j \notin \mathcal{S}} (1 - P_j))$$
(8)

where S is the set that contains K different users. (8) might be used to model a network that each user generates a different traffic load.

P(l|K) is a function of the relay selection scheme. For the relay selection scheme used in [7], the selection of relay nodes is based on a predetermined order. The nodes involved in the collisions are random and the collision slots are also random. Since each node has the same probability to be selected as relay, l is a hypergeometric distributed random variable, i.e.,

$$P(l|K) = \frac{\binom{J-K}{l}\binom{K}{K-1-l}}{\binom{J}{K-1}}$$
(9)

When inserting (7) and (9) into (6), the BER of the ALLIANCES protocol is solely conditioned on the transmission probability.

In Fig. 2 we show the result of evaluating (6) under different traffic loads (dash line). The traffic load λ is defined as the average number of incoming packets per time slot. If each user transmits independently, $\lambda = \sum_{i=1}^{J} \lambda_i$, and $P_i = min(\lambda_i, 1)$. Users do not have a buffer, which means if user *i* has more than one new packets generated in one time slot, it will transmit one and discard all the others. We considered two cases: *Case 1*- users generate identical traffic loads, i.e., $\lambda_i = \lambda/J$ for i = 1, ...J; *Case 2*- users generate different traffic load λ_i . In particular, we use $\lambda_1 = 3\lambda/J$, $\lambda_i = \lambda/3J$ for $i = 2, 3, 4; \lambda_i = \lambda/J$ for i = 5, 6, 7, 8. Variable bit rate traffic is often generated by multimedia applications.

Based on Fig. 2, we can see that for the uniform traffic case, the lowest BER is obtained when $\lambda = J/2$, which is when the expectation of l reaches its maximum value. At high traffic load most nodes are also sources, thus there are not enough non-source relays to offer diversity and the BER increases. For the variable traffic case and at high traffic loads the BER performance is better than that of the uniform traffic case. The reason is that the users with low incoming traffic load have the opportunity to serve as nonsource relays in the collision resolution procedure, thus provide spatial diversity to the system. Thus, ALLIANCES is best suitable for variable bit rate applications. The BER vs traffic load for the aforementioned 2 cases was also computed via simulations and is shown in Fig. 2; simulations result in similar tends as the theoretical expressions, except that now the curves are shifted downward, which is reasonable since the BER expressions serve as an upper bound of performance.

IV. CONCLUSIONS

In this paper we evaluate the ALLIANCES protocol in a Rayleigh flat fading scenario. Both analytical and simulations results show that the spatial diversity introduced by user cooperation improves the bit error rate as compared to collision avoidance



Fig. 2. BER vs Traffic Load, J=8, N=1 bit, SNR=20dB: Theoretical (dashed lines) and simulation results (solid lines). figure

schemes. The results also show that ALLIANCES is best suited for variable rate traffic.

V. APPENDIX

The covariance matrix of the noise term in (3) is $\mathbf{R}_w = diag([\frac{1}{\rho_1^2}, ..., \frac{1}{\rho_K^2}])$ where $\rho_1^2 = \frac{1}{\sigma_w^2}$, $\rho_2^2 = ... = \rho_{l+1}^2 = \frac{1}{\sigma_w^2(1+\sigma_a^2c^2)}$, $\rho_{l+2}^2 = ... = \rho_{\hat{K}}^2 = \frac{1}{\sigma_w^2}$. Let us rewrite (3) as:

$$\mathbf{Z}_{w} \stackrel{\triangle}{=} \mathbf{R}_{w}^{-1/2} \mathbf{Z} = \sigma_{x} \mathbf{R}_{w}^{-1/2} \mathbf{H} \mathbf{X}' + \mathbf{R}_{w}^{-1/2} \mathbf{W}$$
(10)

where $\mathbf{X}' = \frac{1}{\sigma_x} \mathbf{X}$. In (10) the noise term, $\mathbf{R}_w^{-1/2} \mathbf{W}$ is complex Gaussian with zero mean and covariance matrix $\mathbf{I}_{\hat{K}}$.

Let us assume that K and l are fixed. The conditional probability that the ML receiver decides erroneously in favor of $\hat{\mathbf{X}}$ while \mathbf{X} was transmitted equals:

$$P_{e}(\mathbf{X} \to \hat{\mathbf{X}}|K, l) = E\{Q(\sqrt{\frac{\sigma_{x}^{2}}{2}}||\mathbf{R}_{w}^{-1/2}\boldsymbol{H}(\mathbf{X}' - \hat{\mathbf{X}}')||_{F})\} \\ \leq E\{exp(-\frac{\sigma_{x}^{2}}{2}||\mathbf{R}_{w}^{-1/2}\boldsymbol{H}(\mathbf{X}' - \hat{\mathbf{X}}')||_{F}^{2})\}$$
(11)

where $||\cdot||_F$ denotes Frobenius norm; $Q(x) = \int_x^\infty e^{-t^2/2} dt$ and it was used that $Q(x) \le e^{-x^2/2}$. It holds:

$$\begin{aligned} \|\mathbf{R}_{w}^{-1/2}\boldsymbol{H}(\mathbf{X}'-\hat{\mathbf{X}}')\|_{F}^{2} &= trace(\mathbf{R}_{w}^{-1/2}\boldsymbol{H}\mathbf{R}_{\Delta}\boldsymbol{H}^{H}\mathbf{R}_{w}^{-1/2})\\ &= \sum_{i=1}^{\hat{K}}\rho_{i}^{2}\mathbf{h}_{i}\mathbf{R}_{\Delta}\mathbf{h}_{i}^{H} = \rho_{1}^{2}\mathbf{h}_{1}\tilde{\mathbf{R}}_{\Delta}\mathbf{h}_{1}^{H} + \rho_{2}^{2}c^{2}\sum_{i=2}^{l+1}\mathbf{h}_{i}\mathbf{R}_{\Delta}\mathbf{h}_{i}^{H} \end{aligned}$$

where \mathbf{h}_i is the *i*-th row of matrix \boldsymbol{H} and $\tilde{\mathbf{R}}_{\Delta} = \mathbf{R}_{\Delta} + \sum_{i=1}^{\hat{K}-(l+1)} \Phi_{l+i} \mathbf{R}_{\Delta} \Phi_{l+i}$. The second term in $\tilde{\mathbf{R}}_{\Delta}$ is a diagonal matrix whose non-zero diagonal element at (l+i, l+i) equals the corresponding element of \mathbf{R}_{Δ} .

The row vector \mathbf{h}_1 contains channel coefficients between the source nodes and the destination, while \mathbf{h}_i , i = 2, ..., l + 1 can be

expressed as $\mathbf{h}_i = ca_{r_i d} \mathbf{a}_i$, where \mathbf{a}_i contains channel coefficients between the source nodes and the *i*-th relay. Note that the \mathbf{a}_i 's are uncorrelated, and each one is complex Gaussian zero-mean with covariance $\mathbf{R}_a = \sigma_a^2 \mathbf{I}$. The probability density function of \mathbf{a}_i is $f_a(a) = \frac{1}{\pi K \det(\mathbf{R}_a)} exp(-\mathbf{a}\mathbf{R}_{\Delta}^{-1}\mathbf{a}^H)$. It holds:

$$\begin{split} E\{exp(-\frac{\sigma_x^2}{4}||\mathbf{R}_w^{-1/2}H(\mathbf{X}'-\hat{\mathbf{X}}')||_F^2)|_{a_{r_id},\ i=2,...,l+1}\} \\ = & \int exp(-\frac{\sigma_x^2}{4}\rho_1^2\mathbf{h}_1\tilde{\mathbf{R}}_{\Delta}\mathbf{h}_1^H)f_{h_1}(\mathbf{h}_1)d\mathbf{h}_1 \\ & \cdot \prod_{i=2}^{l+1}\int exp(-\frac{\sigma_x^2}{4}\rho_2^2c^2|a_{r_id}|^2\mathbf{a}_i\mathbf{R}_{\Delta}\mathbf{a}_i^H)f_{a_i}(\mathbf{a}_i)d\mathbf{a}_i \\ = & \frac{1}{\sigma_a^{2K}}det(\frac{\sigma_x^2}{4}\rho_1^2\tilde{\mathbf{R}}_{\Delta}+\frac{1}{\sigma_a^2}\mathbf{I})^{-1} \\ & \cdot \prod_{i=2}^{l+1}\frac{1}{\sigma_a^{2K}}det(\frac{\sigma_x^2}{4}\rho_2^2c^2|a_{r_id}|^2\mathbf{R}_{\Delta}+\frac{1}{\sigma_a^2}\mathbf{I})^{-1} \\ = & \prod_{j=1}^{\tilde{r}}(1+\tilde{\lambda}_j\frac{\sigma_x^2}{4}\sigma_a^2\rho_1^2)^{-1}\cdot\prod_{i=2}^{l+1}\prod_{j=1}^{r}(1+\lambda_j\frac{\sigma_x^2}{4}\sigma_a^2\rho_2^2c^2|a_{r_id}|^2)^{-1} \end{split}$$

where λ_i , $\bar{\lambda}_i$ are the eigenvalues of \mathbf{R}_{Δ} , $\bar{\mathbf{R}}_{\Delta}$, respectively. r is the rank of \mathbf{R}_{Δ} , \tilde{r} is the rank of $\mathbf{\tilde{R}}_{\Delta}$. We should note here that both \mathbf{R}_{Δ} and $\mathbf{\tilde{R}}_{\Delta}$ are positive definite, thus their eigenvalues are non-negative.

In (12) it was used that for a positive definite matrix **R** and a complex **x** of size $1 \times K$ it holds: $\int exp(-\mathbf{x}\mathbf{R}^{-1}\mathbf{x}^{H})d\mathbf{x} = \pi^{K}det(\mathbf{R})$. Taking into account that $|a_{r_{id}}|$ is Rayleigh distributed with pdf: $p(|a_{r_{id}}|) = \frac{2|a_{r_{id}}|}{\sigma_a^2}exp(-\frac{|a_{r_{id}}|^2}{\sigma_a^2})$ and using the integral (Integral 3.352, 4 in [12]) we finally get (4).

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