WAVE FIELD SIMULATION WITH THE FUNCTIONAL TRANSFORMATION METHOD

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ABSTRACT

The Functional Transformation Method (FTM) provides a frequency domain based analytic solution of arbitrary linear partial differential equation. For wave field simulations however, its application was so far restricted to simple geometries as the FTM involves a search for the eigenmodes of the model. Recently so called block based modeling algorithms were introduced, that follow a divide-and-conquer approach. A complex geometry is split into several simple elementary blocks. These blocks are solved and discretized separately, while their connection is realized in the discrete system during run-time. In this paper a 2D wave field simulation program based on block based modeling is demonstrated. Elementary block models are solved with the FTM and can be connected together to create complex 2D geometries. The complete system benefits from the advantages of the FTM (e.g. dispersion-free simulations), while the complexity of the geometry fits the needs of typical CAD drawings.

1. INTRODUCTION

For physical models given in terms of Partial Differential Equations (PDEs), the Functional Transformation Method (FTM) provides an elegant analytic solution. In doing so, the FTM has so far been successfully applied to physical models of plates and membranes [1], resulting in real-time capable sound synthesis algorithms based on physical modeling.

Concerning the simulation of room acoustics (for 2D only a special case of the membrane model), the FTM was so far restricted to simple geometries, i.e. rectangular and circular regions as presented in [1]. The search for the eigenmodes of the model, which are needed for the integral transformations of the FTM, gets very evolved for complex geometries.

However, recently so called block based modeling methods were introduced. The complete model is split into several blocks, which are modeled and implemented separately, while their correct physical interaction is taken care of in the discrete realization by a suitable interaction topology. One approach suitable for finite difference schemes and wave guide implementations can be found in [2]. A more general approach which allows almost arbitrary modeling techniques can be found in [3]. Its feasibility for the wave equation with FTM block models has already been proven in [4].

Therefore in this paper a simulation tool for room acoustics based on block based modeling as described in [3] with blocks modeled with the FTM as described in [1] is demonstrated. Focusing on 2D wave fields, a brief review of the approach to block based modeling is given in section 2. The application of the FTM to the specific block models is described in section 3. Section 4 provides more detailed information on the correct interaction of these block models, and the resulting simulation tool is presented in section 5. Section 6 concludes this paper.

2. BLOCK BASED APPROACH

The idea of block based modeling, particularly for solid regions, is to apply a minimum amount of spatial sampling in order to achieve smaller regions, which are easier to model. The principle procedure can be seen in figure 1, where the ground plan of a church is split into simple rectangular and triangular blocks.

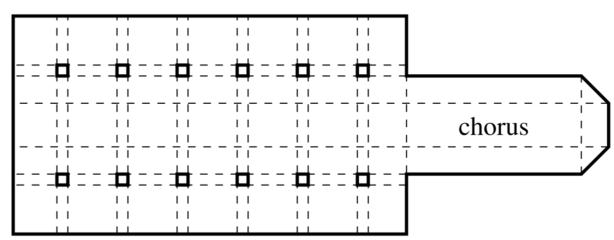

Fig. 1. Ground plan sketch of a typical Gothic church located in Germany. The dashed lines indicate the subdivision into rectangular end triangular block models.

A simulation tool using this block wise description has to face two principle problems: 1) the correct physical interaction of the blocks has to be guaranteed, 2) the interaction condition has to be realized in the discrete system. The solution of these problems according to [3] is described in the sequel.

2.1. Correct Interaction

The starting point to proof correct physical interaction, is a unified system description as done in [3]. As the physical model is described by a set of linear PDEs, one can always introduce additional auxiliary variables to formulate a set of PDE with first order spatial derivatives only. Combining all variables in the vector of outcomes $\mathbf{y} = \mathbf{y}(\vec{x}, t)$ one can describe the set of first order PDEs by one vector PDE with first order derivatives in space and time only

$$\mathbf{A}\mathbf{y} + \mathbf{B}\nabla\mathbf{y} + \mathbf{C}\frac{\partial}{\partial t}\mathbf{y} = \mathbf{f}_{\mathrm{e}}(\vec{x}, t) \ , \tag{1}$$

with ∇ denoting the gradient, resp. the divergence, applied on the elements of $\mathbf{y}$, and space coordinate $\vec{x} \in V$, and time $t \in [0, \infty[$. Each i-th block model, with $\vec{x} \in V_i$, can be described in the form of equation (1) with block-specific system matrices $\mathbf{A}_i, \mathbf{B}_i$, and $\mathbf{C}_i$ and its specific excitation $\mathbf{f}_{\mathrm{e},i}(\vec{x}, t)$.

For a correct physical interaction of two models, there must also be a description in the form of equation (1), that describes the combined model with $V = V_1 \cup V_2 \cup \partial V$, where ∂V is the connection region of V_1 and V_2. Within V_1 and V_2, existence of a description in the form of equation (1) is guaranteed by the block models themselves, but at the connection region is has to be guaranteed by the interaction topology. In detail, one has to assure the existence of $\nabla\mathbf{y}(\vec{x}_{\mathrm{b}}, t)$ for $\mathbf{x}_{\mathrm{b}} \in \partial V$.

In doing so, one has to differentiate between the components normal to the boundary (indicated by the normal vector $\vec{n}_{\mathrm{b}}$), and those in line with the boundary. For the existence of $\nabla\mathbf{y}(\vec{x}_{\mathrm{b}}, t)$ only the continuity of the normal component, i.e. $\vec{n}_{\mathrm{b}}\mathbf{y}(\vec{x}_{\mathrm{b}}, t)$, is necessary as continuity in line with the boundary is guaranteed by the block models. In detail, one has to assure that left-side and right-side boundary value in ∂V is identical

$$\lim_{x_1 \to x_{\mathrm{b}}} \mathbf{y}_1(x_1, t) = \lim_{x_2 \to x_{\mathrm{b}}} \mathbf{y}_2(x_2, t) \ . \tag{2}$$

Equation (2) itself is fulfilled by the definition of suitable boundary conditions, which serve as the inputs to the system (the normal component of the particle velocity in this case), and the definition of suitable output- variables (the pressure at the boundary in this case). Details to the definition of the boundary conditions and the outputs can be found in [4], here it remains to mention, that a correct physical interaction is assured if pressure $p(\vec{x}_{\mathrm{b}}, t)$ and normal component of the particle velocity $\vec{n}_{\mathrm{b}}\vec{v}(\vec{x}_{\mathrm{b}}, t)$ are equal for the connected block models, and this for all points $\vec{x}_{\mathrm{b}} \in \partial V$.

2.2. Discrete Realization

In consequence, the physical interaction of the block models can be traced back to a network with pairs of physical variables (pressure and normal component of particle velocity here), that have to fulfill the so called Kirchhoff rules. These networks correspond to electrical networks, which are well understood and where methods for discrete solutions are well known.

One of these methods for discrete solution is quite of interest in this context, as it preserves the spatial structure of the original model (the connection of blocks) in the discrete system. This method are the Wave Digital Filters (WDFs) [5], that avoid realization problems due to delay free loops via the definition of so called wave variables. These wave variables are causal in the sense, that the reflected wave b is always a response to the incident wave a, in contrast to voltage v and current i. In discrete implementations, this causality of the waves variables is reflected in one sample delay between incident and reflected wave.

However, the usage of wave digital filter principles involves, that the complete modeling and discretization process is done using these principles. In this paper, the FTM should be used to model and discretize the block models, which is not based on WDF principles. A solution of this problem is a recently introduced technique for the interconnection of arbitrary discrete state space structures with wave digital filters referenced in [3]. With a slight modification of the block models discrete structure, connection of FTM blocks to the WDF interaction topology is possible.

3. FTM BLOCK MODELS

The application of the FTM for 2D problems is well documented in [1]. Here a very short review of this process is given with respect to the block based modeling approach.

3.1. Model Description

The physical model for acoustical wave propagation is the well known wave equation, which is based on the two following physical principles

$$\begin{aligned} -\frac{\partial}{\partial t}p(\vec{x}, t) &= \varrho_0 c^2 \nabla \vec{v}(\vec{x}, t) && \text{equ. of continuity} \\ -\nabla p(\vec{x}, t) &= \varrho_0 \frac{\partial}{\partial t}\vec{v}(\vec{x}, t) && \text{equ. of motion ,} \end{aligned}$$

where c is the speed of sound in the air and ϱ_0 is the mass density of air. This two partial differential equations can be easily combined to one vector partial differential equation according to equation (1) with the vector of outcomes

$$\mathbf{y}(\vec{x}, t) := \begin{pmatrix} p(\vec{x}, t) & -\varrho_0 \vec{v}(\vec{x}, t) \end{pmatrix}^T \ . \tag{3}$$

According to the interaction conditions from section 2.1, we define $v_{\mathrm{b}}(\vec{x}_{\mathrm{b}}, t) := -\varrho_0 \vec{n}_{\mathrm{b}} \vec{v}(\vec{x}_{\mathrm{b}}, t)$ to be the inputs of the model, and we solve for the pressure $p(\vec{x}_{\mathrm{b}}, t)$ with $\vec{x}_{\mathrm{b}} \in \partial V$.

3.2. Solution with the FTM

Details on the application of the FTM can be found in [1], an overview of the procedure is given in figure 2. The starting point is a linear PDE, for instance given in the form of equation (1), with suitable Boundary Conditions (BC), here defined according to section 2.1, and suitable Initial Conditions (IC), here homogeneous.

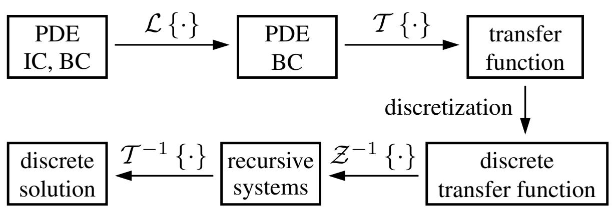

Fig. 2. General procedure of the FTM for solving initial boundary value problems in form of PDEs with initial- (IC) and boundary (BC) conditions. The transformations are explained in the remainder of this section.

The Laplace transformation ($\mathcal{L}\{\cdot\}$) yields a PDE in space only, as all temporal derivatives are replaced by powers of the time-frequency variable s. The following problem specific Sturm-Liouville Transformation (SLT, $\mathcal{T}\{\cdot\}$) acts similar on the spatial derivatives, resulting in a transfer function, both in time and space frequency domain. Discretization with the impulse-invariant transformation, inverse $\mathcal{Z}$-transformation, and inverse SLT yield the desired discrete solution in terms of weighted complex first order recursive systems.

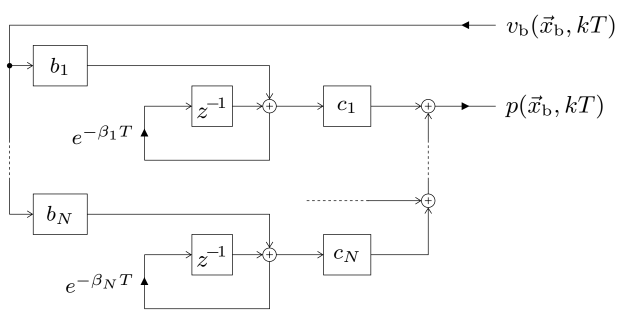

Fig. 3. Structure of the FTM implementation with N complex harmonics. Explicit values for the input weighting constants b_n and output weighting constants c_n can be found in [1].

The resulting structure is depicted in figure 3. T is the sampling interval and k denotes the discrete time step. Each first order resonator with frequency β_μ represent one eigenmode of the system. These eigenmodes are weighted and summed, yielding the desired output. Both, eigenfrequency β_μ and weighting constants c_μ depend on the PDE and on the geometry of the model. So far analytic solutions for rectangular, circular, and triangular regions are available. The input weighting constants b_μ depend on the position and type of excitation, details can be found in [1].

4. INTERCONNECTION

Some specific information about the connection of the discrete block models realization from the previous section are given here.

4.1. Interacting Spatial Harmonics

One problem which is also addressed in-depth in [4], is to guarantee the interaction condition for all boundary points $\vec{x}_\mathrm{b} \in \partial V$, i.e. an infinite number of interaction points. One approach would be to choose a finite number of equally spaced interaction points, however a more reasoned one is provided by the FTM solution. The number of eigenmodes is always finite (N in figure 3), as the eigenfrequencies are limited by the Nyquist frequency. Instead of performing the complete inverse SLT by a weighted sum (see figure 3), one could realize the interaction of the block models via interacting spatial harmonics. On one hand this approach yields always a finite number of interacting variables, and on the other hand it is more efficient, as less multiplications are required to realize the interaction.

4.2. Discrete Realization with WDF Principles

As already mentioned in section 2.2, the structure from figure 3 is slightly modified in order to fit to the wave digital interaction topology. The modified system uses wave variables as in- and outputs. The interaction of these wave variables is realized with so called port adaptors (see [5]), however in this scenario, it is assumed that the speed of sound is identical for both models, what simplifies these adaptors to simple bridging elements.

5. SIMULATION PROGRAM

The complete algorithm, i.e. the block models from section 3 and the discrete realization of the interaction topology according to section 4 is realized in a simulation tool called *"Wave2D"*. Each block model is an object of the same class, so it is possible to simulate an almost arbitrary number of blocks. Possible block geometries are rectangles, circles, and triangles, their arrangement as well as all other program and model parameters can be saved in special script-files. The wave fields are visualized in 3D with hardware supported rendering interfaces.

Two screenshots with the *"Church"*- example can be seen in figure 4. Although more than 169m^2 are simulated with a frequency range up to 11kHz, almost 4 frames per second are calculated and depicted on a Pentium IV with 2.8GHz. Videos of the simulations produced with *"Wave2D"* and further material can be found online [6].

Another example is depicted in figure 5. The FTM enables the exact positioning of sources (loudspeakers) and out-

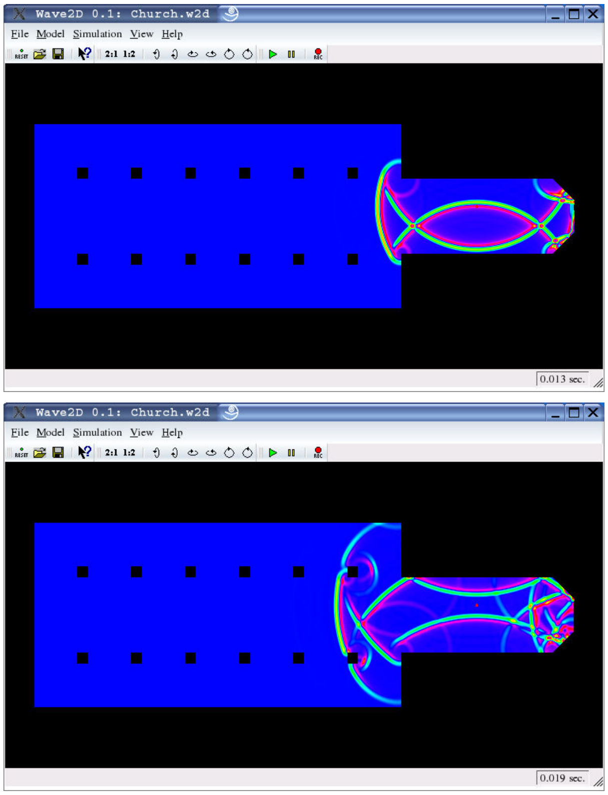

Fig. 4. Impulse propagation in a church simulated with *"Wave2D"*. Complete videos can be found in [6].

puts (microphones), so that advanced multichannel reproduction algorithms could be tested. See [6] for details.

Compared with more popular methods for the simulation of room acoustics, wave digital meshes for instance (see [7]), the proposed approach offers almost the same number of frames per second for comparable scenarios. However, it provides much more accuracy, as the FTM simulations are completely free of dispersion and as the FTM allows exact positioning of sources and outputs.

6. CONCLUSIONS

In this paper the feasibility of the FTM for the simulation of room acoustics has been extended with the help of block based modeling techniques. In doing so a minimum amount of spatial sampling is applied to the room geometry, by splitting the room into blocks that are manageable with the FTM. These blocks are solved with the FTM and realized separately as different objects in the simulation tool. Their correct interaction is assured by the program with the usage of wave digital filter principles. A program was presented that realizes the proposed approach and visualizes the resulting wave fields with smooth animations. The approach offers accurate and fast simulations for geometries with a reasonable amount of complexity.

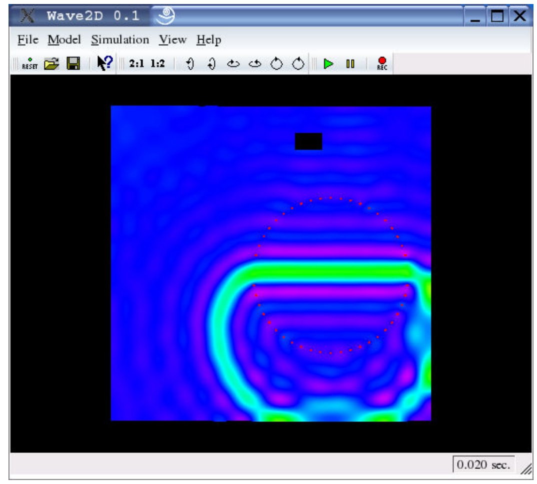

Fig. 5. Simulation of multichannel reproduction systems.

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