FAST FILTERED-X AFFINE PROJECTION ALGORITHM WITH EFFICIENT COMPUTATION OF COEFFICIENT UPDATE

Alberto González, Miguel Ferrer, María de Diego, and Gema Piñero

Audio and Communications Signal Processing Group (i-TEAM)
Universidad Politécnica de Valencia
http://www.gtac.upv.es, agonzal@dcom.upv.es

ABSTRACT

An efficient version of the affine projection (AP) algorithm, which is called FXAP-AC throughout the paper, is presented. This algorithm uses the filtered-x structure instead of the modified structure which is required for the implementation of the AP algorithm when the desired signal is not available, for instance in Active Noise Control (ANC) applications. The conventional AP algorithm for ANC, which is based on the modified structure (MFXAP), provides good performance. However the filtered-x structure is advisable for practical cases since it achieves a meaningful computational saving avoiding the additional filtering needed by the modified structure without significantly affect the performance in practice. In this paper, an efficient computation of coefficient update is developed for the FXAP resulting in the FXAP-AC. This low-cost strategy already proposed in the conventional AP algorithm can be suitably combined with others previously reported to achieve very fast and robust FXAP algorithms. Experimental results validate the use of the proposed algorithm for practical applications.

1. INTRODUCTION

The family of affine projection algorithms (AP) [1] have been developed during last years to be used in practical applications in order to improve the convergence properties of Least Mean Squares (LMS) type adaptive filters but avoiding the high computational complexity of the Recursive Least Squares (RLS) algorithms [2].

The convergence properties of the AP algorithm can be improved by increasing what is called the projection order (N) but also its computational complexity. Computationally efficient strategies have been developed for practical AP algorithms [3] to achieve low computational cost algorithms whereas their good convergence properties remain, thus providing fast AP algorithms (FAP). Different efficient AP filtering methods have been proposed and implemented for different applications, mainly acoustic echo cancellation [4, 5] and sound reproduction and control [6, 7]. Some of these strategies already proposed are based either on efficient calculation of the error vector [3, 8], or low-cost matrix inversions [9, 10, 11], or fast computation of the coefficient update [5] or meaningful combinations between them [12].

The fast computation of the coefficient update (also known as auxiliary coefficient strategy) was first introduced by Gay in [5] for echo cancellation. This strategy involves computing, instead of the adaptive filter coefficients, some auxiliary coefficients which require less operations to be updated. Thus, the desired signal needed for the echo signal cancellation is achieved by means of the auxiliary coefficients and their relationship with the coefficients of the adaptive filter. In the single channel case, this strategy represents a saving of $3N$ multiplications per iteration achieving a meaningful computational saving when the projection order increases. In this paper, we will use this strategy in filtered-x adaptive filtering schemes, specially suitable for ANC.

The application of the AP algorithm to ANC requires to take into account firstly two things: the secondary path between the adaptive filter output and the error sensor whose negative effects on the algorithm performance should be compensated using the suitable adaptive filtering scheme, and the unavailability of the desired signal. Both things together lead us to use the modified structure of the filtered-x adaptive filtering scheme which is more computationally demanding that the basic filtered-x scheme. Most of the reported AP algorithms consider the modified structure [6]. We will name this algorithm as Modified Filtered-x AP (MFXAP) algorithm throughout this paper. On the other hand, a fast AP algorithm based on filtered-x structure was firstly reported by the authors in [8] and it is named Filtered-x AP (FXAP). In this paper, a fast new version of the FXAP algorithm with efficient computation of coefficient update (FXAP-AC) is developed.

In Section 2, we describe the conventional AP algorithm. The FXAP is presented in section 3. Section 4 is devoted to apply the auxiliary coefficient strategy to the FXAP, resulting in the fast FXAP (FXAP-AC). A comparative practical study of the FXAP-AC, the FXAP, the filtered-x LMS (FXLMS) [13] and the MFXAP for a multichannel ANC system, is presented in Section 5.

2. ADAPTIVE AFFINE PROJECTION ALGORITHM

The AP can be derived solving an optimization problem where the solution is constrained to the *minimum perturbation principle* [2], that implies a minimum variation of the weight vector whereas the filter coefficients are constrained so that the desired signal $d[n]$ is exactly generated by filtering the reference signal $x[n]$, see Figure 1. The solution of the constrained optimization criterion gives the following expression of the adaptive filter coefficient update,

$$\mathbf{w}_L[n+1] = \mathbf{w}_L[n] + \mu \mathbf{A}^T[n](\mathbf{A}[n]\mathbf{A}^T[n] + \delta I)^{-1}\mathbf{e}_N[n], \quad (1)$$

where $\mathbf{w}_L[n]$ is a vector comprised of the L adaptive filter coefficients at the nth time instant ($\mathbf{w}_L[n] = [w_1[n], w_2[n], ..., w_L[n]]^T$). In practical applications, a regularization parameter δ is incorporated to the matrix inversion to mitigate matrix inversion problems. A convergence parameter μ is used to adjust the convergence

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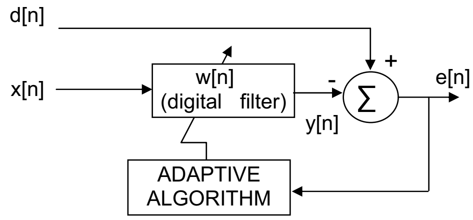


Fig. 1. Adaptive filtering scheme

speed. Matrix $\mathbf{A}[n]$ of dimensions $N \times L$ is defined as follows:

$$\mathbf{A}^T[n] = [\mathbf{x}_L[n], \mathbf{x}_L[n-1], ..., \mathbf{x}_L[n-N+1]], \qquad (2)$$

being $\mathbf{x}_L[n]$ a vector with the last L samples of signal $x[n]$. The error vector, denoted by $\mathbf{e}_N[n]$ in equation (1), is calculated by:

$$\mathbf{e}_N[n] = \mathbf{d}_N[n] - \mathbf{A}[n]\mathbf{w}_L[n], \qquad (3)$$

where $\mathbf{d}_N[n]$ is the $N \times 1$ vector composed by the desired signal samples, ($d_N^T[n] = [d[n], d[n-1], ..., d[n-N+1]]$).

3. ADAPTIVE AFFINE PROJECTION ALGORITHM FOR ANC

Affine projection algorithm was originally proposed for echo cancellation [3, 4], where the desired signal is available. However, this is not the case for ANC systems where the system can only pick up the error signal $e[n]$. Thus, the modified structure has been usually applied in ANC systems [6, 7] because it allows to recover the desired signal needed in the AP algorithm, see equation (3). Nevertheless, the modified structure is more computationally demanding than the adaptive filtering scheme commonly applied in ANC systems, the filtered-x structure, since additional filtering is required [14]. Moreover, the filtered-x scheme offers a good trade-off between convergence properties and computational cost. The application of the filtered-x scheme to the AP algorithm provides the Filtered-x AP (FXAP) algorithm, that can outperform some efficient AP algorithmic variants based on the modified structure in terms of computational complexity whereas it shows useful convergence properties [12].

With regard to the AP algorithm equations described in the previous section, the following additional notation is required to describe the FXAP (for simplicity a single channel ANC system is considered: one input signal, one actuator and one error sensor),

- M: Length of the $\mathbf{h}$ FIR filter modelling the secondary path (acoustic plant between the actuator and the error sensor).
- y[n]: Value at time *n* of the signal at the actuator.
- $x_f[n]$: Value at time *n* of the reference signal filtered by $\mathbf{h}$.
- $\mathbf{h} = [h_1, h_2, ...h_M]^T$, being h_m the *m*th coefficient in the $\mathbf{h}$ FIR filter.
- $\mathbf{v}_L[n]$=$[x_f[n], x_f[n-1], ...x_f[n-L+1]]^T$
- $\mathbf{V}[n] = [\mathbf{v}_L[n] \dots \mathbf{v}_L[n-N+1]]^T$
- $\mathbf{x}_M[n] = [x[n], x[n-1], ...x[n-M+1]]^T$
- $\mathbf{e}_N[n] = [e[n], e[n-1], ... \, e[n-N+1]]^T$

It should be noted, that the error vector $\mathbf{e}_N[n]$ is not exactly the same described by equation (3). As it was previously commented, in ANC systems based in filtered-x structure we can not access the desired signal to calculate the required error vector, therefore the FXAP uses past samples of the error signal $e[n]$ to build $\mathbf{e}_N[n]$. This approximation is accurate enough under the assumption of slow convergence and accurate estimation of the secondary path [8].

According to the preceding notation, the FXAP algorithm can be described as follows, where it is assumed that the signals $e[n]$ and $x[n]$ are available:

- The normalized error vector is computed as:
  $\epsilon_N[n] = [\mathbf{V}[n-1]\mathbf{V}^T[n-1] + \delta I]^{-1}\mathbf{e}_N[n]$
  Multiplications: LN^2+N^2+N+ $O(\frac{N^3}{2})$
- The update equation is given by:
  $\mathbf{w}_L[n] = \mathbf{w}_L[n-1] - \mu\mathbf{V}^T[n-1]\epsilon_N[n]$
  Multiplications: LN+L
- The filtered reference signal is obtained as:
  $\mathbf{x}_f[n] = \mathbf{h}^T\mathbf{x}_M[n]$
  Multiplications: M
- And finally the cancelling signal is obtained as:
  $y[n] = \mathbf{w}_L^T[n]\mathbf{x}_L[n]$
  Multiplications: L

The total number of multiplications per iteration required in the FXAP reaches $LN^2 + N^2 + N + O(\frac{N^3}{2}) + M + NL + 2L$. Among these multiplications, $NL + 2L$ can be saved by using the auxiliary coefficient strategy. On the other hand, other efficient strategies to compute the normalized error vector allow to reduce the number of multiplications even more. As an example, the fast strategy described in [9] to avoid matrix inversions requires a computational cost of only $8N^2+4N^2+2+M$ multiplications instead of $LN^2 + N^2 + N + O(\frac{N^3}{2}) + M$.

Finally, it should also be noted that the temporal index of the reference signals is delayed in the coefficient update equation as well as in the normalized error vector equation. This fact improves significantly the algorithm performance when using a practical sample by sample processing approach since the coefficient update equation depends on two signals highly correlated, the signal derived from the reference signal, $\mathbf{V}[n-1]$, at sampling instant $(n-1)$ and the normalized error vector, belonging to the same instant.

4. FILTERED-X AP ALGORITHM WITH AUXILIARY COEFFICIENTS

4.1. Auxiliary coefficient strategy

The efficient computation of coefficient update involves computing, instead of the coefficients of the adaptive filter, some auxiliary coefficients which require less operations to be updated [5]. Thus, the cancelling signal needed for the active control system is achieved by means of the auxiliary coefficients and their relationships with the coefficients of the adaptive filter,

$$y[n] = \widehat{\mathbf{w}}_L^T[n]\mathbf{x}_L[n] - \mu\mathbf{v}_L^T[n]\overline{\mathbf{V}}^T[n]\widehat{\overline{\mathbf{e}}}_N[n] \qquad (4)$$

where $\overline{\mathbf{V}}[n]$ is a $(N-1) \times L$ matrix with the first $(N-1)$ rows of matrix $\mathbf{V}[n]$ and $\widehat{\mathbf{w}}_L^T[n]$ is a vector comprised of the L auxiliary coefficients with an update equation:

$$\widehat{\mathbf{w}}_L[n] = \widehat{\mathbf{w}}_L[n-1] - \mu\mathbf{v}_L[n-N]e_{N-1}[n] \qquad (5)$$

being $e_{N-1}[n]$ the last element of the auxiliary error vector,

$$\widehat{\mathbf{e}}_N[n] = \epsilon_N[n] + \begin{bmatrix} 0 \\ \widehat{\overline{\mathbf{e}}}_N[n-1] \end{bmatrix} \quad (6)$$

where $\widehat{\overline{\mathbf{e}}}_N[n]$ in (4) and (6) refers to the first $N-1$ elements of auxiliary error vector $\widehat{\mathbf{e}}_N[n]$.

From equations (4) and (5) it seems that computational saving gained by using the auxiliary coefficient update is lost by the cost due to the computation of the cancelling signal, equation (4). However, the vector-matrix product $\mathbf{v}_L^T[n]\overline{\mathbf{V}}^T[n]$ can be recursively computed as:

$$[\mathbf{v}_L^T[n]\overline{\mathbf{V}}^T[n]]^T = \overline{\mathbf{r}}[n] = \overline{\mathbf{r}}[n-1] + v[n]\overline{\mathbf{v}}_N^T[n] - v[n-L]\overline{\mathbf{v}}_N^T[n-L] \quad (7)$$

where $\overline{\mathbf{v}}_N^T[n]$ is the $(N-1) \times 1$ vector with the first $(N-1)$ elements of the filtered reference signal vector $\mathbf{v}_L[n-1]$.

4.2. Algorithm description

The Filtered-x AP algorithm with the auxiliary coefficient strategy embedded (FXAP-AC) can be finally described as follows.

- The normalized error vector is calculated as,
  $\epsilon_N[n] = [\mathbf{V}[n-1]\mathbf{V}^T[n-1] + \delta I]^{-1}\mathbf{e}_N[n]$
  Multiplications: LN^2+N^2+N+O($\frac{N^3}{2}$) (O($\frac{N^3}{2}$) cost can be avoided by means of efficient matrix inversion [9])
- Following, the auxiliary error vector is,
  $\widehat{\mathbf{e}}_N[n] = \epsilon_N[n] + \begin{bmatrix} 0 \\ \widehat{\overline{\mathbf{e}}}_N[n-1] \end{bmatrix}$
- Next, the auxiliary coefficient update,
  $\widehat{\mathbf{w}}_L[n] = \widehat{\mathbf{w}}_L[n-1] - \mu\mathbf{v}_L[n-N]e_{N-1}[n]$
  Multiplications: L+1
- The filtered reference signal is obtained as,
  $x_f[n] = \mathbf{h}^T\mathbf{x}_M[n]$
  Multiplications: M
- The vector $\overline{\mathbf{r}}$[n] is given by
  $\overline{\mathbf{r}}[n] = \overline{\mathbf{r}}[n-1] + v[n]\overline{\mathbf{v}}_N^T[n] - v[n-L]\overline{\mathbf{v}}_N^T[n-L]$
  Multiplications: 2(N-1)
- Finally, the cancelling signal is obtained,
  $y[n] = \widehat{\mathbf{w}}_L^T[n]\mathbf{x}_L[n] - \mu\overline{\mathbf{r}}[n]^T\widehat{\overline{\mathbf{e}}}_N[n-1]$
  Multiplications: L+(N-1)+1

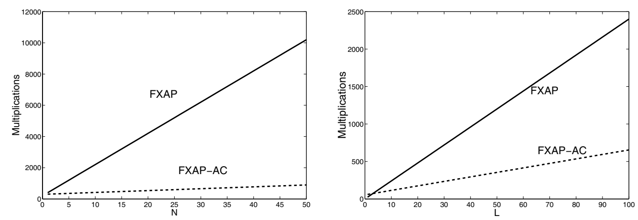
5. SIMULATION RESULTS

5.1. Multichannel ANC system

In order to test the performance of the proposed FXAP-AC algorithm compared to other versions of the AP and the FXLMS in a practical system, a multichannel ANC system with one primary source ($I = 1$), two secondary sources ($J = 2$) and two error sensors ($K = 2$) (1:2:2 ANC system) is considered. Several simulations have been carried out using: a white gaussian noise source, real acoustic paths measured in a real room and a length of $M = 250$ coefficients for the filters modelling the secondary paths.

Algorithm	multiplications per iteration	Typical case
FXAP	IJK[LN^2+N^2+N+O($\frac{N^3}{2}$)+M]+ IJ(NLK+2L)	7370
FXAP-AC	IJK[LN^2+N^2+N+O($\frac{N^3}{2}$)+M]+ IJ[K(L+N)+L+1]+IJK[2(N-1)]	6524

Table 1. Number of multiplications per channel and per iteration of the FXAP and the FXAP-AC algorithms for the general case and also for a 1:2:2 ANC system, $N = 50$, $L = 50$ and $M = 250$.



(a) (b)

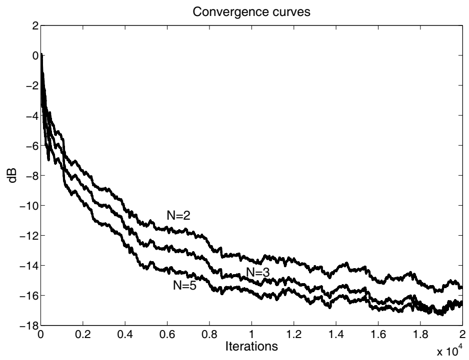
Fig. 2. Comparison of the computational cost of the FXAP and the FXAP-AC algorithms in a 1:2:2 ANC system for: a) different projection orders and b) different length of adaptive filter.

5.2. Computational complexity

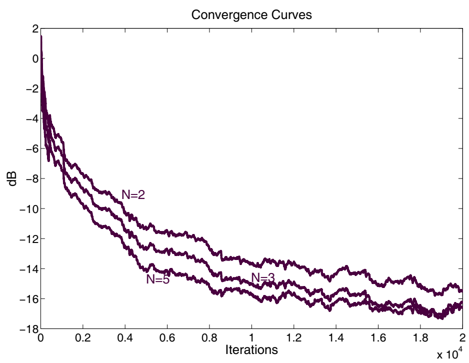
Table 1 illustrates the number of multiplications per channel and per iteration required by the FXAP and the FXAP-AC algorithms for the general case and for the 1:2:2 ANC system. As it was expected, the FXAP-AC shows less computational complexity than the FXAP algorithm. In Figure 2, a computational cost comparison for different projection orders ($L = 50$) and for different length of adaptive filters ($N = 50$) in a 1:2:2 ANC system is presented. It must be noted that the computational saving of the FXAP-AC algorithm increases when N and/or L increase. More details related AP algorithms in terms of computational cost can be found in [15].

5.3. Convergence curves

In order to validate the new approach for practical applications, the convergence speed should also be studied. Different convergence curves [15] have been obtained. The FXAP and the FXAP-AC exhibit pretty similar results, see Figure 3. Moreover, when the projection order increases the residual error decreases in both algorithms as it was expected. On the other hand, it seems convenient to compare algorithm performances using maximum value of the convergence parameter μ. A projection order of $N = 5$ and the algorithms FXLMS, FXAP, FXAP-AC and the MFXAP, were chosen in the simulations shown in Figure 4. In addition to the similarity between the FXAP and the FXAP-AC curves, the MFXAP algorithm shows a slightly faster convergence speed but with a higher computational cost than the FXAP [12]. Furthermore, the residual error is less in the filtered-x algorithms (FXLMS, FXAP and FXAP-AC), since their convergence parameters are smaller than those of the MFXAP.



(a)



(b)

Fig. 3. Convergence curves of the FXAP algorithm (a) compared to the proposed FXAP-AC (b) of order $N = 2$, 3 and 5 for random noise. Signal in error sensor 2.

6. CONCLUSIONS

The FXAP-AC algorithm based on the the filtered-x structure and the auxiliary coefficient strategy has been introduced for multichannel ANC, providing a significant computational saving over the FXAP and the MFXAP for similar convergence properties. Moreover, the FXAP-AC showed a better performance than others AP algorithm versions. Therefore this algorithm has shown to be very suitable for real practical multichannel ANC systems.

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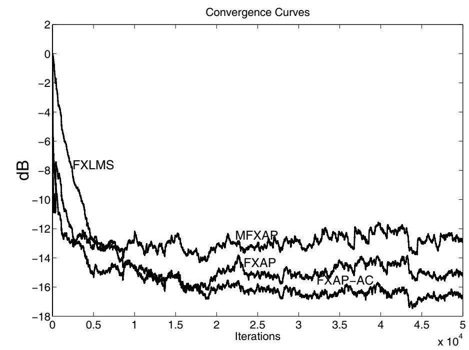

Fig. 4. Convergence comparison of the FXLMS, and AP algorithms of order $N = 5$: FXAP, FXAP-AC and MFXAP. Signal in error sensor 2.

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