ON THE ADAPTATION OF THE LINEAR BICHARACTERISTIC SCHEME TO BLOCK-BASED PHYSICAL MODELING FOR DIGITAL SOUND SYNTHESIS OF STRING INSTRUMENTS

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ABSTRACT

Block-based physical modeling is one of the most promising methods for digital sound synthesis. This technique proposes to model and discretize each instrument element separately and then, to implement their interaction. In this paper, the use of the Linear Bicharacteristic Scheme (LBS) or upwind leapfrog is proposed for digital sound synthesis of string instruments. It provides an efficient and accurate alternative stencil to the classical leapfrog scheme of the Finite Difference Time Domain (FDTD) method. Moreover, the conversion of dependent wave equation variables into characteristic variables makes this method suitable to interact with Wave Digital Filter models and others paradigms. This technique is extensively presented and finally justified with some examples.

1. INTRODUCTION

Recently, physical modeling has arisen as an interesting alternative to digital sound synthesis. In this way, several paradigms offer different advantages (i.e., see [1] for details). Moreover, block-based physical modeling grows as one of the most promising research line [2] in this field. In this case, the physical model is divided in different parts, and therefore, each one is separately modeled and implemented. Furthermore, each part can be modeled by different modeling techniques [3].

One of the most employed paradigm for string simulation is the FDTD method [4]. Although it offers several advantages for string simulation, the inherent dispersion forces to increase the computional cost to solve it [5]. In this paper, the Linear Bicharacteristic Scheme (LBS), or upwind leapfrog scheme [6], arises as an alternative scheme to decrease the effort for reducing the dispersion effect. LBS is a well-known scheme for unsteady aeroacoustic and electromagnetism applications. It has a more compact stencil compared with the classical leapfrog scheme. Clustering the stencil around the preferred directions (characteristics), it enables high accuracy with a low order of operations (see Fig. 1 for details).

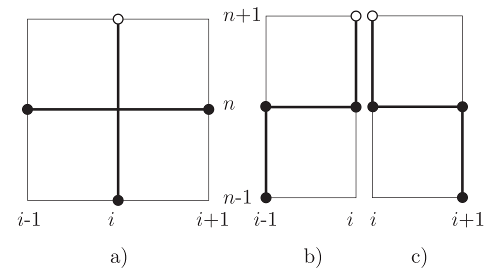

Fig. 1. Stencil of : a) the FDTD leapfrog scheme, b) the LBS right-going component and c) left-going component.

Furthermore, in this work, LBS is not only proposed for digital audio synthesis and to reduce the computational cost, but also this technique is modified in order to identify bicharacteristic variables with Wave Digital Filter (WDF) variables [7]. As will be shown in section 4, this approach modifies the original method introducing additional refinements that provides a flexible and transparent method to interconnect LBS with WDF or other paradigms [3].

2. BLOCK-BASED MODEL

2.1. Wave equation

The fundamental of physical modeling is the description of physical laws by means of differential equation. In this paper, a simplified model of a string has been chosen, using the linearized Mass Conservation (1) and Euler (2) equations and including some non frequency-dependent losses terms.

$$\frac{\partial p(x,t)}{\partial x} = -\rho \frac{\partial u(x,t)}{\partial t} - \alpha^* u(x,t), \quad (1)$$
$$\frac{\partial u(x,t)}{\partial x} = -\frac{1}{\rho c^2} \frac{\partial p(x,t)}{\partial t} - \alpha p(x,t), \quad (2)$$

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where $p(x,t)$ and $u(x,t)$ are the pressure and the particle velocity component, ρ and c are the density and speed of sound respectively, and α^* and α define the non frequency-dependent losses terms. Note that the losses terms have not physical meaning, but can be combined to obtain an approach to the desired attenuation. Furthermore, they are susceptible to include PML absorbing boundary conditions [**?**].

2.2. Block-based interaction

A complete description and characterization of a partial differential equation (PDE) problem (such in this case, Eq. (1) and (2)) requires a set of additional information: initial conditions (IC) for temporal derivatives and boundary conditions (BC) for spatial derivatives. In fact, the interaction between blocks is expressed as a BC.

In a block-based model, the PDE that governs each block can be carried out by means of different models. Assuming a seamless transition from one model to the other, there has to be a function that solves a global PDE on the combined region. This is achieved when the differentiability is assured in all points of the interface region between blocks (see [3] for details). An usual and simple way consists into arranging the interface points into pairs of ports variables and connecting them to the appropriate pair of ports variables that share the same boundary region.

3. WAVE DIGITAL FILTER MODELS

In order to achieve the interaction defined at section 2.2, Wave Digital Filters (WDF) are used. WDF provide an smart and efficient method to solve continuous networks in the discrete time domain [7]. Their main advantage in this context is the discretization process. The discretization is carried out separately for each network element by the bilinear transformation. Potential computational problems, e.g. delay-free-loops, are avoided by the definition of the so called *wave variables* (W-variables)

$$a = p + R \cdot u, \quad (3)$$
$$b = p - R \cdot u. \quad (4)$$

The variables a and b are called the incident and the reflected wave variables, respectively. This transition from the *Kirchhoff variables* (K-variables) p and u to the W-variables is the key point to a block-based system [1]. By a proper choice of port resistances R, WDFs offer the opportunity to make independent the numerical method of each block from their interaction by selecting the appropriate adaptor elements (see [7]).

[1]Note that the *impedance analogy* [8] has been used, where classical electric notation of WDF has been substituted by acoustic variables.

4. LINEAR BICHARACTERISTIC SCHEME

In this section, first, some properties of the LBS are summarized in order to understand their advantages. After that, the details of LBS implementation are presented, centered in the identification of characteristic variables with W-variables.

4.1. LBS Fourier Analysis

Several complete LBS Fourier analysis have already been carried out (a detailed compilation of these results can be found in [9]). Some of the most important information regards Fourier analysis is summarized in this paper. The stability condition for the LBS is $\nu \leq 1$, where ν is the Courant number, defined as $\nu = c\Delta t/\Delta x$, where c is the speed of sound and Δt and Δx are the time and spatial sampling, respectively.

As is mentioned previously, LBS offers some advantages about classical leapfrog implementation. In the FDTD case, the leading error term of the phase speed error in the leapfrog scheme is given by

$$\Theta_{\mathrm{FDTD}} = \frac{4\pi^2}{6N^2}\nu(\nu^2 - 1), \quad (5)$$

where N is the number of points per wavelength.

In LBS case, the leading error term of the phase speed error is

$$\Theta_{\mathrm{LBS}} = \frac{4\pi^2}{12N^2}\nu(1-\nu)(1-2\nu). \quad (6)$$

Fig. 2 compares normalized phase speed at different ν, according to Eq. (5) and (6). It can be observed that for achieving less than 1 % phase speed error, about $N = 18$ for the FDTD method and about $N = 7$ for the LBS is required. Moreover, Eq. (6) shows that the zero dispersion in the LBS can be achieved with $\nu = 1$ and $\nu = 0.5$. These results show that LBS can be 2-3 times more economical than the FDTD method for the same level of accuracy [9].

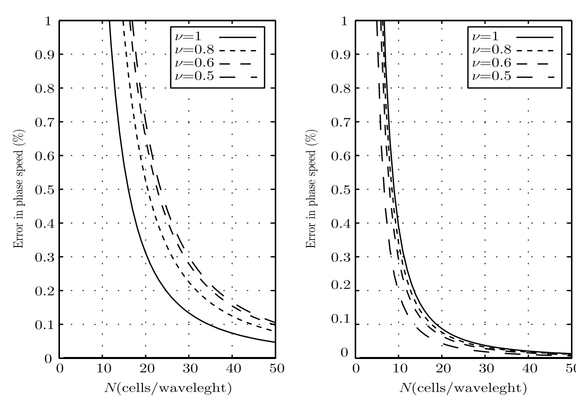

Fig. 2. Percentage error in phase speed versus grid resolution: a) FDTD and b) LBS. Plot parameter is Courant number ν [10].

4.2. Implementation

The LBS method operates transforming the dependent variables p and u into characteristic variables. Moreover, in this paper, these variables are directly identified with W-variables a and b. To transform Eq. (1) and (2) into characteristic form, first Eq. (2) is multiplied by ρc and then substracted from (1) multiplied by c, to give

$$c\frac{\partial(p+\rho cu)}{\partial x}+\frac{\partial(p+\rho cu)}{\partial t}+\alpha^* cu+\alpha\rho c^2 p=0, \quad (7)$$
$$c\frac{\partial(p-\rho cu)}{\partial x}-\frac{\partial(p-\rho cu)}{\partial t}+\alpha^* cu-\alpha\rho c^2 p=0. \quad (8)$$

Using definitions (3) and (4) and using the definition of the *acoustic impedance* [8] $R=\rho c$, these equations can be rewritten as

$$\frac{\partial a}{\partial t}+c\frac{\partial a}{\partial x}+\frac{k_1}{2}a+\frac{k_2}{2}b=0, \quad (9)$$
$$\frac{\partial b}{\partial t}-c\frac{\partial b}{\partial x}+\frac{k_1}{2}b+\frac{k_2}{2}a=0, \quad (10)$$

where

$$k_1 = \left(\alpha c+\frac{\alpha^*}{\rho}\right), \quad (11)$$
$$k_2 = \left(\alpha c-\frac{\alpha^*}{\rho}\right). \quad (12)$$

To develop the discretized algorithm for a 1-D system, the stencil of Fig. 1 a) and b) is followed for the LBS. To obtain a solution without dissipation (this means no amplitude errors), it is necessary that the stencil has central symmetry in order to obtain a reversible scheme in time. A scheme without dissipation means that integrating from given data at t = 0 to a solution at $t=T$, and then, with time reversed, integrating back to $t=0$, the data are exactly recovered in amplitude (apart from roundoff errors). The source terms require some special attention in order to avoid an unstable result. In this paper, they have been implemented following [10]. It found an efficient and accurate scheme indexing the self source term in Eq. (9), this means a, at time step $n+1$ and to index the coupled source term b at time level n. Eq. (9) follows the same procedure.

Using the stencil of Fig. 1 and the source term indexing scheme outlined above, the finite difference equations results

$$\frac{(a_i^{n+1}-a_i^n)+(a_{i-1}^n-a_{i-1}^{n-1})}{2\Delta t} + c\left(\frac{a_i^n-a_{i-1}^n}{\Delta x}\right) + \frac{k_1}{2}a_i^{n+1}+\frac{k_2}{2}b_i^n=0, \quad (13)$$

$$\frac{(b_i^{n+1}-b_i^n)+(b_{i+1}^n-b_{i+1}^{n-1})}{2\Delta t} - c\left(\frac{b_{i+1}^n-b_i^n}{\Delta x}\right) + \frac{k_1}{2}b_i^{n+1}+\frac{k_2}{2}a_i^n=0. \quad (14)$$

These equations are rewritten as

$$a_i^{n+1}=f_1^n/(1+k_1\Delta t), \quad (15)$$
$$b_i^{n+1}=f_2^n/(1+k_1\Delta t), \quad (16)$$

where f_1^n and f_2^n are the residuals defined by

$$f_1^n = a_{i-1}^{n-1}+(1-2\nu)(a_i^n-a_{i-1}^n)-k_2\Delta t b_i^n, \quad (17)$$
$$f_2^n = b_{i+1}^{n-1}-(1-2\nu)(b_{i+1}^n-b_i^n)-k_2\Delta t a_i^n. \quad (18)$$

5. RESULTS

In order to demonstrate the viability of LBS for digital sound synthesis of string instruments (1-D wave equation), an example is presented. Fig. 3 shows the structure followed in the example (see [1] for detailed WDF elements description). It represents a very simplified string modeled by means of the LBS approach presented in the paper. The string extremes are modeled with a WDF mass and a WDF resistance. Two parallel adaptators are used in order to connect both WDF with the string model.

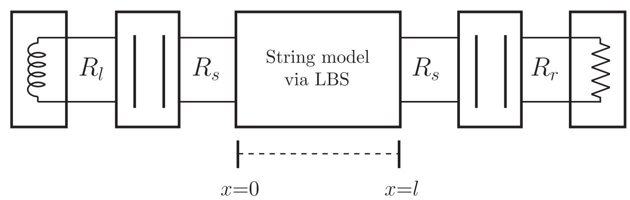

Fig. 3. String model implemented.

The values of the parameters do not correspond with any real example, they are selected in order to facilitate visualization of the properties. The length of the string is l=1 m, with $\rho=1.21$ kg/m^3 and $c=680$ m/s, as values of the density and the speed of sound, respectively. It means that the string resistance is R_s=822.8 kg/(m^2·s). The sampling frequency is $f_s=44.1$ kHz. The WDF components have a value of $R_r=5R_s$ and $R_l=2Lf_s$, with $L=0.1$ kg·s/m^2. The attenuation values are $\alpha^*=200$ kg/(m^3·s) and $\alpha=0.001$ s/(kg·m). The source has been implemented by means of a gaussian pulse with a bandwidth $W=f_s/6$.

The use of LBS takes its major advantages when $\nu=0.5$ (it is necessary the half of discrete points compared with $\nu=1$, it is non-dispersive and it have lower phase error), so this is the value of Courant number used in this example.

Fig. 4 shows the simulation result. It represents the pressure variable result of the wave equation under BC specified in Fig. 3.

As is expected, the simulation does not show dispersion effects for the Courant number selected. Furthermore, the effect caused by the WDF at the extremes of the string can

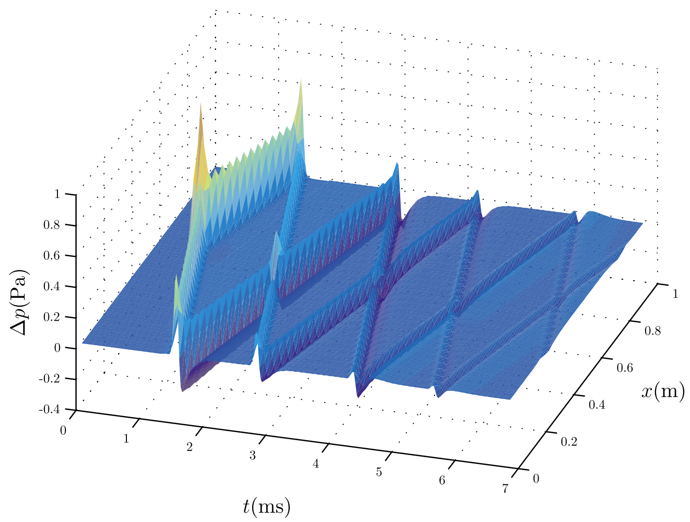

Fig. 4. Simulation of the one-dimensional wave-equation with LBS technique.

be observed specially in the first reflection (approximately at t=1.4 ms). As can be perceived, the effect of WDF inductance (at $x = 0$) acts like a low-pass filter, whereas WDF resistor (at x=1) produces a dissipation of energy. More complex WDF structures, even nonlinearities, can be used at boundaries using the same procedure [11]. The simulations carried out with ν=1 show similar results.

These results look promising and they make suitable the use of this scheme into block-based physical modeling with mixed strategies. In fact, it allows the use of the classical FDTD properties [5] with an additional reduction of computational cost, due to the conversion between *Kirchhoff* and *wave variables* is avoided [2] and the inherent properties of LBS scheme.

6. CONCLUSIONS

In this paper, the use of linear bicharacteristic scheme into block-based physical modeling mixing strategies is presented. This scheme appears as an alternative to the classical leap-frog scheme for the FDTD paradigm, providing a more economical method keeping the same accuracy.

Furthermore, LBS is modified in order to identify bicharacteristic variables with W-variables, allowing a direct connection with WDF and without additional calculations in order to interact between them, obtaining successful results. Extensions to two or three-dimensional problems should be straightforward.

7. REFERENCES

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