EFFICIENT DESIGNS FOR BROADBAND LINEAR ARRAYS

Michael M. Goodwin

Audio Research Department
Creative Advanced Technology Center
Scotts Valley, CA

ABSTRACT

Linear electroacoustic arrays are useful for various applications in audio acquisition and reproduction. For robust broadband performance, a filter network is typically incorporated to achieve roughly frequency-invariant beamforming. In this paper, we are concerned with the design of cost-effective broadband arrays of limited size for consumer audio applications. Filter-based broadband beamformers are suboptimal for such applications due to high cost and the large number of elements generally needed to achieve invariance. Here, three alternative design methods are presented which lead to effective far-field broadband performance of small arrays at low cost: allpass weighting, optimal nonuniform spacing, and optimal delay dispersion. Performance improvements are demonstrated for each method and various design tradeoffs are considered.

1. INTRODUCTION

Linear electroacoustic arrays are useful for a variety of applications in audio acquisition and reproduction. In hands-free telecommunication, for instance, an adaptive beamformer can be used to locate and track a moving talker while combining the microphone signals in some optimal manner to enhance the acquired speech. In such cases, the processing leverages the redundancy of the desired signal across the various array microphones. Here, we are interested in exploiting broadcast redundancy in an array of small loudspeakers (or *microdrivers*) to achieve acoustic power output similar to a large loudspeaker but at lower cost and with a form factor more amenable to consumer audio deployment. Furthermore, and in marked contrast to many array scenarios, we are interested in using a loudspeaker array not for directional sound transmission but rather for rendering broadband audio over a wide listening area at an acceptable level of quality. Essentially, we would like to take advantage of the array form factor without being penalized by the *de facto* directionality.

Various constraints can be enumerated based on the goal of wide-angle fidelity as well as the target deployment in low-cost consumer products. There are both performance constraints and marketplace cost considerations to incorporate in the array design:

- Maximal output level. The broadside output gain should be as high as possible given the number of array elements.
- Flat broadside response. The correctly positioned listener should have an uncompromised listening experience.
- Robust off-broadside response. The response degradations for off-broadside listeners should be minimized.
- Small number of elements. Low-cost consumer audio products have practical and necessary size limits.
- Minimal implementation cost. For low-cost productization, it is necessary to keep the cost of the parts as low as possible.

In this paper, we discuss array designs which address these constraints and demonstrate the improvements that can be achieved by the various methods. Similar constraints led to the design approach proposed in [1], which is described here in Section 2.2.

1.1. Array response formulation

In the far field, *i.e.* at a distance where the signals emanating from the array elements can be reasonably approximated as plane waves, the response of a uniformly spaced N-element linear array at frequency $\omega = 2\pi f$ and listening angle θ (from broadside) is given by

$$A(\omega, \theta) \;=\; \sum_{n=0}^{N-1} a_n e^{-jn\omega \frac{d}{c}\sin\theta} \;=\; \text{DTFT}\{a_n\}\big|_{\Omega = \frac{\omega d}{c}\sin\theta} \quad (1)$$

where c is the speed of sound, d is the interelement spacing, and the a_n are element weights. Considering the mapping $\Omega = \frac{\omega d}{c}\sin\theta$, it is clear that the the DTFT of the taps, denoted by $A(\Omega)$ hereafter, completely characterizes the array response.

Given the array response formulation, we can express the first three constraints of Section 1 in mathematical terms:

$\sum_n a_n = N$	Maximal output level
$A(\omega, 0) = A(\omega_0, 0)$	Flat broadside response
$\lvert A(\omega,\theta) - A(\omega_0,\theta)\rvert < \epsilon(\omega,\theta)$	Robust off-broadside response

In this paper, we will demonstrate full-range optimizations, but in principle these constraints could be defined for limited ranges $\theta_0 \leq \theta \leq \theta_1$ and $\omega_0 \leq \omega \leq \omega_1$, and the design optimizations could be carried out accordingly. In the first constraint, we are implying the limit $|a_n| \leq 1$ without loss of generality; thus, satisfying the maximal level constraint with the basic array described in Eq. (1) calls for the use of uniform excitation ($a_n = 1$). As for the second constraint, the broadside response is specified entirely by the tap DTFT value at $\Omega = 0$, namely $A(\omega, 0) = \sum_n a_n$. Since it meets the first two constraints, then, we consider uniform weighting further. We shall see that it provides a clear illustration of the difficulties posed by the third constraint, which is expressed here as a idealization; in practice, we will attempt to determine the array configuration, within the degrees of design freedom, which achieves the minimum deviation bound, which may potentially be region-dependent for advanced designs or to account for perceptual considerations. As for the remaining constraints from the introduction, we will restrict the simulations to small arrays ($N = 4$) although the methods are scalable to larger systems, and the final cost constraint will not be quantified but rather kept in mind as a design concern. Note that the two first constraints above can be taken together as an indication of the broadband efficiency of the system; a fully efficient N-element array will reproduce a broadband input signal faithfully at broadside with amplitude

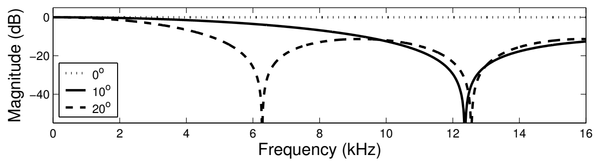

Fig. 1. Frequency response (at various angles) of a 4-element array with uniform weighting and uniform 4*cm* interelement spacing.

gain N. Also, we note here that the response magnitudes in the simulations are normalized with respect to the maximum level N; this allows for a fair comparison between the various methods.

For a uniform array, Eq. (1) yields the array response magnitude

$$|A(\omega,\theta)| \;=\; \left|\frac{\sin\left(\frac{N\omega d}{2c}\sin\theta\right)}{\sin\left(\frac{\omega d}{2c}\sin\theta\right)}\right|. \tag{2}$$

This response is plotted in Figure 1 for $\theta = 0$ (broadside), 10, and 20 degrees. The off-broadside response degradations can be understood in terms of the frequency-dependent mapping of the DTFT $|A(\Omega)| = |\frac{\sin N\Omega/2}{\sin \Omega/2}|$ into the visible range of the array. At $\omega = 0$, the angular array response is a constant given by the value of the DTFT at $\Omega = 0$. As the frequency ω increases, progressively more of the DTFT is mapped into the visible range; the broadside lobe becomes narrower and the higher-Ω features of the DTFT (*e.g.* nulls) begin to appear in the angular response. The frequency response at a fixed angle, then, has a lowpass behavior corresponding to the narrowing of the main lobe; and, analogously to the case for ω described above, at larger θ more of the high-Ω features of the DTFT are evident in the array frequency response. Note that such response degradation occurs for any weights whose DTFT exhibits attenuation and nulls at large Ω and is not specific to uniform weighting.

To address the response degradations of uniform arrays, the array geometry and processing can be generalized as in Figure 2, which includes nonuniform element spacing, arbitrary elemental delays, and frequency-dependent weighting. The far-field response is then

$$A(\omega,\theta) \;=\; \sum_{n=0}^{N-1} a_n(\omega) e^{-j\omega\left(\frac{x_n}{c}\sin\theta - T_n\right)} \tag{3}$$

where x_n, T_n, and $a_n(\omega)$ are the position, delay, and weighting of the n-th element. Comparing this to the simplification of Eq. (1) as a mapped DTFT, we see that Eq. (3) can only be similarly simplified if the taps are not frequency-dependent and if the delays and positions are both affine functions of the element index. For the case of frequency-dependent weighting, the DTFT mapping applies in a limited way: the angular response at a given ω_0 is specified by the DTFT of $a_n(\omega_0)$ evaluated at $\Omega = \frac{\omega_0 d}{c}\sin\theta$.

In Figure 2, the elemental delays are shown implemented using a delay line for each array element. For the sake of efficiency, in practice a single tapped delay line and a router would be used instead of separate delay lines or, for that matter, lumping the delays into the weighting filters $a_n(\omega)$. Using a single tapped delay line is standard in delay-sum beamforming, where the delays are most commonly progressive in nature, *e.g.* delays $T_n = \frac{x_n}{c}\sin\theta_0$ which result in a steering of the beam pattern to angle θ_0, for instance in fixed beam steering as in end-fire arrays or in adaptively steered talker-tracking systems [2, 3]. Here, though, we are not restricted to such progressive delays; we are allowing for an irregular correspondence (not necessarily progressive or even sequential) between the delay line and the array elements.

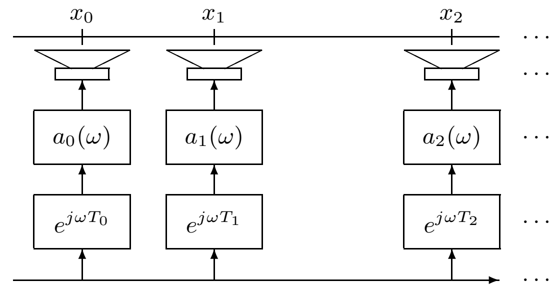

Fig. 2. A generalized array processor with element weights $a_n(\omega)$, delays T_n, and positions x_n.

1.2. Frequency-invariant beamforming

Off-broadside array response degradations can be explained in terms of the frequency dependence of the array pattern. From that perspective, the solution is to introduce some compensation to counteract the frequency variation and maintain a constant beam pattern. A number of such *frequency-invariant beamforming* methods have been described in the literature; for instance, an exact solution for continuous linear arrays and a sampling method for discrete arrays is given in [4], and a general least-squares framework for discrete array geometries is developed in [5]. Such methods are largely based on establishing a frequency dependence for the element weights, sometimes in conjunction with symmetric unequal spacing [4, 6], such that the array pattern is maintained across frequency. In some designs, the invariance is achieved for one octave, and subarrays for adjacent octaves are nested to form a broadband array [7, 8, 9].

Essentially, the invariant beamformers developed for linear arrays involve progressively tapering the outer elements such that the array becomes effectively shorter at higher frequencies. The top plot in Figure 3 shows the response improvement achieved if filters based on the results in [4] are incorporated in a 4-element array with 4*cm* spacing, specifically $a_n(\omega) = \frac{\sin\alpha\omega m}{\alpha\omega m}$ where $m = n - \frac{N-1}{2}$ and α is a design parameter. Comparing this to the uniformly weighted array of Figure 1, we see that the off-axis response has been significantly improved, but at the cost of a lowpass broadside response given by $\sum_n a_n(\omega)$. If a global pre-filter $G(\omega) = \left[\sum_n a_n(\omega)\right]^{-1}$ is introduced as in [9], the broadside response can be flattened. Note however that a broadside loss is still incurred since the response is then normalized to meet a frequency-dependent maximum level constraint $G(\omega)a_n(\omega) \leq 1$. Such a normalization may not be physically necessary given the inherent decreased high-frequency sensitivity of typical loudspeaker elements, but it is included here so as to maintain fair comparisons.

In the Figure 3 simulation, it is assumed that the ideal elemental and compensation filters are realizable. In practice, these are nontrivial filters. The approach is thus problematic for our application given the cost-related constraints: a filter is required for each array element (or each pair of elements for symmetric filtering) and for global compensation, which implies a nonneglible material cost for filter components or a digital signal processor. Furthermore, the tapering of the array at high frequencies means that the outer array elements are only being used at low frequencies; the compensation helps to account for this, but in any event some of the array elements are not in full broadband use, and this is undesirable from the perspective of cost efficiency. Moreover, frequency-invariant beamformers, though applicable to small arrays as demonstrated, generally are able to perform more robustly for arrays with a number of elements beyond that suitable for low-cost consumer products.

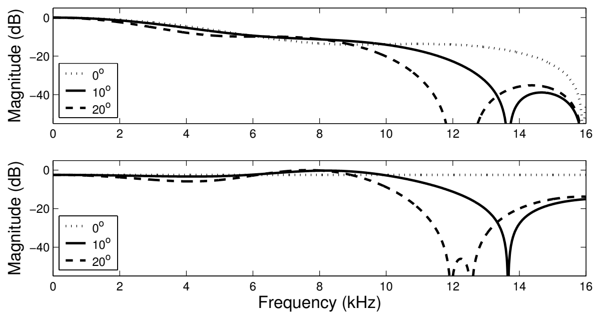

Fig. 3. Frequency response (at various angles) of a 4-element frequency-invariant beamformer with uniform 4*cm* spacing (top) and with a broadside compensation filter (bottom).

2. DESIGN METHODS

Frequency-invariant beamformers based on elemental filters conflict with our design constraints as described above. To reduce the processing cost and increase the efficiency, in the following sections we consider approaches based on the degrees of design freedom offered by static (frequency-independent) weights, nonuniform spacing, and arbitrary elemental delays. Three design approaches are considered in light of the enumerated constraints: allpass filter truncation, nonuniform spacing, and delay diffusion.

2.1. Allpass arrays

Given that the response of the basic array of Eq. (1) for any (ω, θ) corresponds to a sample of the DTFT $A(\Omega)$, the most straightforward approach to removing the frequency dependence of the array response is to design a_n such that $|A(\Omega)| \approx |A(0)|\ \forall\ \Omega$. Namely, if a_n is a good FIR approximation of an allpass filter, *i.e.* has a roughly constant magnitude response, then the array response will also be approximately invariant with respect to both angle and frequency.

In the audio engineering literature, arrays with allpass behavior have been discussed in the limited scope of Bessel arrays, *i.e.* arrays weighted by coefficients generated via Bessel series [10]. The allpass response is typically explained as a feature of the Bessel series; however, a more appropriate explanation is that the Bessel approach can serve as an effective tool for deriving approximate FIR allpass filters. One inherent drawback is that Bessel arrays are limited to odd N. In Figure 4, the zeros of the 5-element Bessel weighting $\{\frac{1}{2}, 1, 1, -1, \frac{1}{2}\}$ are depicted; note that such rational approximations of Bessel-generated weights are common in practice since they allow for efficient implementation using only out-of-phase and series-parallel connections of the array elements (and no additional components), and in some cases even outperform the raw Bessel weights. Now, if a conjugate pair of the Bessel zeros are replaced with a single real zero, a 4-element allpass array is realized. This *ad hoc* design leads to invariance comparable to the Bessel array as illustrated in the frequency responses of Figure 5; clearly, a restriction to an odd number of elements is unnecessary for allpass array design.

Standard FIR filter design algorithms have proven generally ineffective for designing short allpass approximations due to the low filter order; there is thus an opportunity for new optimization methods to be explored, especially for the task of finding rational a_n which enable efficient series-parallel implementation. There may however be diminishing returns in such optimization given the effectiveness of the simple z-plane design shown here.

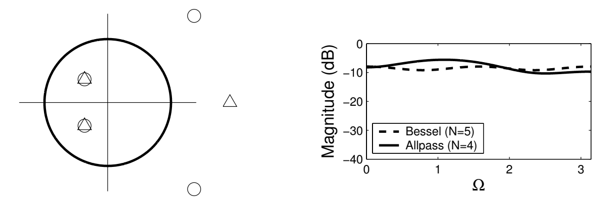

Fig. 4. Zero locations with respect to the unit circle for a 5-element Bessel array (O) and an *ad hoc* 4-element allpass array (△). The characteristic DTFTs are given in the graph; the responses have been normalized by $N = 5$ and $N = 4$, respectively.

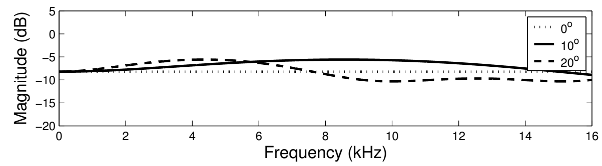

Fig. 5. Frequency response (at various angles) of a 4-element allpass array with uniform 4*cm* spacing.

Allpass weighting effectively addresses the constraints of a flat broadside response and a robust off-broadside response. However, good allpass designs require out-of-phase taps, *i.e.* some of the a_n will be positive and some will be negative. There is thus typically a substantial gain reduction with respect to uniform weighting, so the maximal gain constraint is not met.

2.2. Nonuniform spacing

To satisfy the gain constraint explicitly, we can restrict the design to $a_n = 1$ and consider varying only the element positions x_n. Setting $T_n = 0$ and assuming without loss of generality that the x_n can be expressed as integral points on an underlying grid with arbitrarily close spacing Δ, namely $x_n = \xi_n \Delta$, we can rewrite Eq. (3) as

$$A(\omega, \theta) = \sum_{m=0}^{\xi_{N-1}} \alpha_m e^{-jm\omega \frac{\Delta}{c} \sin\theta} = \mathrm{DTFT}\{\alpha_m\}\big|_{\Omega = \frac{\omega\Delta}{c}\sin\theta} \tag{4}$$

where the sequence α_m is a sparse representation of the array on the underlying grid: $\alpha_m = 1$ if $m \in \{\xi_n\}$ and 0 otherwise. In [1], an exhaustive search for the optimal spacing is carried out using nested loops which cover all valid configurations. In the innermost loop, the algorithm leverages the ability to characterize the response fully using the DTFT of the sparse binary sequence α_m. The DTFT whose minimum magnitude over the target operating range is a maximum over the set of valid configurations is determined, and the corresponding array design is selected as optimal. This optimization amounts to finding the array design with the minimum ϵ bound for the off-broadside response constraint.

As in the allpass array design, it is informative to consider the locations of the zeros of the sequence α_m. For a uniformly spaced 4-element array (with $a_n = 1$), the zeros are on the unit circle as shown in Figure 6. In this design, the optimized spacing is given by doubling the interelement distance to the end element. This introduces another zero; as indicated in the figure, though, none of the zeros of the optimized array sequence are on the unit circle; instead, they are distributed as conjugate pairs and reciprocals. It should

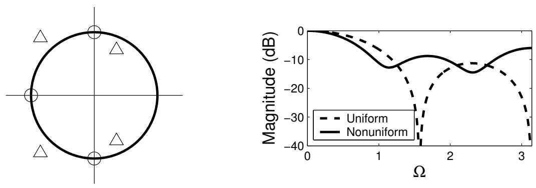

Fig. 6. Zero locations for a 4-element uniformly weighted array with uniform spacing (O) and optimal nonuniform spacing (△). The characteristic DTFTs are given in the graph.

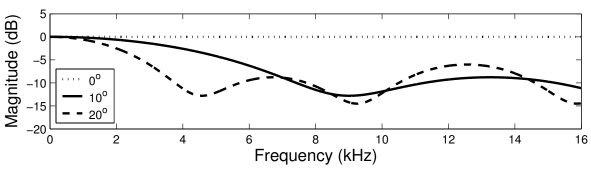

Fig. 7. Frequency response (at various angles) of a 4-element array with optimized nonuniform spacing.

be noted here that the optimal spacing to reduce off-broadside nulls is always asymmetric (for full-band optimization). Any symmetric equal-valued sequence will have at least one zero on the unit circle and thus will exhibit a null in the array response (except in limited angle and frequency ranges to which the null does not get mapped).

2.3. Arbitrary delays

Considering the exponent in Eq. (3), we see that the position of the elements leads to an angle-dependent time delay $\frac{x_n}{c}\sin\theta$ between the respective plane waves in the array sum. The delays T_n, on the other hand, are independent of angle. Here, we consider using such *delay dispersion* to optimize the response of a uniform array:

$$A(\omega,\theta) \;=\; \sum_{n=0}^{N-1} e^{-j\omega\left(\frac{nd}{c}\sin\theta - T_n\right)} \tag{5}$$

where T_n is a constrained multiple of the sampling period of the processor. As in the spacing optimization, we can construct nested loops to cover all delay configurations consistent with the length of the processor delay line. In the inner loop, however, we cannot characterize the array response with a single DTFT since T_n is not an affine function of n; since the delays can be arbitrary, the element index cannot be factored out to yield an Ω mapping, and there is thus not a single characteristic function on which an optimization can be carried out. Instead, the full array response over the target angle and frequency range must be evaluated in order to find the minimum magnitude for a given delay configuration; this is substantially more intensive than the single DTFT computation in the spacing search. Maximizing the minimum magnitude response over all realizable sets of delays gives the optimal configuration as in the spacing search.

The achievable improvement of a 4-element array for a small range of delays is illustrated in Figure 8. Note that the incorporation of the delays results in a reduction of the broadside response at high frequencies. This can be compensated for as in the frequency-invariant beamformers of Section 1.2; in some cases, partial compensation is a good design choice to prevent excessive high-frequency boosting at off-broadside angles.

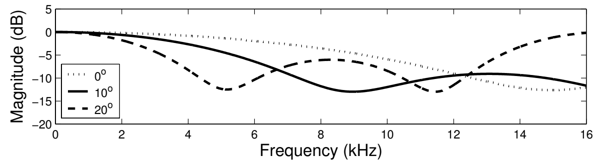

Fig. 8. Frequency response (at various angles) of a 4-element uniformly weighted array with $4cm$ uniform spacing and optimized sample delays $\{1, 0, \text{-}1, 1\}$ for a sampling rate of $48kHz$.

3. CONCLUSIONS AND FUTURE WORK

We have presented three effective methods for designing broadband linear arrays that address performance and cost constraints: allpass weighting, nonuniform spacing, and delay dispersion. We have considered *ad hoc* designs and exhaustive searches for min-max optimization, leveraging the characterization of the far-field array response by the DTFT where possible. Indeed, searches for optimal allpass weights or nonuniform spacings are similar to the search for low-autocorrelation sequences for digital communication and other applications. Such searches are also carried out via exhaustive analysis, although we anticipate that advances in efficient optimization (such as genetic algorithms) may prove applicable. Further work may address such optimization issues for array design. More importantly, joint optimization of the design parameters in the general array processor is of interest, both for single-band designs and for multi-band crossover-filtered structures. This poses an intense exhaustive search even for small arrays, so simplifications to the optimization are a key concern.

4. REFERENCES

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