PROPORTIONATE FREQUENCY DOMAIN ADAPTIVE ALGORITHMS FOR BLIND CHANNEL IDENTIFICATION

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ABSTRACT

We present fast-converging adaptive blind channel identification algorithms for acoustic room impulse responses. These new algorithms exploit the fast-convergence of the improved proportionate normalized least-mean-square (IPNLMS) algorithm and address the problem of delay inherent in frequency domain algorithms by employing the multi-delay filter (MDF) structure. Simulation results for both speech and white Gaussian noise show that the proposed algorithms outperform current frequency domain blind channel estimation algorithms.

1. INTRODUCTION

Blind channel identification (BCI) for single-input multipleoutput (SIMO) systems is an important technique with extensive applications in signal processing and communications. The identified channel can be utilized, after inversion, to remove the degradation introduced by the propagating channel. Techniques for BCI based upon second order statistics [1] [2] and higher order statistics [3] have been studied. Multichannel identification techniques are increasingly popular. The normalized multichannel frequency domain LMS (NM-CFLMS) algorithm [4] has been shown to be effective in identifying room impulse responses which are of particular interest in acoustic dereverberation. However, NMCFLMS lacks robustness to additive noise and can suffer misconvergence even in moderate noise conditions. This has been studied in [5].

We propose fast-converging BCI algorithms which exploit the quasi-sparse nature of acoustic impulse responses (AIRs). In addition the proposed algorithms reduce the delay inherent in frequency domain BCI approaches such as NMCFLMS [4]. Since acoustic impulse responses have leading zeros due to bulk delay and the late reflections in AIRs have very little energy, AIRs can be considered as quasi-sparse. This motivates us to employ proportionate adaptive algorithms such that the convergence can be accelerated. The proposed improved proportionate NMCFLMS (IPNMCFLMS) algorithm exploits the fast convergence due to proportionality control of the IPNLMS algorithm [6] while the proposed multichannel multi-delay filter (MCMDF) algorithm benefits from the frequent update of filter coefficients and the reduction in delay due to the MDF structure [7]. The proposed improved proportionate MCMDF (IPMCMDF) algorithm inherits fastconvergence and reduction in delay from IPNMCFLMS and MCMDF hence achieving better performance over previous methods.

2. REVIEW OF THE NMCFLMS ALGORITHM



Fig. 1. Relationship between input and output in a SIMO model.

With reference to Fig. 1 and defining s(n) and $\mathbf{b}_i(n)$ as the source signal and background noise respectively, the *i*th channel output signal $\mathbf{x}_i(n)$ is given by

$$\mathbf{x}_i(n) = \mathbf{H}_i(n)\mathbf{s}(n) + \mathbf{b}_i(n), \ i = 1, 2, \dots, M,$$
 (1)

where M is the number of channels while

$$\mathbf{x}_{i}(n) = [x_{i}(n) \ x_{i}(n-1) \ \dots \ x_{i}(n-L+1)]^{T},$$
(2)

$$\mathbf{h}_{i}(n) = [h_{i,0}(n) \ h_{i,1}(n) \ \dots \ h_{i,L-1}(n)]^{T},$$
(3)

$$\mathbf{H}_{i}(n) = \begin{vmatrix} n_{i,0}(n) & \cdots & n_{i,L-1}(n) & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & h_{i}(n) & \dots & h_{i}(n) \end{vmatrix}$$
(4)

$$\begin{bmatrix} 0 & \cdots & h_{i,0}(n) & \cdots & h_{i,L-1}(n) \end{bmatrix}_{L \times (2L-1)}$$

$$\mathbf{s}(n) - [\mathbf{s}(n) \ \mathbf{s}(n-1) & \mathbf{s}(n-2L+2)]^T$$
(5)

$$\mathbf{b}_{i}(n) = [b_{i}(n), b_{i}(n-1), \dots, b_{i}(n-L+1)]^{T},$$
(6)

such that L is the length of the longest impulse response and the superscript T denotes vector transposition. We assume

that the additive noise on M channels is uncorrelated, i.e., $E\{b_i(n)b_j(n)\} = 0$ for $i \neq j$, $E\{b_i(n)b_i(n - n')\} = 0$ for $n \neq n'$, and $E\{b_i(n)s(n)\} = 0$ where $E\{\cdot\}$ is defined as the expectation operator. For channel identifiability [8], we also assume that (i) the channel transfer function $H_i(z)$ does not contain any common zeros, and (ii) the autocorrelation matrix of the source signal, $\mathbf{R}_{ss} = E\{\mathbf{s}(n)\mathbf{s}^T(n)\}$, is full rank.

A blind multichannel system can be identified, in the absence of noise, using the cross-relationship between the i^{th} and j^{th} channel outputs given, for $i \neq j$, by [4]

$$\mathbf{x}_i^T(n)\mathbf{h}_j(n) = \mathbf{x}_j^T(n)\mathbf{h}_i(n), \quad i, j = 1, \dots, M.$$
(7)

An *a priori* error exists if noise is present, or the channels are estimated with error, given, for $i \neq j$, by

$$e_{ij}(n) = \mathbf{x}_i^T(n)\mathbf{\hat{h}}_j(n-1) - \mathbf{x}_j^T(n)\mathbf{\hat{h}}_i(n-1), \ i, j = 1, \dots, M,$$
(8)

where $\hat{\mathbf{h}}_i(n)$ is the estimated i^{th} channel impulse response. Using (8), BCI algorithms such as NMCFLMS are derived by minimizing the cost function $J(n) = \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} e_{ij}^2(n)$ with respect to the estimated impulse response $\hat{\mathbf{h}}_i(n)$ for $i = 1, \dots, M$. The NMCFLMS [4] algorithm is a fast-converging and efficient algorithm for multichannel frequency-domain BCI and is given, for each m^{th} frame, by:

$$\mathbf{e}_{ij}(m) = [e_{ij}(mL-L) \dots e_{ij}(mL+L-1)]^T, \qquad (9)$$

$$\underline{\epsilon}_{ij}^{01}(m) = \mathbf{F}_{2L} \mathbf{W}_{2L \times 2L}^{01} \mathbf{e}_{ij}(m)$$

$$= \mathcal{W}_{2L \to 0L}^{01} \times$$

$$[\mathcal{D}_{x_i}(m)\mathcal{W}_{2L\times L}^{10}\hat{\mathbf{h}}_j(m) - \mathcal{D}_{x_j}(m)\mathcal{W}_{2L\times L}^{10}\hat{\mathbf{h}}_i(m)], (10)$$

$$\mathcal{P}_{i}(m) = \lambda \mathcal{P}_{i}(m-1) + (1-\lambda) \sum_{j=1, j \neq i}^{m} \mathcal{D}_{x_{j}}^{*}(m) \mathcal{D}_{x_{j}}(m), \quad (11)$$

$$\hat{\underline{\mathbf{h}}}_{i}^{10}(m) = \hat{\underline{\mathbf{h}}}_{i}^{10}(m-1) - \rho [\mathcal{P}_{i}(m) + \delta \mathbf{I}_{2L \times 2L}]^{-1} \times \sum_{j=1}^{M} \mathcal{D}_{x_{j}}^{*}(m) \underline{\epsilon}_{ji}^{01}(m), \quad i = 1, \dots, M,$$
(12)

where * denotes complex conjugate, ρ is the step-size, λ is the forgetting factor and δ is the regularization constant. Defining $\mathbf{I}_{L\times L}$, $\mathbf{0}_{L\times L}$, and \mathbf{F}_L as the identity, null and Fourier matrices of dimension $L \times L$ respectively, $\mathbf{W}_{2L\times L}^{10} = [\mathbf{I}_{L\times L} \ \mathbf{0}_{L\times L}]^T$, $\mathbf{W}_{2L\times 2L}^{01} = \begin{bmatrix} \mathbf{0}_{L\times L} & \mathbf{0}_{L\times L} \\ \mathbf{0}_{L\times L} & \mathbf{I}_{L\times L} \end{bmatrix}$, $\mathbf{W}_{2L\times L}^{10} = \mathbf{F}_{2L}\mathbf{W}_{2L\times L}^{10}\mathbf{F}_L^{-1}$, $\mathbf{W}_{2L\times 2L}^{01} = \mathbf{F}_{2L}\mathbf{W}_{2L\times 2L}^{01}\mathbf{F}_{2L}^{-1}$, $\hat{\mathbf{h}}_i(m) = \mathbf{F}_L\hat{\mathbf{h}}_i(m)$, $\hat{\mathbf{h}}_i^{10}(m) = \mathbf{F}_{2L}\begin{bmatrix} \hat{\mathbf{h}}_i(m) \\ \mathbf{0}_{L\times 1} \end{bmatrix}$ and $\mathcal{D}_{x_j}(m) = \text{diag}\{\mathbf{F}_{2L}[x_j(mL-L) \ \ldots \ x_j(mL+L-1)]^T\}$. In [5], the direct path component of the estimated impulse response is constrained in order to improve NMCFLMS's robustness to noise and, in a similar manner, we employ this constraint in our proposed algorithms.

3. NEW ALGORITHMS FOR BCI

3.1. IPNMCFLMS ALGORITHM

We now formulate an improved proportionate NMCFLMS (IPNMCFLMS) algorithm for BCI. The IPNLMS algo-

rithm [6] achieves fast convergence by updating each filter coefficient with individual step-sizes proportional to the estimated impulse response. It incorporates the proportionality into NLMS using a controlling factor α . A diagonal step-size control matrix $\mathbf{Q}_i(m)$, for $i = 1, \ldots, M$, is given by

$$\mathbf{Q}_{i}(m) = \operatorname{diag}\{q_{i,0}(m) \ q_{i,1}(m) \dots q_{i,L-1}(m)\}, \ (13)$$

$$q_{i,l}(m) = \frac{1-\alpha}{2L} + (1+\alpha)\frac{|\hat{h}_{i,l}(m)|}{2\|\hat{\mathbf{h}}_{i}(m)\|_{1} + \varrho}, \quad (14)$$

where $\|.\|_1$ is the l_1 -norm operator for $l = 0, 1, \ldots, L-1$ and ϱ is a constant. Following the approach in [9] we update the filter coefficients of NMCFLMS in the time domain by first defining the matrix

$$\widetilde{\mathbf{G}}_{L\times 2L}^{10} = \mathbf{W}_{L\times 2L}^{10} \mathbf{F}_{2L}^{-1},\tag{15}$$

where $\mathbf{W}_{L\times 2L}^{10} = [\mathbf{I}_{L\times L} \ \mathbf{0}_{L\times L}]$. The IPNMCFLMS update equation for BCI can be expressed as

$$\hat{\mathbf{h}}_{i}(m) = \hat{\mathbf{h}}_{i}(m-1) - \rho_{\mathrm{I}} L \mathbf{Q}_{i}(m) \widetilde{\mathbf{G}}_{L\times 2L}^{10} \times [\mathcal{P}_{i}(m) + \delta_{\mathrm{IP}} \mathbf{I}_{2L\times 2L}]^{-1} \sum_{j=1}^{M} \mathcal{D}_{x_{j}}^{*}(m) \underline{\epsilon}_{ji}^{01}(m), (16)$$

where $\rho_{\rm I}$ is the step-size and $\delta_{\rm IP}$ is the regularization constant.

3.2. MCMDF ALGORITHM

We next devise the multichannel multi-delay filtering (MCMDF) algorithm for BCI. The MDF structure [7] reduces the problem of delay in frequency domain algorithm implementation by partitioning the adaptive filter of length L into K blocks such that L = KN where N is the block length. Let m be the frame index and we define input matrix $\mathbf{X}_i(m)$ and a priori error $\mathbf{e}_{ij}(m)$ for $i \neq j$:

$$\mathbf{X}_{i}(m) = [\mathbf{x}_{i}(mN) \dots \mathbf{x}_{i}(mN+N-1)], i = 1, 2, \dots, M,$$

$$\mathbf{e}_{ij}(m) = \mathbf{X}_{i}(m)\hat{\mathbf{h}}_{j}(m-1) - \mathbf{X}_{j}(m)\hat{\mathbf{h}}_{i}(m-1)$$

$$= [e_{ij}(mN) \dots e_{ij}(mN+N-1)]^{T}.$$
(17)

Defining k as the block index, the diagonal data matrix $\mathbf{D}_{x_i}(m)$ for channel i is

$$\mathbf{D}_{x_i}(m) = \operatorname{diag}\{\mathbf{F}_{2N}\mathcal{X}_i(m)\},\\ \mathcal{X}_i(m) = [x_i(\tau+N) \ x_i(\tau-N+1) \dots x_i(\tau+N-1)]^T, (18)$$

where $\tau = mN - kN$. Note that the first element of the diagonal of $\mathbf{D}_{x_i}(m)$ is arbitrary, but it is normally equal to the first sample of the previous block k - 1 [10]. We now define the frequency domain quantities: $\mathbf{\hat{h}}_{i,k}(m) = \mathbf{F}_{2N} \begin{bmatrix} \mathbf{\hat{h}}_{i,k}(m) \\ \mathbf{0}_{N\times 1} \end{bmatrix}$, $\mathbf{\underline{e}}_{ij}(m) = \mathbf{F}_{2N} \begin{bmatrix} \mathbf{0}_{N\times 1} \\ \mathbf{e}_{ij}(m) \end{bmatrix}$, $\mathbf{W}_{2N\times 2N}^{01} = \begin{bmatrix} \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} & \mathbf{I}_{N\times N} \end{bmatrix}$, $\mathbf{W}_{2N\times 2N}^{10} = \begin{bmatrix} \mathbf{1}_{N\times N} & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} & \mathbf{0}_{N\times N} \end{bmatrix}$, $\mathbf{W}_{2N\times 2N}^{10} = \mathbf{F}_{2N} \mathbf{W}_{2N\times 2N}^{10} \mathbf{F}_{2N}^{-1}$ and $\mathbf{G}_{2N\times 2N}^{01} = \mathbf{F}_{2N} \mathbf{W}_{2N\times 2N}^{01} \mathbf{F}_{2N}^{-1}$, where $\mathbf{\hat{h}}_{i,k}(m)$ is the k^{th} subfilter of the i^{th} channel, for

 $k = 0, \dots, K - 1$ and $i = 1, \dots, M$. The MCMDF adaptive algorithm for BCI is then given by:

$$\underline{\mathbf{e}}_{ij}(m) = \mathbf{G}_{2N \times 2N}^{01} \sum_{k=0}^{K-1} \mathbf{D}_{x_i}(m-k) \underline{\mathbf{\hat{h}}}_{j,k}(m-1) - \mathbf{G}_{2N \times 2N}^{01} \sum_{k=0}^{K-1} \mathbf{D}_{x_j}(m-k) \underline{\mathbf{\hat{h}}}_{i,k}(m-1), \quad (19)$$

$$\mathbf{S}_{i}(m) = \lambda \mathbf{S}_{i}(m-1) +$$

$$(1-\lambda) \sum_{j=1, j \neq i}^{M} \mathbf{D}_{x_{j}}^{*}(m) \mathbf{D}_{x_{j}}(m), \qquad (20)$$

$$\underline{\hat{\mathbf{h}}}_{i,k}(m) = \underline{\hat{\mathbf{h}}}_{i,k}(m-1) - \beta \ \mathbf{G}_{2N\times 2N}^{10} \times \left[\mathbf{S}_{i}(m) + \delta_{\mathrm{MDF}}\right]^{-1} \sum_{j=1}^{M} \mathbf{D}_{x_{j}}^{*}(m-k) \underline{\mathbf{e}}_{ji}(m), \ (21)$$

where β is the step-size and $0 \ll \lambda < 1$ is the forgetting factor. Defining σ_x^2 as the input signal variance, $\mathbf{S}_i(0) = \sigma_{x_i}^2/100$ is the initialization [10] and δ_{MDF} is the regularization constant which is set to one fifth of the total power over all channels at the first frame [4].

3.3. IPMCMDF ALGORITHM

We additionally propose the improved proportionate multichannel MDF (IPMCMDF) algorithm for BCI which incorporates proportionate updates as in IPNMCFLMS whilst achieving reduced delay by exploiting the MCMDF approach. To incorporate proportionality, we update the MCMDF filter coefficients in the time-domain similar to Section 3.1. In this case, $\tilde{\mathbf{G}}_{N\times 2N}^{10}$ is computed for each block k and hence is of dimension $N \times 2N$. Moreover the partitioned control elements of block k in channel i are defined, for $k = 0, \ldots, K-1$, as

$$\mathbf{q}_{i,k}(m) = [q_{i,kN}(m) \dots q_{i,kN+N-1}(m)],$$
 (22)

where each element in this block is determined by

$$q_{i,kN+r}(m) = \frac{1-\alpha}{2L} + (1+\alpha)\frac{|\hat{h}_{i,kN+r}(m)|}{2\|\hat{\mathbf{h}}_{i}(m)\|_{1} + \varrho},$$
(23)

while r = 0, 1, ..., N - 1 is the tap-index for each of the K blocks. The IPMCMDF update equation for the k^{th} subfilter of channel *i* is then given by

$$\hat{\mathbf{h}}_{i,k}(m) = \hat{\mathbf{h}}_{i,k}(m-1) - \beta_{\mathrm{I}} L \mathbf{Q}_{i,k}(m) \mathbf{G}_{N \times 2N}^{10} \times [\mathbf{S}_{i}(m) + \delta_{\mathrm{IPMDF}}]^{-1} \sum_{j=1}^{M} \mathbf{D}_{x_{j}}^{*}(m-k) \underline{\mathbf{e}}_{ji}(m), (24)$$

where β_{I} is the step-size for IPMCMDF, δ_{IPMDF} is the regularization constant and $\mathbf{Q}_{i,k}(m) = \text{diag}\{\mathbf{q}_{i,k}(m)\}$, while $\underline{\mathbf{e}}_{ji}(m)$ and $\mathbf{S}_{i}(m)$ are computed using (19) and (20) respectively.

4. SIMULATIONS AND RESULTS

We now present simulation results to compare the performance of IPNMCFLMS, MCMDF and IPMCMDF algorithms for BCI against the NMCFLMS algorithm [4] in the context of acoustic room impulse response identification. The direct path constraint of [5] has been employed throughout to achieve noise robustness. The dimensions of the room are $(5 \times 4 \times 3)$ m and impulse responses are generated using the method of images [11] with reverberation time $T_{60} = 0.1$ s which are then truncated to length L = 128. A linear microphone array containing M = 5 microphones with uniform separation d = 0.2 m is used. The source and the first microphone are placed at (1.0, 1.5, 1.6) m and (2.0, 1.2, 1.6) m, respectively. Input signal is either white Gaussian noise (WGN) or a male speech signal while an uncorrelated zero-mean additive WGN is added to achieve the SNR specified for each experiment. The sampling frequency is 8 kHz and the SNR is 20 dB unless otherwise specified. Defining $\mathbf{h} = [\mathbf{h}_1^T \ \mathbf{h}_2^T \ \dots \ \mathbf{h}_M^T]^T$, the SNR for this BCI application is given [4] as SNR $\triangleq 10 \log_{10}[\sigma_s^2 \|\mathbf{h}\|^2 / (M \sigma_b^2)]$ where σ_s^2 and σ_b^2 are the signal and noise powers, respectively, while the following parameters are chosen for all simulations: $\alpha = -0.75$, $\lambda = [1 - 1/(3L)]^N$, $\mathbf{S}_i(0) = \sigma_{x_i}^2/100$ for MCMDF, $\mathbf{S}_i(0) = (1-\alpha)\sigma_{x_i}^2/200$ for IPMCMDF, $\delta_{\text{IP}} = \frac{(1-\alpha)\delta}{2L}$, $\delta_{\text{IPMDF}} = \frac{(1-\alpha)\delta_{\text{MDF}}}{2L}, \ \hat{\mathbf{h}}_i(0) = [1 \quad 0 \dots 0]^T / \sqrt{M} \text{ for } i = 1, \dots, M.$ The normalized projection misalignment (NPM) [4] is used as performance measure and is given, for frame m, by

NPM(m)=20 log₁₀
$$\left(\left\| \mathbf{h} - \frac{\mathbf{h}^T \hat{\mathbf{h}}(m)}{\hat{\mathbf{h}}^T(m) \hat{\mathbf{h}}(m)} \hat{\mathbf{h}}(m) \right\| / \|\mathbf{h}\| \right) dB$$
, (25)

where $\|.\|$ is the l_2 norm and $\hat{\mathbf{h}}(m) = [\hat{\mathbf{h}}_1^T(m) \ \hat{\mathbf{h}}_2^T(m) \dots \hat{\mathbf{h}}_M^T(m)]^T$. Figure 2 shows the variation in convergence with block-size N for MCMDF using WGN input sequence. It can be seen that the convergence rate increases for smaller N since the adaptive filter coefficients are updated more frequently.

Figure 3 shows a comparison of convergence between NMCFLMS, IPNMCFLMS, MCMDF and IPMCMDF using a WGN input sequence. The block-size for MCMDF and IPMCMDF is N = 32 while the step-sizes for all algorithms are adjusted such that they reach same asymptotic NPM. This corresponds to $\rho = 0.5$, $\rho_I = 0.4$, $\beta = 0.8$ and $\beta_I = 0.6$. The MCMDF algorithm converges faster than NM-CFLMS because the filter coefficients are being updated more frequently due to the MDF structure. The IPNMCFLMS and IPMCMDF algorithms exhibit even higher rate of convergence compared to NMCFLMS and MCMDF due to the exploitation of the quasi-sparse nature of AIRs. During convergence, IPMCMDF achieves approximately 1 dB improvement in NPM over IPNMCFLMS and approximately 5 dB improvement over NMCFLMS.

Figure 4 shows an additional result using a male speech input sequence with an SNR = 40 dB where step-sizes for all algorithms are adjusted to achieve same asymptotic NPM which correspond to $\rho = 0.02$, $\rho_I = 0.12$, $\beta = 0.02$ and $\beta_I = 0.12$. It can be seen that the relative performance of all algorithms is similar to that obtained using WGN input with IPMCMDF achieving the highest rate of convergence.



Fig. 2. Variation of convergence with N for MCMDF.



Fig. 3. Variation of NPM with WGN sequence input at SNR=20 dB.

5. CONCLUSION

We have developed IPNMCFLMS, MCMDF and IPMCMDF algorithms for BCI. The IPNMCFLMS algorithm offers fast convergence by exploiting proportionality control of IPNLMS while the MCMDF algorithm benefits from frequent update of filter coefficients due to its MDF structure. The IPMCMDF algorithm has beneficial properties of both IP-NMCFLMS and MCMDF. In addition, MCMDF and IPM-CMDF offer a reduction in the delay inherent in frequency domain BCI algorithms due to the MDF structure. Simulation results show for both WGN and speech signal inputs, the proposed algorithms offer improvement in NPM by approximately 2 to 5 dB over the NMCFLMS algorithm.

6. REFERENCES

G. Xu, H. Liu, L. Tong, and T. Kailath, "A least-squares approach to blind channel identification," *IEEE Trans. Signal*



Fig. 4. Variation of NPM with a male speech signal input.

Processing, vol. 43, no. 12, pp. 2982-2993, Dec. 1995.

- [2] E. Moulines, P. Duhamel, J.-F. Cardoso, and S. Mayrargue, "Subspace methods for blind identification of multichannel FIR filters," *IEEE Trans. Signal Processing*, vol. 43, no. 2, pp. 516–525, Feb. 1995.
- [3] G. Giannakis and J. Mendel, "Identification of nonminimum phase systems using higher order statistics," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, pp. 7360–7377, Mar. 1989.
- [4] Y. Huang and J. Benesty, "A class of frequency-domain adaptive approaches to blind multichannel identification," *IEEE Trans. Speech Audio Processing*, vol. 51, no. 1, pp. 11–24, Jan. 2003.
- [5] M. K. Hasan, J. Benesty, P. A. Naylor, and D. B. Ward, "Improving robustness of blind adaptive multichannel identification algorithms using constraints," in *Proc. 13th European Signal Processing Conf.*, 2005.
- [6] J. Benesty and S. L. Gay, "An improved PNLMS algorithm," in Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing, vol. 2, 2002, pp. 1881–1884.
- [7] J. S. Soo and K. K. Pang, "Multidelay block frequency domain adaptive filter," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 38, no. 2, pp. 373–376, 1990.
- [8] L. Tong, G. Xu, and T. Kailath, "A new approach to blind identification and equalization of multipath channels," in *Proc. 25th Asilomar Conf. on Signals, Systems, and Computers*, vol. 2, 1991, pp. 856–860.
- [9] A. W. H. Khong, J. Benesty, and P. A. Naylor, "An improved proportionate multi-delay block adaptive filter for packetswitched network echo cancellation," in *Proc. 13th European Signal Processing Conf.*, 2005.
- [10] J. Benesty, T. Gansler, D. R. Morgan, M. M. Sondhi, and S. L. Gay, Advances in Network and Acoustic Echo Cancellation. Springer, 2001.
- [11] J. B. Allen and D. A. Berkley, "Image method for efficiently simulating small-room acoustics," *J. Acoust. Soc. Amer.*, vol. 65, no. 4, pp. 943–950, Apr. 1979.