A QUARTIC ALGORITHM FOR SQUINT SAR IMAGING

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ABSTRACT

In this paper some improvements of Non-linear Chirp Scaling (NCS) approach are proposed and a modified algorithm based on the NCS is described. The improvements include three main aspects. A quartic polynomial model is adopted to represent SAR signal in Range-Doppler domain and 2-D frequency domain to improve precision. A constant scaling factor is employed to replace the reference frequency in the NCS, which makes NCS more flexible. A second chirp scaling operation is added to the end of the NCS so as to remove scaling effect on range direction of SAR image. Those improvements make NCS applicable to process highly squinted SAR data from a large range swath at a fine resolution.

1. INTRODUCTION

In conventional Synthesis Aperture Radar (SAR) system, the antenna is pointed at broadside, i.e. pointing direction of antenna is nearly perpendicular to flight path [1]. When there is an offset angle between the pointing direction of antenna and broadside, the SAR system is working under the squint mode and the offset angle is the so called squint angle. Squint mode working could increase the flexibility of SAR system and may gather more information about scene when the backscatter of which is squint angle dependent. The squint mode SAR can also be applied in constellation SAR system, which is composed by several independent SAR sensors distributed in the space and most of them work under squint mode.

There are a number of algorithms developed for strip SAR imaging, such as Range-Doppler (R-D) algorithm [2][3], Chirp-Scaling (CS) algorithm [4], Extended Chirp-Scaling (ECS) algorithm [5][6], Nonlinear Chirp-Scaling (NCS) algorithm [7][8] and Extended Exact Transfer Function (EETF) algorithm [9][10], etc. In those algorithms, a focusing filter is constructed at a reference range and only the SAR data at the range can be focused perfectly. To yield a large depth of focus, a range variant processing, which is designed based on some approximation of range-azimuth coupling term, is needed to compensate the difference between the signal at reference range and other range cells. The algorithms above make efforts to improve the precise of approximation to achieve a good performance.

NCS is an effective algorithm to focus squinted SAR data. In NCS [7], a cubic-order polynomial of f_r, the range frequency, is adopted to model the coupling term in 2-D frequency domain and the range dependent chirp rate is represented by a linear function of range. The cubic term of f_r is removed at reference range in 2-D frequency domain. The range variant property of the migration curve and the chirp rate is removed by a non-linear chirp scaling processing in R-D domain. Due to the elaborate structure, NCS has ability to process highly squinted SAR data [7][8]. But there are still some shortcomings. First, a reference frequency of azimuth should be chosen and it varies with the squint angle and the system parameters. Secondly, the output image is scaled in range direction and the quantity of scaling relates with the choice of the reference frequency. Besides, the cubic model is not precise enough when there is an ultra large squint angle

In the algorithm of EETF in [9], the transfer function of the SAR system is computed at a reference range under some assumptions. The focusing of the SAR data is done by anti-filtering. EETF also consider the difference of azimuth signal at various range cells, and a range-variant filter is employed to compensate the difference in R-D domain. Because the range-variant property of the coupling term and the range-migration is not taken into account when constructing the transfer function, EETF can achieve the best performance only at the reference range, but the depth of focus is too limited to make the algorithms applicable to process the SAR data from a large range swath at one time. EETF divides SAR data into several blocks along range direction and the processing parameters are updated every block.

With reference to NCS and EETF, a quartic Non-linear Chirp Scaling algorithm (NCS4) is proposed and some improvements are made to enhance performance of the new algorithm. In NCS4, a quartic model is adopted to approximate phase term of SAR data in R-D domain. A constant factor β is introduced to replace the reference frequency defined in NCS. β is invariant with the squint angle. Due to the quartic model of phase term, β can be chosen much close to 1 and there will be little scaling effect along range direction. Anther chirp-scaling can be employed to remove the scaling effect completely [5].

NCS4 can process the SAR data with an ultra-large squint angle from wide range swath at one time.

The geometric relationship and data of squinted SAR are analyzed in section 2. The algorithm of NCS4 is described in section 3. The choice of β, the constant factor, is discussed in Section 4. Some simulation results are given in Section 5 and conclusion in Section 6.

2. GEOMOTRY AND SIGNAL MODEL

Figure. 1 shows geometric relationship of a squinted SAR.

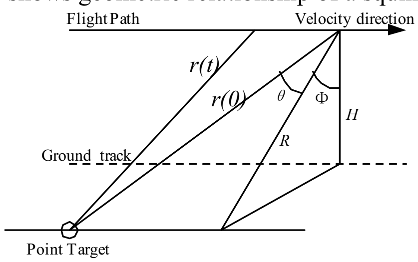


Figure 1. The geometric relationship of the squint mode SAR

The spacecraft moves along the X direction at an altitude of H with a constant velocity v, which is a relative velocity between the earth and the spacecraft. The radar antenna transmits and receives pulses in a squinted direction defined by the squint angle θ and the look angle Φ. Assuming that the closest slant distance from a point target to the SAR platform at zero squint angle is R=H/cosΦ, the instantaneous distance at slow time (azimuth time) t is:

$$r(t) = \sqrt{R^2 + (t+t_s)^2 v^2}, t_s = R\tan\theta / v \cdot \qquad (1)$$

The range of t is from $-T/2$ to $T/2$, where T is aperture time. In Figure 1, $r(0)$ is the instantaneous slant range at t=0, i.e. the center of aperture. The SAR signal from a point target can be expressed as:

$$s(t,\tau,R) = a(t,\tau,R)\exp[-j\pi k(\tau - \frac{2r(t)}{c})^2 - j\frac{4\pi r(t)}{\lambda}], \qquad (2)$$

where $a(t, \tau, r)$ is the function responsible for all variation in amplitude, k is chirp rate of transmitted signal, τ is fast time (range time), λ is wave length and c is velocity of light [4]. With reference of [11] and omitting the amplitude function in (2), SAR signal in 2-D frequency domain can be expressed as:

$$S(f_a, f_r, R) = \exp[-j\frac{4\pi R}{\lambda}\sqrt{(1+\frac{f_r\lambda}{c})^2 - (\frac{f_a\lambda}{2v})^2}] \times \exp(j\frac{\pi f_r^2}{k} + j2\pi f_a t_s), \qquad (3)$$

where f_r and f_a are the frequency of range and azimuth, respectively. Phase term of (3) can be expended into a cubic polynomial of f_r as:

$$(2\pi f_a t_s - \frac{4\pi R\sqrt{A}}{\lambda}) - 2\pi f_r \tau_R + \frac{\pi f_r^2}{Km_R} + \pi Kc_R f_r^3 \qquad (4)$$

where

$$\begin{aligned} &A = 1 - (\frac{f_a\lambda}{2v})^2, \ \tau_R = \frac{2R}{c\sqrt{A}} \\ &Km_R = \frac{k}{1+Rk\gamma_1}, \ \gamma_1 = \frac{2\lambda(1-A)}{c^2(\sqrt{A})^3} \\ &Kc_R = R\gamma_2, \ \gamma_2 = \frac{2\lambda^2(A-1)}{c^3(\sqrt{A})^5} \end{aligned} . \qquad (5)$$

The subscript 'R' of Km_R, Kc_R and τ_R denotes range R. From (4), the SAR signal can be divided into two parts: the coupling of azimuth and range,

$$\exp[j(-2\pi f_r \tau_R + \frac{\pi f_r^2}{Km_R} + \pi Kc_R f_r^3)], \qquad (6)$$

and residual part,

$$\exp(j2\pi f_a t_s - j\frac{4\pi R\sqrt{A}}{\lambda}) \cdot \qquad (7)$$

With the similar method described in [7], (6) can be transformed into R-D domain as:

$$\exp\{-j\pi[Km_R(\tau-\tau_R)^2 + Kc_R(\tau-\tau_R)^3]\} \cdot \qquad (8)$$

To express the variance of Km and Kc with range R, they can be modeled by:

$$\begin{aligned} &Km_R = Km_{ref} + ks_1\Delta\tau + ks_2\Delta\tau^2 \\ &Kc_R = Kc_{ref} + ks_3\Delta\tau \end{aligned}, \qquad (9)$$

where $\Delta\tau = \frac{2(R-R_{ref})}{c\sqrt{A}}$, R_{ref} is the reference range, Km_{ref} and Kc_{ref} are the values of Km and Kc at R_{ref} and ks_1, ks_2 and ks_3 are expressed as follows:

$$\begin{aligned} &ks_1 = -\gamma_1 Km_{ref}^2 c\sqrt{A}/2 \\ &ks_2 = (\gamma_1 c\sqrt{A})^2 Km_{ref}^3/4 \\ &ks_3 = \frac{1}{2}c\sqrt{A}\gamma_2 \end{aligned} . \qquad (10)$$

3. ALGORITHM DESCRIPTION

The delay curve at an arbitrary range should be scaled to same shape as the curve at reference range. With the relationship shown in Figure.2, the scaling factor can be expressed as:

$$\alpha(f_a) = \sqrt{1-(\frac{\lambda f_a}{2v})^2} / \sqrt{1-(\frac{\lambda f_{dc}}{2v})^2} \cdot \qquad (11)$$

The relationship between τ_{ref}, τ_R, τ_s and $\Delta\tau$ is

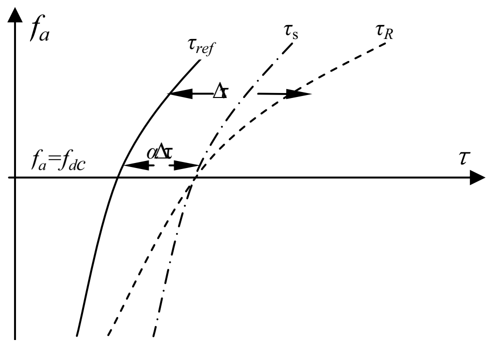
$$\begin{aligned} &\tau_{ref} = \tau_s - \alpha\Delta\tau \\ &\tau_R = \tau_s - (\alpha-1)\Delta\tau \end{aligned} . \qquad (12)$$

The chirp scaling function will take the form of:

$$CS_1 = \exp[-j\pi p_1(\tau-\tau_{ref})^2 - j\pi p_2(\tau-\tau_{ref})^3] \times \exp[-j\pi p_3(\tau-\tau_{ref})^4] . \qquad (13)$$

To yield the expressions of p_1, p_2 and p_3, two parameters, referred as Y_1 and Y_2, will be introduced by filtering the signal with respect to f_r in 2-D frequency domain with

$$M_{pre} = \exp[j\pi(Y_1 f_r^3 + Y_2 f_r^4)] \qquad (14)$$



τ_{ref} is the delay curve of the reference range, τ_R is the delay curve of an arbitrary range. τ_s is the desired curve by scaling. f_a is the azimuth frequency and f_{dc} is the center frequency.

Figure 2. The illustration of scaling on the range delay curve

The chirp scaling result can be yielded by multiplying (8) with (13). To eliminate Δτ, the coefficients of the terms, which contain $\Delta\tau^n(\tau-\tau_s)^m$, should be set to zero. A fact should be noticed that Y_1 and Y_2 are always multiplied by (α-1). To avoid the solved Y_1 and Y_2 being infinitive when α=1, the scaling factor $\alpha(f_a)$ is multiplied with a constant factor β to make it far away from 1 in the azimuth band width. The solved parameters are shown as:

$$\begin{cases} p_1 = Km_{ref}\dfrac{(1-\beta\alpha)}{\beta\alpha} \\ p_2 = \dfrac{1-\beta\alpha}{3\beta\alpha}ks_1 \\ p_3 = -\dfrac{2ks_2 + 9Y_1Km_{ref}^2 ks_1(\beta\alpha-1) + 3ks_3Km_{ref}^3(\beta\alpha-1)}{12\beta\alpha} \\ Y_1 = \dfrac{\beta\alpha-2}{3(\beta\alpha-1)Km_{ref}^3}ks_1 \\ Y_2 = \dfrac{4p_3\beta\alpha + 3Y_1Km_{ref}^2 ks_1 + ks_3Km_{ref}^3}{4Km_{ref}^4(\beta\alpha-1)} \end{cases} . \quad (15)$$

The result of chirp scaling can be divided into two parts: the terms about (τ-τ_s) for imaging, i.e.

$$\exp\{-j\pi[(Km+p_1)(\tau-\tau_s)^2 + (Y_1Km^3+p_2)(\tau-\tau_s)^3 + (p_3 - Y_2Km^4)(\tau-\tau_s)^4]\} \quad (16)$$

and the remainder phase of chirp scaling. By removing the high-order (above 2) phase term of f_r in 2-D frequency domain, the data are ready to be focused in range by multiplying with

$$\exp\{-j\pi(Km+p_1)(B-\beta\alpha\Delta\tau)^2\}. \quad B = \tau - \tau_{ref} \quad (17)$$

To remove the constant factor β, another chirp scaling procedure is employed. The chirp scaling function is

$$CS_2 = \exp[-j\pi(Km_{ref}+p_1)(\beta-1)B^2]. \quad (18)$$

Result of the second chirp scaling algorithm is

$$\exp[j\pi\frac{\alpha f_r^2}{Km_{ref}} - j2\pi(\tau_{ref}+\alpha\Delta\tau)f_r]. \quad (19)$$

Multiplied with

$$M_R = \exp(-j\pi\frac{\alpha f_r^2}{Km_{ref}} + j2\pi(\tau_{ref} - \frac{2R_{ref}}{c\cos\theta}f_r) \quad (20)$$

in 2-D frequency domain, eq(19) is de-chirped, and the SAR data are focused in range. The range focused SAR data take the form of:

$$\exp[-j2\pi(\frac{2R\sqrt{A}}{\lambda} - f_a t_s + \frac{2R}{c\cos\theta}f_r)] \quad (21)$$

in 2-D frequency domain. Multiplied with

$$M_A = \exp[j\frac{4\pi R\sqrt{A}}{\lambda}] \quad (22)$$

in R-D domain, eq(21) will be focused in azimuth. In time domain, the focused point target is located at $2R/(c\cos\theta)$ in range and $R\tan\theta/v$ in azimuth. There is also residual phase term from the two chirp scaling procedures and the phase term can be expressed as:

$$\begin{aligned} ph_r &= \exp[j\pi(a_2\Delta\tau^2 + a_3\Delta\tau^3 + a_4\Delta\tau^4)]\times \\ &\quad \exp\{-j\pi(Km+p_1)\beta(\beta-1)[\frac{2(R-R_{ref})}{c\cos\theta}]^2\} \end{aligned} \quad (23)$$

$$\begin{aligned} a_2 &= Km(\beta\alpha-1)^2 + (\beta\alpha)^2 p_1 \\ a_3 &= ks_1(\beta\alpha-1)^2 + Y_1Km^3(\beta\alpha-1)^3 + p_2(\beta\alpha)^3 \\ a_4 &= ks_2(\beta\alpha-1)^2 + 3Y_1Km^2ks_1(\beta\alpha-1)^3 \\ &\quad -Y_2Km^4(\beta\alpha-1)^4 + Km^3ks_3(\beta\alpha-1)^3 + p_3(\beta\alpha)^4 \end{aligned}$$

A flow chart of NCS4 is show in Figure 3.

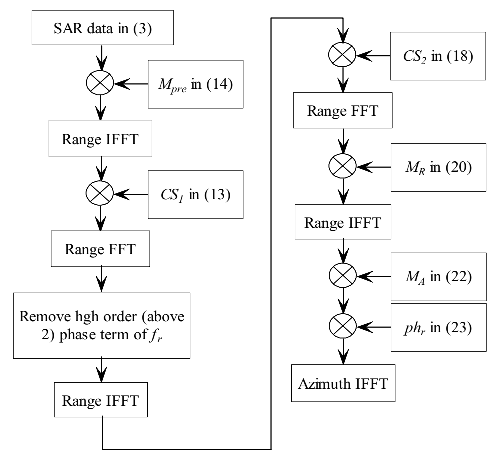


Figure 3. Flow chart of NCS4

4. CHOICE OF β

Being a ratio, the scaling factor α in (11) is nearly stable under various squint angles. Figure 4 shows a simulation result of α under various squint angles. The parameters used for simulation are listed in Table I.

The choice of β will affect the correction of Fourier transform on SAR data. For Fourier transform of polynomial phase signal, the second-order term is required to be the dominant part of the phase [7][11]. It can be noticed that the denominator of the second-order phase term is linear with β while the denominators of the third-order and the fourth-order are quadratic with β. It should also be noticed that too large a $|\beta|$ may make the band width of (18) out of the sampling frequency. Thus, the choice of β is to make its absolute value as large as possible while keeping the band width under the sampling frequency. In fact when $\beta\approx 1$, the secondary chirp scaling can be omitted. Specially, when β=-1, the image will not be scaled other than an inverse in the range direction.

Table I. Parameters for simulation

PRF	10kHz	Slant Distance	700km
SAR Velocity	7,540m/s	Range Band Width	108MHz
Wave Length	0.03m	Aperture time	1.6384s

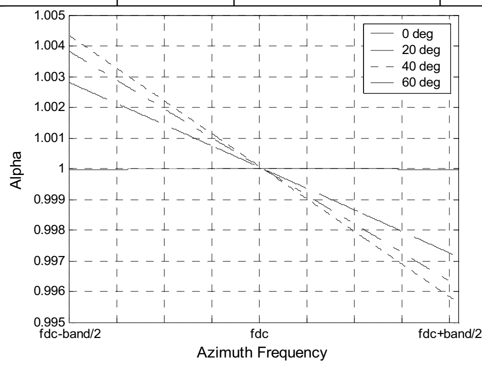


Figure 4. Simulation of α under various squint angles.

5. SIMULATION

In this section, a simulation is given to compare NCS4 with NCS. The parameters for the simulation are listed in Table II. There are three point targets in the scene at the same azimuth, and the distance between any two adjacent point targets is 5 km. The results are shown in Figure 5. NCS can only focus SAR data around the center of scene while the targets at margin are degraded. On the contrary, the results from NCS4 are focused identically.

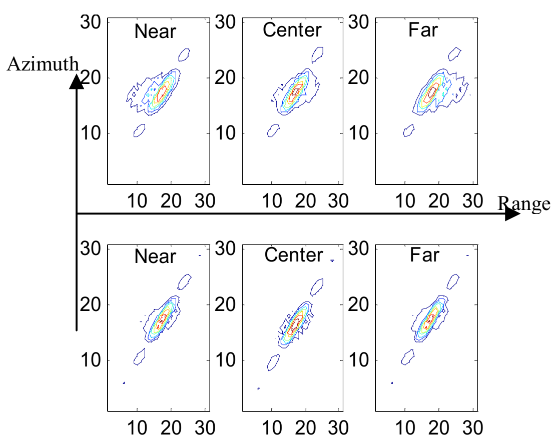


Figure 5. The comparison of the results from the two algorithms; The upper row is results from NCS and lower is from NCS4.

Table II. Simulation Parameters

Aperture time	1.6384s	Wave length	0.03m
Squint angle	50°	Distance	782,167m
Range Bandwidth	108MHz	Velocity	7540m/s
Sampling frequency	250MHz	PRF	10kHz

6. CONCLUSION

From the simulation results above, NCS4 can achieve a satisfactory performance in processing SAR data with the squint angle up to 50 degree and a range swath of 50km. If $|\beta|$ is chosen properly, NCS4 can process the SAR data with a range swath as wide as 100km.

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