CLOSED FORM PARAMETERS ESTIMATION FOR 3-D NEAR FIELD SOURCES*

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ABSTRACT

A new algorithm is proposed for bearing estimation of the narrow-band near field sources in the spherical coordinates (azimuth, elevation and range), which can distinguish $2p$ independent sources with a cross array of $2p+3$ elements by exploiting the time-domain and space-domain correlations of impinging signals jointly. The number of distinguishable sources in our algorithm is more than double of that in conventional algorithms when the number of array elements is the same. In addition, our algorithm exploits the eigen values and vectors simultaneously, and then the calculated parameters only need simple pairing and the multidimensional searching can be omitted completely. Simulation results demonstrate the novelty, validity and effectiveness of the proposed algorithm.

1. INTRODUCTION

Lots of researches on array processing were focused on the far-field sources. When a source locates in the $Fresnel$ region of the array aperture[1], the far-field assumption is not valid and a more accurate approximation is applied in this case, which considers the second Taylor expansion of the non-linear propagation delay. After exploiting this expansion, the new steering vectors are characterized by not only the impinging DOAs, but the ranges as well. Then, the accurate depiction of the space signature of the signals allows for the joint estimation.

In the conventional near-field signal processing, the number of distinguishable sources is much fewer than that in far-field scenarios. For example, a ULA with $2p+1$ elements can distinguish $2p$ sources in 2-D far-field scenarios, but only p sources in the near-field ones in most proposed algorithms [1-2]. The number of distinguishable sources of the near-field signals is only half of the far-field signals. The algorithms in [3-4] can estimate more sources, but they have some constraints on the sources' frequencies.

In addition, the estimation in the 3-D near-field scenarios is a multidimensional problem, where the multidimensional search is an extremely time-consuming operation. Many algorithms [3-4], based on the ESPRIT or matrix pencils can avoid this step, but the pair operation they proposed is also a time-consuming one.

In this paper, a novel bearing estimation algorithm is proposed for 3-D near-field sources, which exploits both the space-domain and time-domain correlations. As a result, it can distinguish $2p$ sources with a cross array of $2p+3$ elements, more than double of the most conventional ones when the array elements are the same number. For instance, the algorithm in [5] can only distinguish p sources with $4p+2$ elements. In addition, this algorithm does not need the multidimensional search and its pair operation is very simple. Besides them, its computational burden is comparable with those based on the second-order statistics ones, for it only needs the same number of snapshots as the other second-order statistics ones.

2. SYSTEM MODEL

Consider M near-field independent sources imping upon a cross array aligned with the X and Y axes (shown in Fig. 1). $2p+1$ elements are on the X axis and their center is designated as the phrase reference point and the origin of the coordinate system. 3 elements are on the Y axis, and their center is also the origin of the coordinate system. Two array branches both have interelement spacing d. Then, the signal received by the $(l,0)$-th and $(0,l)$-th sensor are expressed as

$$x_{l,0}(t)=\sum_{m=1}^{M}s_m(t)e^{-j\tau_{xl}(m)}+n_{l,0}(t), l=-p,\dots,p \quad (1)$$

$$x_{0,l}(t)=\sum_{m=1}^{M}s_m(t)e^{-j\tau_{yl}(m)}+n_{0,l}(t), l=-1,0,1 \quad (2)$$

where $s_m(t)$ denotes the m-th source signal, $\{n_{l,0}(t),n_{0,l}(t)\}$ are the additive white Gaussian noise (AWGN), and $\{\tau_{xl}(m),\tau_{yl}(m)\}$ are delays of the m-th impinging signal propagation time difference between sensor "0" and sensor $(l,0)$ and $(0,l)$, respectively. According to [5], (1)(2) can be approximated as:

$$x_{l,0}(t)=\sum_{m=1}^{M}s_m(t)e^{-j(\omega_{mx}l+\phi_{mx}l^2)}+n_{l,0}(t) \quad (3)$$

$$x_{0,l}(t)=\sum_{m=1}^{M}s_m(t)e^{-j(\omega_{my}l+\phi_{my}l^2)}+n_{0,l}(t) \quad (4)$$

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where ω_{mx}, ϕ_{mx}, ω_{my}, and ϕ_{my} are defined as

$$\omega_{mx} = -\frac{2\pi d \sin\theta_m \cos\alpha_m}{\lambda}, \phi_{mx} = \frac{\pi d^2(1-\sin^2\theta_m\cos^2\alpha_m)}{\lambda r_m}$$
$$\omega_{my} = -\frac{2\pi d \sin\theta_m \sin\alpha_m}{\lambda}, \phi_{mx} = \frac{\pi d^2(1-\sin^2\theta_m\sin^2\alpha_m)}{\lambda r_m}$$

where θ_m, α_m, and r_m are the azimuth, elevation and range of the m-th source, respectively; λ denotes the signal wavelength.

Our aim is to estimate these parameters $\{(\omega_{mx}, \phi_{mx})\}$, $\{(\omega_{my}, \phi_{my})\}$ $(m = 1, \ldots, M)$ and to further deduce the original parameters $\{\theta_m, \alpha_m, r_m\}$ $(m = 1, \ldots, M)$. Throughout the rest of this paper, the following hypotheses are assumed to be hold:

$\mathcal{H}1$: The source signals $s_m(t)$, $m = 1, 2, \ldots, M$ are mutually independent signals. They are narrow-band and stationary processes.

$\mathcal{H}2$: The additive noises, $n_{l,0}(t)$, $l = -p, \ldots, p$, $n_{0,1}(t)$ and $n_{0,-1}(t)$ are independent and zero-mean Gaussian processes with covariance σ^2, and are independent of the source signals.

$\mathcal{H}3$: The impinging DOAs (azimuth and elevation) of the sources are not equal, i.e., $\theta_i \neq \theta_j$ and $\alpha_i \neq \alpha_j$ for $i \neq j$.

$\mathcal{H}4$: The interelement spacing of the array is $d \leq \frac{\lambda}{4}$; Additionally, the number of sources is $M \leq 2p$.

3. BLIND ESTIMATION ALGORITHM

The estimation algorithm consists of two steps: In the first step, ω_{mx} and ϕ_{mx} are estimated from signals received by the elements on the X axis, i.e., from $x_{l,0}$ $(l = -p, \ldots, p)$; and in the second step, ω_{my} are calculated from the correlation between the signals impinging on the X axis and that impinging on the Y axis. After a simple pairing process, the original parameters of sources can be obtained in a closed form.

3.1. Estimate the ω_{mx} and ϕ_{mx}

According to $\mathcal{H}1$ and $\mathcal{H}2$ in section 2, a set of space-domain correlation variables can be defined as:

$$\begin{aligned} r_{-l-1,-l}(\tau) &= E\{x_{-l-1,0}(t+\tau)x^*_{-l,0}(t)\} \\ &= E\left\{\sum_{m=1}^{M} s_m(t+\tau)e^{-j\omega_{mx}(-l-1)-j\phi_{mx}(-l-1)^2}\right. \\ &\quad \left.\sum_{i=1}^{M} s^*_i(t)e^{j\omega_{ix}(-l)+j\phi_{ix}(-l)^2}\right\} + \sigma^2\delta(-l-1+l) \\ &= \sum_{m=1}^{M} r_{sm}(\tau)e^{j(\omega_{mx}-\phi_{mx})}e^{-2jl\phi_{mx}} \end{aligned} \quad (5)$$

where $r_{sm}(\tau) \stackrel{\triangle}{=} E\{s_m(t+\tau)s^*_m(t)\}$ and $\delta(\cdot)$ is the Dirac function.

Similarly,

$$\begin{aligned} r_{l+1,l}(\tau) &= E\{x_{l+1,0}(t+\tau)x^*_{l,0}(t)\} \\ &= \sum_{m=1}^{M} r_{sm}(\tau)e^{-j(\omega_{mx}+\phi_{mx})}e^{-2jl\phi_{mx}} \end{aligned} \quad (6)$$

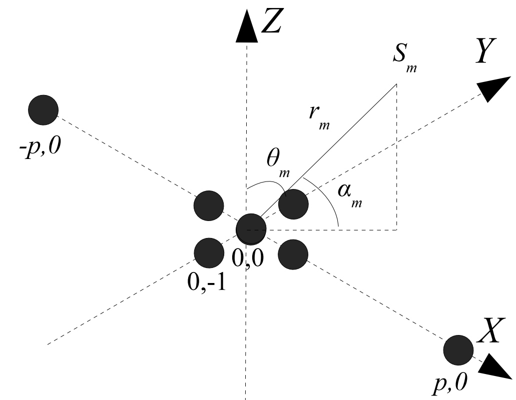

Fig. 1 System odel of near field scenarios

Concatenate $r_{-l-1,-l}(\tau)$ and $r_{l+1,l}(\tau)$ for $l = -p, -p+1, \ldots, p-1$, two vectors $\mathbf{r}_1(\tau)$, $\mathbf{r}_2(\tau)$ can be constructed with length $2p$. They are

$$\mathbf{r}_1(\tau) = [r_{p-1,p}(\tau), \ldots, r_{-1,0}(\tau), \ldots, r_{-p,-p+1}(\tau)]^T \quad (7)$$
$$\mathbf{r}_2(\tau) = [r_{-p+1,-p}(\tau), \ldots, r_{1,0}(\tau), \ldots, r_{p,p-1}(\tau)]^T \quad (8)$$

Alternative forms for (7)(8) are

$$\mathbf{r}_1(\tau) = \mathbf{B\Phi\Omega}^*\mathbf{r_s}(\tau) \quad (9)$$
$$\mathbf{r}_2(\tau) = \mathbf{B\Phi\Omega}\mathbf{r}_s(\tau) \quad (10)$$

where

$$\mathbf{B} = [\,\mathbf{b}(\phi_{1x}) \quad \ldots \quad \mathbf{b}(\phi_{Mx})\,] \qquad 2p \times M$$
$$\mathbf{b}(\phi_{mx}) = [\,e^{-2jp\phi_{mx}} \quad e^{-2j(p-1)\phi_{mx}} \quad \ldots \quad e^{2j(p-1)\phi_{mx}}\,]^T$$
$$\mathbf{r}_s(\tau) = [\,r_{s1}(\tau) \quad \ldots \quad r_{sM}(\tau)\,]^T$$
$$\mathbf{\Omega} = \mathrm{diag}\{e^{-j\omega_{1x}}, e^{-j\omega_{2x}}, \ldots, e^{-j\omega_{Mx}}\}$$
$$\mathbf{\Phi} = \mathrm{diag}\{e^{-j\phi_{1x}}, e^{-j\phi_{2x}}, \ldots, e^{-j\phi_{Mx}}\}$$

By sampling $\mathbf{r}_1(\tau)$ and $\mathbf{r}_2(\tau)$ uniformly at N $(N > M)$ lags τ_n $(\tau_n = T_s, 2T_s, \ldots, NT_s)$, the "pseudo snapshots" can be collected as follows:

$$\mathbf{R_1} = [\,\mathbf{r}_1(T_s) \quad \mathbf{r}_1(2T_s) \quad \ldots \quad \mathbf{r}_1(NT_s)\,] \quad (11)$$
$$\mathbf{R_2} = [\,\mathbf{r}_2(T_s) \quad \mathbf{r}_2(2T_s) \quad \ldots \quad \mathbf{r}_2(NT_s)\,] \quad (12)$$

And also we have

$$\mathbf{R_1} = \mathbf{B\Phi\Omega}^*\mathbf{R}_s \quad (13)$$
$$\mathbf{R_2} = \mathbf{B\Phi\Omega R}_s \quad (14)$$

where $\mathbf{R}_1$ and $\mathbf{R}_2$ are with dimensions $2p \times N$; $\mathbf{R}_s = [\mathbf{r}_s(T_s), \mathbf{r}_s(2T_s), \ldots, \mathbf{r}_s(NT_s)]$.

If we define two matrices: $\mathbf{A} \stackrel{\triangle}{=} \mathbf{B\Phi\Omega}^*$ and $\mathbf{\Psi} \stackrel{\triangle}{=} (\mathbf{\Omega})^{\mathbf{2}}$, (13) and (14) can be rewritten as

$$\mathbf{R}_1 = \mathbf{AR}_s \quad (15)$$
$$\mathbf{R}_2 = \mathbf{A\Psi R}_s \quad (16)$$

(15) and (16) are basic equations of the ESPRIT algorithm [6]. We can estimate $\mathbf{\Psi}$ directly, thus $\omega_{1x}, \ldots, \omega_{Mx}$ can be deduced. But another set of parameters $\phi_{1x}, \ldots, \phi_{Mx}$ are also important. To estimate them, an ESPRIT-like algorithm $-$ DOA-Matrix algorithm [7] is exploited in this case, which can estimate $\mathbf{A}$ and $\mathbf{\Psi}$ jointly in a closed form by the EVD operation, without the simultaneous diagonalization of multiple matrices. In order to estimate $\mathbf{A}$, we define a signature matrix as

$$\mathbf{R} = \mathbf{R}_2[\mathbf{R}_1]^- \tag{17}$$

where $[\bullet]^-$ denotes the pseudo-inverse, i.e., $\mathbf{R}_1[\mathbf{R}_1]^- = \mathbf{I}$.

Lemma: If $\mathbf{A}$ is full column rank, $\mathbf{R}_s$ is nonsingular, and $\mathbf{\Psi}$ has no equal elements on the main diagonal line, the nonzero eigen values of $\mathbf{R}$ equal to the M diagonal elements of $\mathbf{\Psi}$, and corresponding eigenvectors equal to the M columns of $\mathbf{A}$, i.e.

$$\mathbf{RA} = \mathbf{A\Psi} \tag{18}$$

The detail proof is in [8].

According to the lemma, $\mathbf{A}$ is also estimated in a closed form. It is directly to estimate $\phi_{1x}, \ldots, \phi_{Mx}$ from $\mathbf{A}$ because to any column of $\mathbf{A}$, it associates one ϕ_{mx} only, and furthermore, the quotient of two adjacent rows is just $e^{-2j\phi_{mx}}$. Note that the ω_{mx} and ϕ_{mx} are one-to-one correspondences because they are from the same pair of eigen value and vector, i.e., two parameters are paired automatically.

3.2. Estimate the ω_{my}

ω_{my} are also estimated by exploiting the rotational invariant of two matrices. First, two sets of space-domain variables are defined as

$$\begin{aligned} r_l^-(\tau) &= E\{x_{0,-1}(t)x_{l,0}^*(t+\tau)\} \\ &= \sum_{m=1}^{M} r_{sm}(\tau)e^{j\omega_{my}}e^{j(-\phi_{my}+\omega_{mx}l+\phi_{mx}l^2)} \end{aligned} \tag{19}$$

$$\begin{aligned} r_l^+(\tau) &= E\{x_{0,1}(t)x_{l,0}^*(t+\tau)\} \\ &= \sum_{m=1}^{M} r_{sm}(\tau)e^{-j\omega_{my}}e^{j(-\phi_{my}+\omega_{mx}l+\phi_{mx}l^2)} \end{aligned} \tag{20}$$

Concatenate $r_l^-(\tau)$ and $r_l^+(\tau)$ for $l = -p, \ldots, p$, two vectors $\mathbf{r}_l^-(\tau)$ and $\mathbf{r}_l^+(\tau)$ can be constructed with length $2p+1$, i. e.,

$$\begin{aligned} \mathbf{r}_l^-(\tau) &= [\, r_{-p}^-(\tau) \quad \ldots \quad r_p^-(\tau)\,]^T \\ \mathbf{r}_l^+(\tau) &= [\, r_{-p}^+(\tau) \quad \ldots \quad r_p^+(\tau)\,]^T \end{aligned} \tag{21}$$

Similarly with (7)(8), they can be rewritten as

$$\mathbf{r}_l^-(\tau) = \mathbf{CD}\mathbf{r}_s(\tau), \qquad \mathbf{r}_l^+(\tau) = \mathbf{CD}^*\mathbf{r}_s(\tau) \tag{22}$$

where$\mathbf{C} = [\mathbf{c}(1), \ldots, \mathbf{c}(M)]$ with dimensions $2p+1 \times M$; $\mathbf{c}(m) = [e^{j(-\phi_{my}+\omega_{mx}(-p)+\phi_{mx}(-p)^2)}, \ldots, e^{j(-\phi_{my}+\omega_{mx}p+\phi_{mx}p^2)}]$; and $\mathbf{D} = \mathrm{diag}\{e^{j\omega_{1y}}, \ldots, e^{j\omega_{My}}\}$.

Just like (11)-(18), we can estimate ω_{my} and $\mathbf{C}$ by sampling $\mathbf{r}_l^-(\tau)$, $\mathbf{r}_l^+(\tau)$ at the time domain, and exploiting the DOA-Matrix algorithm.

Because $\mathbf{C}$ can be calculated directly, the pairing operation of (ω_{mx}, ϕ_{mx}) and ϕ_{my} is relatively simple. By calculating the phrase difference of the adjacent rows in $\mathbf{C}$ and comparing with the estimate (ω_{mx}, ϕ_{mx}) pair, we can deduce the right pair of three parameters. In fact, our algorithm only needs a two-parameter pair in this three-parameter estimation.

The proposed algorithm differs from not only those MUSIC-like algorithms which exploit the eigen vectors only and need the operation of multidimensional searching or rooting, but also those ESPRIT-like algorithms which exploit eigen values only and need more complex pairing operations. In addition, our algorithm exploits both the space-domain and the time-domain correlations, differing from those exploiting the space-domain correlation only. Therefore, it can estimate more than double independent sources of what other algorithms did.

Our algorithms can be summarized as:

1. Construct $\mathbf{R}_1$ and $\mathbf{R}_2$ according to (11) and (12), where $r_{p-1,p}(T_s), \ldots, r_{-p,-p+1}(NT_s)$ and $r_{-p+1,-p}(T_s)$, …, $r_{p,p-1}(NT_s)$ are calculated by (5) and (6).
2. Calculate $\mathbf{R}$ from (17).
3. Perform the EVD on $\mathbf{R}$, Obtain its eigen values and vectors.
4. $\omega_1, \ldots, \omega_M$ is calculated from $\mathbf{\Psi}$ while $\phi_1, \ldots, \phi_M$ is from $\mathbf{A}$. They are paired automatically.
5. Construct $\mathbf{r}_l^+(\tau)$ and $\mathbf{r}_l^-(\tau)$. Estimate ω_{my} and $\mathbf{C}$ by means of the similar methods in step 1-3.
6. Pair the estimate parameters $\{\omega_{mx}, \phi_{mx}\}$ and ω_{my} according to the known structure of $\mathbf{C}$.
7. Calculate the original source parameters $\{\theta_m, \alpha_m, r_m\}$.

4.SIMULATIONS

In simulations, we adopt a cross array of 7 sensors ($p = 2$) with element spacing $d = \lambda/4$ to measure our algorithm. In contrast, the Abed-Meriam's method [5] is also simulated, which does not work in this array configuration. To solve this, a cross array of 9 sensors, 5 sensors are on every axis, is exploited to measure it. $N_t = 100$ independent Monte-Carlo simulations are performed. The performance is measured by the Root Mean Square Error (RMSE) defined as

$$\mathrm{RMSE}(\mathbf{x}) = \frac{1}{\|\mathbf{x}\|}\sqrt{\frac{1}{N_t}\sum_{i=1}^{N_t}\|\hat{\mathbf{x}}(i) - \mathbf{x}\|^2} \tag{23}$$

where N_t is the number of Monte-Carlo trials, and $\| \bullet \|$ represents the Frobenius norm; and $\mathbf{x}$ represents the exact values of parameters, and $\hat{\mathbf{x}}(i)$ represents the estimated values in the i-th Monte-Carlo trials, respectively. $\mathbf{x}$ can be such as the impinging DOA (azimuth and elevation), $\{\theta_i\}$ and $\{\alpha_i\}$ $(i = 1, 2, \ldots)$, the range $\{r_i\}$, $(i = 1, 2, \ldots)$. In each trial, 1024 real snapshots and 30 pseudo snapshots are collected.

The performance of two algorithms are shown in Fig. 2∼4 when $M = 1, 2, 3$, respectively. All parameters of sources in the three cases are listed in Table 1. The SNR is defined as $\mathrm{SNR} = 1/\sigma^2$. From the simulations, we observe that the performance of our algorithm is close to Abed-Meraim's [5] when $M = 1$ although it exploits 9 sensors.

When $M = 2$, the performance of ours is better than Abed-Meraim's. Because 2 is the maximum number of distinguishable sources in Abed-Meraim's method, and no subspace is for noise, its estimated eigen value is not very accurate.

Table 1 Sources' Parameters when $M = 1, 2, 3$

M	r	$\theta(^\circ)$	$\alpha\ (^\circ)$
1	1/4λ	20	40
2	1/4λ	20	40
	2/5λ	30	50
3	1/4λ	20	40
	2/5λ	30	50
	1/3λ	40	30

When $M = 3$, Abed-Meraim's method does not work while our algorithm still distinguishes every source well and the performance is acceptable.

5.CONCLUSIONS

This paper proposed a 3-D near-field sources estimation algorithm exploiting both the space-domain and the time-domain correlations. This algorithm equivalently enlarges the aperture of the array, therefore, the number of its distinguishable sources exceeds double of what conventional algorithms offered when the elements are the same number. Furthermore, it exploits the eigen values and vectors simultaneously, and consequently, it does not need the time-consuming multidimensional search and the pair operation is rather simple. Besides them, its computational burden is comparable with those based on the second-order statistics algorithms. The simulation results show the effectiveness of our algorithm.

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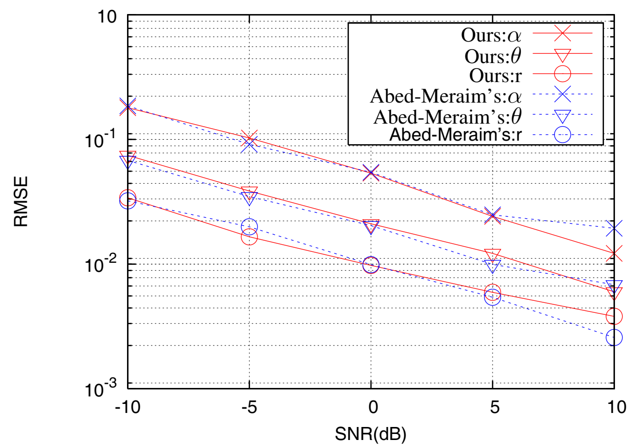

Fig. 2 The performance of two algorithms when $M = 1$

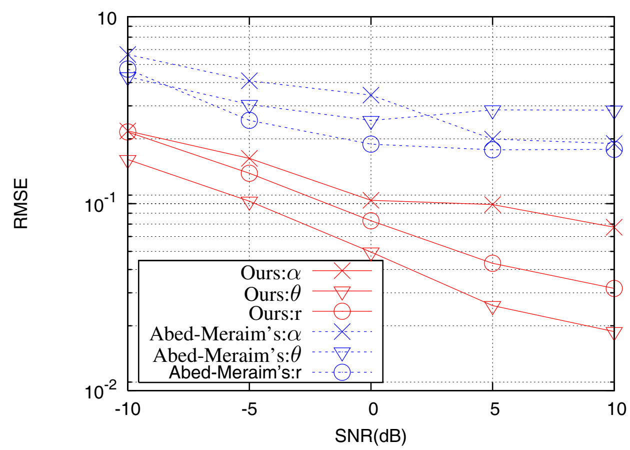

Fig. 3 The performance of two algorithms when $M = 2$

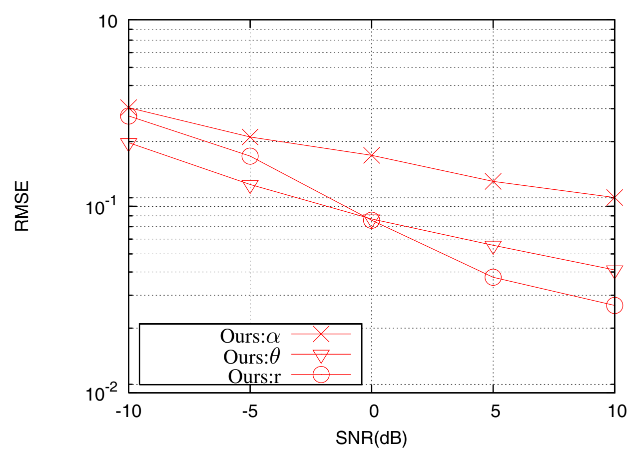

Fig. 4 The performance of our algorithm when $M = 3$