Multichannel Parametric Rao Detector

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***Abstract*— The parametric Rao test for a multichannel adaptive signal detection problem is derived by modeling the disturbance signal as a multichannel autoregressive (AR) process. Interestingly, the parametric Rao test takes a form identical to that of the recently introduced parametric adaptive matched filter (PAMF) detector. The equivalence offers new insights into the performance and implementation of the PAMF detector. Specifically, the Rao/PAMF detector is asymptotically (for large samples) a parametric generalized likelihood ratio test (GLRT), due to an asymptotic equivalence between the Rao test and the GLRT. The asymptotic distribution of the Rao test statistic is obtained in closed-form, which follows an exponential distribution under H_0 and, respectively, a non-central Chi-squared distribution with two degrees of freedom under H_1. The non-centrality parameter of the non-central Chi-squared distribution is determined by the output signal-to-interference-plus-noise ratio (SINR) of a temporal whitening filter. Since the asymptotic distribution under H_0 is independent of the unknown parameters, the Rao/PAMF asymptotically achieves constant false alarm rate (CFAR). Numerical results show that these results are accurate in predicting the performance of the parametric Rao/PAMF detector even with moderate data support.**

I. Introduction

Space-time adaptive processing (STAP) based multichannel signal detectors have been successfully used to mitigate the effects of clutter and/or interference in radar, remote sensing, and communication systems [1]-[3]. However, traditional STAP detectors, including the well-known RMB detector by Reed, Mallett, and Brennan [4], Kelly's generalized likelihood ratio test (GLRT) [5], the adaptive matched filter (AMF) detector [6], and the adaptive coherence estimator (ACE) detector [7], usually involve estimating and inverting a large-size space-time covariance matrix of the disturbance signal (viz., clutter, jamming, and noise) for each test cell using training data. This entails *high complexity* and *large training requirement*. While the first difficulty may create real-time implementation burdens, the second implies that such covariance-matrix based techniques may not be used in heterogeneous (due to varying terrain, high platform altitude, bistatic geometry, conformal array, among others) or dense-target environments, which offer limited training data.

Addressing the above issues has become an important topic in recent multichannel signal detection research. One effective way to reduce the computational and training requirement is to utilize a suitable parametric model for the disturbance signal and exploit the model for signal detection. For example, multichannel autoregressive (AR) models have been found to be very effective in representing the spatial and temporal correlation of the disturbance [8]. A parametric detector based on such a multichannel AR model is developed in [8], which is referred to as the *parametric adaptive matched filter* (PAMF). The PAMF detector has been shown to significantly outperform the aforementioned covariance-matrix based detectors for small training size at reduced complexity [8] [9].

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Although intuitively sound, the PAMF detector was obtained in a heuristic approach by modifying the AMF test statistic. Specifically, it replaces the joint space-time whitening incurred by the AMF detector with a two-step approach that involves temporal whitening via an inverse moving-average (MA) filter followed by spatial whitening. The test threshold, false alarm and detection probabilities were determined primarily by computer simulation, due to limited analysis available for the PAMF detector.

In this paper, we present the *parametric Rao test* for the multichannel signal detection problem and perform the asymptotic analysis of the test statistic. The generic Rao test is known to offer a standard solution to a class of parameter testing problems. It is easier to derive and implement than the GLRT, and is also asymptotically (for large-sample) equivalent to the latter. Other attributes of a generic Rao test can be found in [10]. We show that, interestingly, the parametric Rao test takes a form identical to that of the PAMF detector in [8]. Moreover, if the ML estimator is utilized, the parametric Rao/PAMF detector is asymptotically a parametric GLRT. Asymptotic analysis shows that the parametric Rao/PAMF detector achieves constant false alarm rate (CFAR). Numerical results show that our asymptotic results are accurate in predicting the performance of the Rao/PAMF detector even when the data size is modest.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; superscripts $(\cdot)^T$, and $(\cdot)^H$ denote transpose and complex conjugate transpose, respectively; $\mathcal{CN}(\boldsymbol{\mu}, \mathbf{R})$ denotes the multivariate complex Gaussian distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\mathbf{R}$; $\mathbb{C}$ denotes the complex number field.

II. Data Model and Problem Statement

The problem under consideration involves detecting a known multichannel signal with unknown amplitude in the presence of spatially and temporally correlated disturbance (e.g., [1]):

$$\begin{aligned} H_0 : &\quad \mathbf{x}_0(n) = \mathbf{d}(n), \quad n = 0, 1, \ldots, N-1, \\ H_1 : &\quad \mathbf{x}_0(n) = \alpha \mathbf{s}(n) + \mathbf{d}(n), \quad n = 0, 1, \ldots, N-1, \end{aligned} \tag{1}$$

where all vectors are $J \times 1$ vectors, J denotes the number of spatial channels, and N is the number of temporal observations. Henceforth, $\mathbf{x}_0(n)$ is called the *test signal*, $\mathbf{s}(n)$ is the signal to be detected with amplitude α, and $\mathbf{d}(n)$ is the *disturbance signal* that may be correlated in space and time. In addition to the test signal, it is assumed that a set of target-free *training* or *secondary* data vectors $\mathbf{x}_k(n)$, $k = 1, 2, \ldots, K$ and $n = 0, 1, \ldots, N-1$, are available to assist signal detection.

Let $\mathbf{s} = \left[\mathbf{s}^T(0), \mathbf{s}^T(1), \ldots, \mathbf{s}^T(N-1)\right]^T$, and $\mathbf{d}$ and $\mathbf{x}_k$ are formed similarly from $\mathbf{d}(n)$ and $\mathbf{x}_k(n)$, respectively. Then, (1) can be more compactly written as

$$\begin{aligned} H_0 : &\quad \mathbf{x}_0 = \mathbf{d}, \\ H_1 : &\quad \mathbf{x}_0 = \alpha \mathbf{s} + \mathbf{d}. \end{aligned} \tag{2}$$

Clearly, the composite hypothesis testing problem (1) or (2) is also a *two-sided parameter testing problem* that tests $\alpha = 0$ against $\alpha \neq 0$. The general assumptions in the literatures ([1]-[2], [4]-[8]) are as follows:

- **AS1:** the signal vector $\mathbf{s}$ is deterministic and *known* to the detector;
- **AS2:** the signal amplitude α is complex-valued, deterministic, and *unknown*;
- **AS3:** the secondary data $\{\mathbf{x}_k\}_{k=1}^K$ and the disturbance signal $\mathbf{d}$ (equivalently, $\mathbf{x}_0$ under H_0) are independent and identically distributed (i.i.d.) with distribution $\mathcal{CN}(\mathbf{0}, \mathbf{R})$, where $\mathbf{R}$ is the *unknown* space-time covariance matrix.

While Assumptions **AS1** to **AS3** are standard (e.g., [1]-[2], [4]-[7]), we further assume the following:

- **AS4**: the disturbance signal $\mathbf{d}(n)$ can be modeled as a multi-channel AR(P) process with *known* model order P but *unknown* AR coefficient matrices and spatial covariance.

Based on Assumption **AS4**, the secondary data $\{\mathbf{x}_k\}_{k=1}^K$ are represented as

$$\mathbf{x}_k(n) = -\sum_{p=1}^{P} \mathbf{A}^H(p)\mathbf{x}_k(n-p) + \boldsymbol{\varepsilon}_k(n), \tag{3}$$

where $\{\mathbf{A}^H(p)\}_{p=1}^P$ denote the $J \times J$ AR coefficient matrices, $\boldsymbol{\varepsilon}_k(n)$ denote the driving multi-channel spatial noise vectors that are temporally white but spatially colored Gaussian noise: $\boldsymbol{\varepsilon}_k(n) \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q})$, where $\mathbf{Q}$ denotes the $J \times J$ spatial covariance matrix. Meanwhile, the test signal $\mathbf{x}_0$ is given by

$$\begin{aligned} &\mathbf{x}_0(n) - \alpha\mathbf{s}(n) \\ &= -\sum_{p=1}^{P} \mathbf{A}^H(p)\left\{\mathbf{x}_0(n-p) - \alpha\mathbf{s}(n-p)\right\} + \boldsymbol{\varepsilon}_0(n), \end{aligned} \tag{4}$$

where $\alpha = 0$ under H_0, $\alpha \neq 0$ under H_1, and $\boldsymbol{\varepsilon}_0(n) \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q})$. Let $\tilde{\mathbf{s}}(n)$ denote a regression on $\mathbf{s}(n)$ and $\tilde{\mathbf{x}}_0(n)$ a regression on $\mathbf{x}_0(n)$ *under* H_1:

$$\tilde{\mathbf{s}}(n) = \mathbf{s}(n) + \sum_{p=1}^{P} \mathbf{A}^H(p)\mathbf{s}(n-p), \tag{5}$$

$$\tilde{\mathbf{x}}_0(n) = \mathbf{x}_0(n) + \sum_{p=1}^{P} \mathbf{A}^H(p)\mathbf{x}_0(n-p). \tag{6}$$

Then, the driving noise in (4) can be alternatively expressed as

$$\boldsymbol{\varepsilon}_0(n) = \tilde{\mathbf{x}}_0(n) - \alpha\tilde{\mathbf{s}}(n). \tag{7}$$

The problem of interest is to develop a decision rule for the above composite hypothesis testing problem using the test and training signals as well as exploiting the multichannel parametric AR model.

III. Prior Solutions

A number of solutions to the above problem have been developed. If the space-time covariance matrix $\mathbf{R}$ is known exactly, the optimum detector that maximizes the output SINR is the matched filter (MF) [6]:

$$T_{\text{MF}} = \frac{\left|\mathbf{s}^H\mathbf{R}^{-1}\mathbf{x}_0\right|^2}{\mathbf{s}^H\mathbf{R}^{-1}\mathbf{s}} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{\text{MF}}, \tag{8}$$

where γ_{MF} denotes the MF threshold. The MF detector is obtained by a GLRT approach (e.g., [10]), by which the ML estimate of the unknown amplitude α is first estimated and then substituted back into the likelihood ratio to form a test statistic. It should be noted that the MF detector *cannot* be implemented in real applications since $\mathbf{R}$ is unknown. However, it provides a baseline for performance comparison when considering any realizable detection scheme.

In practice, the unknown $\mathbf{R}$ can be replaced by some estimate, such as the sample covariance matrix obtained from the secondary data: $\hat{\mathbf{R}} = \frac{1}{K}\sum_{k=1}^K \mathbf{x}_k\mathbf{x}_k^H$.Using $\hat{\mathbf{R}}$ in (8) leads to the so-called AMF detector [6]:

$$T_{\text{AMF}} = \frac{\left|\mathbf{s}^H\hat{\mathbf{R}}^{-1}\mathbf{x}_0\right|^2}{\mathbf{s}^H\hat{\mathbf{R}}^{-1}\mathbf{s}} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{\text{AMF}}, \tag{9}$$

where γ_{AMF} denotes the AMF threshold. The AMF test is a CFAR detector, which is a desirable property in radar systems. However, they also entail a large training requirement. In particular, the sample covariance matrix $\hat{\mathbf{R}}$ has to be inverted, which imposes a constraint on the training size $K \geq JN$ to ensure a full-rank $\hat{\mathbf{R}}$. The Reed-Brennan rule [4] suggests that at least $K \geq (2JN-3)$ target-free secondary data vectors are needed to obtain performance within 3 dB from the optimum MF detector. Such a training requirement may be difficult to met, especially in non-homogeneous or dense-target environments. Besides excessive training, the computational complexity of these detectors is also high, since $\hat{\mathbf{R}}$ has to be computed and inverted for each test signal.

While the AMF test may be called a *covariance-matrix based technique* as it involves computing and inverting $\hat{\mathbf{R}}$, the recently introduced PAMF detector [8] utilizes a multichannel AR(P) model that allows spatial/temporal whitening to be implemented in a multichannel time-series fashion (see [8] for details):

$$T_{\text{PAMF}} = \frac{\left|\sum_{n=P}^{N-1} \hat{\tilde{\mathbf{s}}}_{\text{P}}^H(n)\hat{\mathbf{Q}}_{\text{P}}^{-1}\hat{\tilde{\mathbf{x}}}_{0,\text{P}}(n)\right|^2}{\sum_{n=P}^{N-1} \hat{\tilde{\mathbf{s}}}_{\text{P}}^H(n)\hat{\mathbf{Q}}_{\text{P}}^{-1}\hat{\tilde{\mathbf{s}}}_{\text{P}}(n)} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{\text{PAMF}}, \tag{10}$$

where $\hat{\mathbf{Q}}_{\text{P}}$ denotes an estimate of the spatial covariance matrix $\mathbf{Q}$, $\hat{\tilde{\mathbf{x}}}_{0,\text{P}}(n)$ and $\hat{\tilde{\mathbf{s}}}_{\text{P}}(n)$ are the *temporally* whitened test signal and steering vector, respectively, using an inverse AR(P) filter (i.e., a multichannel MA filter) whose parameters, along with $\hat{\mathbf{Q}}_{\text{P}}$, are estimated from the secondary data. In contrast to simultaneous spatio-temporal whitening used in the AMF test, the PAMF detector performs whitening in two distinct steps: temporal whitening followed by spatial whitening. The parametric approach saves in both training and computation, since the parameters to be estimated are significantly fewer compared to covariance matrix based approaches.

IV. The Parametric Rao Test

A. Test Statistic

The parametric Rao test is derived in [9] and given by [1]

$$T_{\text{Rao}} = \frac{2\left|\sum_{n=P}^{N-1} \hat{\tilde{\mathbf{s}}}^H(n)\hat{\mathbf{Q}}^{-1}\hat{\tilde{\mathbf{x}}}_0(n)\right|^2}{\sum_{n=P}^{N-1} \hat{\tilde{\mathbf{s}}}^H(n)\hat{\mathbf{Q}}^{-1}\hat{\tilde{\mathbf{s}}}(n)} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma_{\text{Rao}}, \tag{11}$$

where γ_{Rao} denotes the test threshold, which can be set by using the results in Section IV-B, and $\hat{\tilde{\mathbf{s}}}(n)$ and $\hat{\tilde{\mathbf{x}}}_0(n)$ denote, respectively, the steering vector and test signal that have been whitened temporally:

$$\hat{\tilde{\mathbf{s}}}(n) = \mathbf{s}(n) + \sum_{p=1}^{P} \hat{\mathbf{A}}^H(p)\mathbf{s}(n-p), \tag{12}$$

$$\hat{\tilde{\mathbf{x}}}_0(n) = \mathbf{x}_0(n) + \sum_{p=1}^{P} \hat{\mathbf{A}}^H(p)\mathbf{x}_0(n-p), \tag{13}$$

[1]Although the factor of 2 on the test statistic can be absorbed by the test threshold, it is retained to keep the asymptotic distribution of the test statistic more compact. See Section IV-B.

where $\hat{\mathbf{A}}^H(p)$ denotes the ML estimate of the AR coefficient matrix $\mathbf{A}^H(p)$. Additional spatial whitening is provided by $\hat{\mathbf{Q}}^{-1}$, which is the inverse of the ML estimate of the spatial covariance matrix to be specified next.

To present the ML estimates more compactly, let $\mathbf{A}^H = \left[\mathbf{A}^H(1), \quad \mathbf{A}^H(2), \quad \cdots, \quad \mathbf{A}^H(P)\right] \in \mathbb{C}^{J \times JP}$, which contains all the coefficient matrices involved in the P-th order AR model, and $\mathbf{y}_k(n) = \left[\mathbf{x}_k^T(n-1), \quad \mathbf{x}_k^T(n-2), \quad \cdots, \quad \mathbf{x}_k^T(n-P)\right]^T$, $k = 0, 1, \cdots, K$, which contains the regression subvectors formed from the test signal $\mathbf{x}_0$ or the k-th training signal $\mathbf{x}_k$. Then, the ML estimates of the AR coefficients $\mathbf{A}^H$ and the spatial covariance matrix $\mathbf{Q}$ are obtained by

$$\hat{\mathbf{A}}^H = -\hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1}, \tag{14}$$

$$\hat{\mathbf{Q}} = \frac{1}{(K+1)(N-P)} \left(\hat{\mathbf{R}}_{xx} - \hat{\mathbf{R}}_{yx}^H \hat{\mathbf{R}}_{yy}^{-1} \hat{\mathbf{R}}_{yx}\right) \tag{15}$$

where the correlation matrices are computed by $\hat{\mathbf{R}}_{xx} = \sum_{n=P}^{N-1} \sum_{k=0}^{K} \mathbf{x}_k(n)\mathbf{x}_k^H(n)$, $\hat{\mathbf{R}}_{yy} = \sum_{n=P}^{N-1} \sum_{k=0}^{K} \mathbf{y}_k(n)\mathbf{y}_k^H(n)$, and $\hat{\mathbf{R}}_{yx} = \sum_{n=P}^{N-1} \sum_{k=0}^{K} \mathbf{y}_k(n)\mathbf{x}_k^H(n)$.

Remark 1: The PAMF detector also involves estimating the AR coefficients $\mathbf{A}^H$ and the spatial covariance matrix $\mathbf{Q}$ [8]. Several estimators were suggested, e.g., the Strand-Nuttall algorithm and the least-squares (LS) estimator. The LS estimator yields better performance. Our ML estimates differ from the LS estimates of [8] in that we have used both the test and training signals to obtain these estimates, whereas the LS estimator in [8] utilizes only the training signals for parameter estimation. Our approach is similar to Kelly's GLRT [5], which also employs both the test and training signals for parameter estimation. However, we shall stress that Kelly's GLRT does not exploit the multichannel parametric model as shown in (3) and (4).

Remark 2: By comparing the parametric Rao test statistic (11) with the PAMF test statistic (10), we can quickly see that if both detectors use the ML estimator for parameter estimation, they are identical except for a scaling factor of 2. Hence, *the PAMF detector is a parametric Rao detector*. Since the parametric Rao test is asymptotically equivalent to the *parametric GLRT*, the PAMF detector with the ML parameter estimates is also an asymptotic parametric GLRT. As we shall see in Section IV-B, the equivalence offers additional insights into the performance and implementation of the PAMF detector.

B. Asymptotic Analysis

We can show that the asymptotic distribution of the Rao/PAMF test statistic is given by

$$T_{\text{Rao}} \overset{a}{\sim} \begin{cases} \chi_2^2, & \text{under } H_0, \\ \chi_2'^2(\lambda), & \text{under } H_1, \end{cases} \tag{16}$$

where χ_2^2 denotes the central Chi-squared distribution with 2 degrees of freedom and $\chi_2'^2(\lambda)$ the non-central Chi-squared distribution with 2 degrees of freedom and non-centrality parameter λ:

$$\lambda = 2|\alpha|^2 \sum_{n=P}^{N-1} \tilde{\mathbf{s}}^H(n)\mathbf{Q}^{-1}\tilde{\mathbf{s}}(n), \tag{17}$$

where $\tilde{\mathbf{s}}(n)$ is the temporally whitened steering vector given by (5). Note that λ is related to the SINR at the output of the temporal whitening filter. Recall that a χ_2^2 random variable is equivalent to an exponential random variable with probability density function (PDF) given by

$$f_{\chi_2^2}(x) = \frac{1}{2}\exp\left(-\frac{1}{2}x\right), \quad x \geq 0. \tag{18}$$

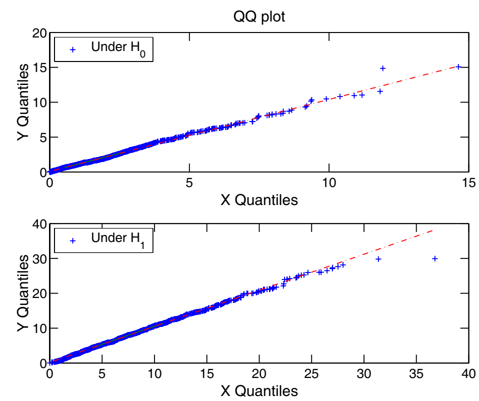

Fig. 1. The quantile-quantile (QQ) plot of the parametric Rao/PAMF test statistic and its asymptotic distribution under H_0 (upper plot) and H_1 (lower plot), respectively, with $J = 4$, $N = 32$, and $K = 8$. Specifically, the x-axis shows the ordered samples of the parametric Rao/PAMF test statistic, while the y-axis shows the ordered samples of the asymptotic distribution.

The PDF of $\chi_2'^2(\lambda)$ is given by [10]

$$f_{\chi_2'^2(\lambda)}(x) = \frac{1}{2}\exp\left[-\frac{1}{2}(x+\lambda)\right] I_0\left(\sqrt{\lambda x}\right), \quad x \geq 0, \tag{19}$$

where $I_0(u)$ is the modified Bessel function of the first kind and zero-th order [10].

The above distributions can be employed to set the Rao test threshold for a given probability of false alarm, as well as to compute the detection and false alarm probabilities, etc. For a given threshold, the probability of false alarm is given by

$$P_{\text{f}} = \int_{\gamma_{\text{Rao}}}^{\infty} f_{\chi_2^2}(x)dx = \exp\left(-\frac{1}{2}\gamma_{\text{Rao}}\right), \tag{20}$$

which can easily be inverted to find the test threshold γ_{Rao} for a given P_{f}. In addition, the probability of detection is given by

$$P_{\text{d}} = \int_{\gamma_{\text{Rao}}}^{\infty} \frac{1}{2}\exp\left[-\frac{1}{2}(x+\lambda)\right] I_0\left(\sqrt{\lambda x}\right) dx \tag{21}$$

for a given test threshold γ_{Rao}.

Remark 3: The asymptotic distribution under H_0 is independent of the *unknown* parameters. The probability of false alarm in (20) depends only on the test threshold, which is a design parameter. It is evident that the Rao/PAMF test asymptotically achieves CFAR.

V. Numerical Results

In the following, we present our numerical results of the parametric Rao/PAMF detector obtained by computer simulation and by the above asymptotic analysis. In addition, the performance of the MF and AMF detectors, which can be computed analytically, is included for comparison. The SINR is defined as

$$\text{SINR} = |\alpha|^2 \mathbf{s}^H \mathbf{R}^{-1}\mathbf{s}, \tag{22}$$

where $\mathbf{R}$ is the $JN \times JN$ joint space-time covariance matrix of the disturbance $\mathbf{d}$.

First, we consider the asymptotic distribution of the parametric Rao/PAMF test statistic obtained in Section IV-B. Figure 1 depicts the quantile-quantile (QQ) plot of the Rao/PAMF test statistic under both hypotheses against the corresponding asymptotic distribution when $J = 4$, $N = 32$, and $K = 8$, a case with limited training. It is seen that even with a relatively small data size, the asymptotic distribution matches well the sample test statistics, with only some minor deviation at the tail portion.

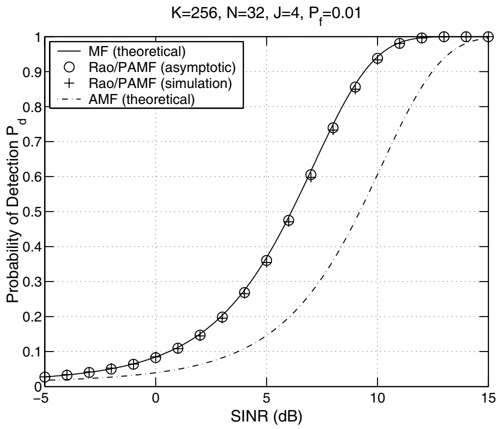

Fig. 2. The probability of detection P_{d} versus the input SINR when $P_{\mathrm{f}} = 0.01$, $J = 4$, $N = 32$, and $K = 256$.

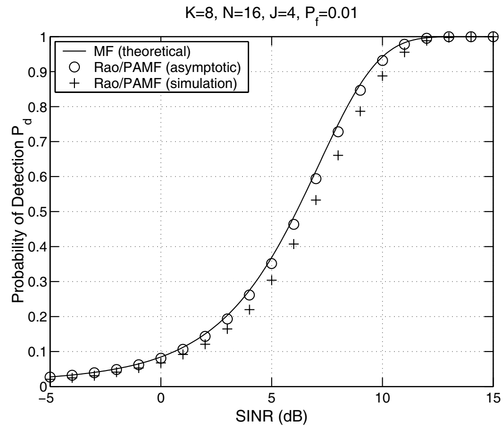

Fig. 3. The probability of detection P_{d} versus the input SINR when $P_{\mathrm{f}} = 0.01$, $J = 4$, $N = 16$, and $K = 8$. Note that the AMF detector is not included since it cannot be implemented for such a small K.

Figures 2 and 3 depict the probability of detection P_{d} versus SINR for the MF, AMF, and the parametric Rao/PAMF detectors. In particular, Figure 2 corresponds to the case with adequate training, for which the Reed-Brennan rule [4] is satisfied, whereas Figure 3 corresponds to the case with limited training. An examination of these figures reveals that the asymptotic analysis, in general, provides a quite accurate prediction of the performance of the parametric Rao/PAMF detectors. Even for the case with $K = 8$ and $N = 16$, the gap is about 0.5 dB, as shown in Figure 3. Moreover, we can see that the parametric Rao/PAMF detector is very close to the optimum MF detector; the parametric Rao/PAMF detector outperforms the AMF detector by 2 to 3 dB when the Reed-Brennan rule is marginally satisfied which agrees with earlier observations made in [8].

So far we have assumed that the model order P of the multichannel AR process is known (cf. Assumption **AS4**). In practice various model selection techniques can be used to estimate P. It is not unusual for these techniques to under- or over-estimate the model order by a small number (relative to the true model order P) [11]. Hence, it would be of interest to find out how the parametric Rao/PAMF detector performs when an inaccurate model order estimate is used. Figure 4 depicts the performance of the Rao/PAMF detector with true, under-estimated, and over-estimated model order. It shows that using an inaccurate model order estimate degrades the detection performance, but the degradation is not significant.

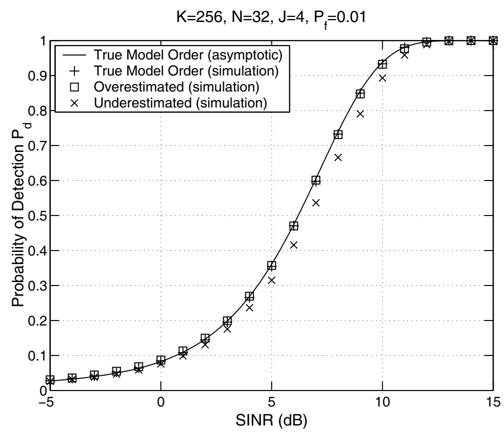

Fig. 4. The probability of detection P_{d} versus the input SINR of the parametric Rao/PAMF detector when the model order of the multichannel AR process used for computing the test statistic is true ($P = 2$), under-estimated (assuming $P = 1$), and over-estimated (assuming $P = 3$), along with $P_{\mathrm{f}} = 0.01$, $J = 4$, $N = 32$, and $K = 256$.

VI. CONCLUSIONS

We have developed a parametric Rao test for the multichannel adaptive signal detection problem by exploiting a multichannel AR model. The parametric Rao/PAMF test has been shown to be an asymptotic parametric GLRT. The asymptotic analysis of the test statistic shows that the Rao/PAMF test asymptotically achieves CFAR. Moreover, we can set the test threshold for a given P_{f} by utilizing the PDF's (closed form) in Section IV-B instead of computer simulations. Finally, our asymptotic analysis has been shown to provide fairly accurate prediction of the performance of the parametric Rao/PAMF test.

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