# UNDERWATER NOISE MODELING AND DIRECTION-FINDING BASED ON CONDITIONAL HETEROSCEDASTIC TIME SERIES

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## ABSTRACT

In this paper, we propose a new method for practical non-Gaussian and non-stationary underwater ambient noise modeling and direction-finding approach. In this application, measurement of ambient noise in natural environment shows that noise can sometimes be significantly non-Gaussian and time-varying features such as variances. Therefore, signal processing algorithms such as direction-finding that are optimized for Gaussian noise, may degrade significantly in this environment. Generalized Autoregressive Conditional Heteroscedasticity(GARCH) models are feasible for heavy tailed PDFs and time-varying variances of stochastic process and also has flexible forms. We use a more realistic GARCH(1,1) based noise model in the Maximum Likelihood Approach for the estimation of Direction-Of-Arrivals (DOAs) of impinging sources and show using experimental data that this model is suitable for the additive noise in an underwater environment.

## 1. INTRODUCTION

A passive SONAR generally employs array processing techniques to resolve problems such as localization of targets[1, 2]. In this way, additive noise model has an important role for these methods. As a matter of fact, all of them such as direction-finding utilize assumptions for noise model, that have very essential part in these methods. In the underwater environment, the measurements of ambient noise shows that we have non-Gaussian process[3, 4]. Natural and man-made sources such as reverberation and industrial noise that cause ambient noise distribution exhibit performances far away from the Gaussian model. These factors are more in coastal and shallow waters. Thus, the algorithms that are optimized for Gaussian distribution will degrade in actual experiments. All this mentioned factors give a stochastic and time-varying nature to the additive noise. Thus, a proper model presentation which could best and simply describe the different features of the realistic additive noise affecting the desired signal is an important part of an SONAR signal processing. In the last decade, after the works by Engle [5] and Bullerslev [6] there has been a growing interest in time series modeling of changing variance or Heteroscedasticity. These models have found a great number of applications in non-stationary time series such as financial records. Generalized autoregressive conditional Heteroscedasticity; e.g., GARCH [6], is a time series modeling technique that uses past variances and the past variance forecasts to forecast future variances. GARCH models account for two main characteristics, excess kurtosis; i.e., heavy tailed probability distribution, and the volatility clustering; i.e., large changes tend to follow large changes and small changes tend to follow small ones, compatible to a large extent to the ambient noises in a natural environment. We suggested this more realistic dynamic model for additive noise modeling in array signal processing [7]. Now, We offer this model for the underwater ambient noise in passive SONAR due to the facts that the commonly used model for environmental additive noise exhibits heavier tail than the standard normal distribution [4], and the conditional-Heteroscedasticity suggests a time series model in which successive disturbances are uncorrelated but dependent, that is, a more logical modeling for the dynamic of the additive noise[6]. Hence, the GARCH model is a good offer for the additive noise model in the source localization problem for passive SONAR signal processing, therefore, in this paper, we propose to assume a GARCH noise model as ambient noise. This paper is organized as follows: section (2) we present the GARCH time series. The considering of the proposed model for the underwater noise and so the approach of direction-finding based on Maximum Likelihood in conjunction with GARCH(1,1) is given in section (3), and section (4) is devoted to Cramér-Rao Bound and the simulation results of the proposed method come in section (5). Conclusions are provided in the end section of this paper.

## 2. GARCH TIMES SERIES

The exploitation of time-series properties is one the approaches in the signal modeling and parameter estimation. For example ARMA time-series have wide applications in signal processing such as SONAR signal processing and noise modeling [8, 9]. One of them that has been used in the past decade, Conditional Heteroscedasticity time series was first introduced by [5] in the context of modeling United Kingdom inflation as known Autoregressive Conditional Heteroscedasticity (ARCH). Such models are characterized by being conditionally Gaussian, additionally represented by a non-constant and state-dependent variance. However, in [5, 6, 10], it is shown that a time-varied variance over time is more useful than a constant for modeling non-Gaussian and non-stationary phenomena such as economic series. Generalization of ARCH is proposed in [6] as called Generalized Autoregressive Conditional Het-

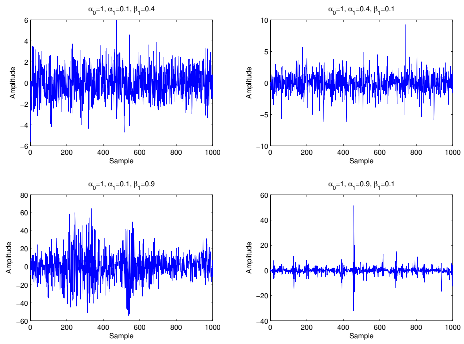

**Fig. 1**. Some realizations of the GARCH(1,1) with different coefficients

eroscedasticity (GARCH), generally speaking, in Heteroscedasticity we consider time series with time varying variance; GARCH models account for heavy tailed PDF as excess kurtosis and volatility clustering a type of heteroscedasticity. Now, we let $\epsilon(k)$ denote a real-valued discrete-time stochastic process, the GARCH $(p, q)$ process is then given by [6],

$$\epsilon(k) \;=\; \eta(k)\sigma(k), \qquad \eta(k) \sim \mathcal{N}(0,1), \tag{1}$$

$$\sigma^2(k) \;=\; \alpha_0^2 + \sum_{i=1}^{q} \alpha_i^2 \epsilon^2(k-i) + \sum_{i=1}^{p} \beta_i^2 \sigma^2(k-i), \tag{2}$$

where $\eta(k)$ is a sequence of independent and identically distributed random variables with zero mean and variance of one, and $\mathcal{N}$ denotes the standard normal probability density function. For example, Figure (1) shows some realizations of the GARCH(1,1) with different coefficients. The flexibility of GARCH process are displayed in this figure, so that some different time series such as impulsive data can be modelled. The estimation of orders $p$ and $q$ has an important role in the GARCH modeling of the time series. In this way, some methods are proposed such as Likelihood Ratio Tests [11], Akaike (AIC) and Bayesian (BIC) information criteria [12]. The Likelihood Ratio Tests would be used to determine to support the use of a specific GARCH model for a time series. In the Akaike and Bayesian information criteria approach, we can compare the alternative models and select the better one for fitting data.

## 3. PROPOSED METHOD

### 3.1. Underwater Noise Modeling

Generally, in a passive system such as SONAR we consider received additive noise that conclude additive noise and interferences. In the underwater environment, these are two major factors that can limit the performance of general methods in the practical experiments. In different applications such as SONAR, the time-varying characteristic is generally due to time-varying nature of the medium channel, environment, noise and interferences [13]. For example, underwater acoustic channel is a time-varying and multi-path channel specially in shallow water. It varies due to differential season, area and situation of sea face. The channel variations can be due to the spatial movement of the source and/or changes in the propagation conditions such as sound speed profile. All this mentioned factors give a stochastic and time-varying nature to the additive noise. As a result of the above time-varying

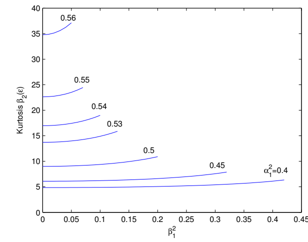

**Fig. 2**. Kurtosis for GARCH(1,1)

events, it can be assumed that the additive noise has time-varying variance in the receiver. Moreover, measurements of the ambient noise in related application such as underwater environment shows that the noise can sometimes be significantly non-Gaussian [3, 4] due to natural and man-made sources such as reverberation and industrial noise. It can be shown for the widely accepted model of additive noise and interference excess kurtosis can be observed. Thus, the assumed noise model that covers the properties of additive noise such as time-varying variance and heavy-tail PDF is more attractive. Under the above assumptions and important features of the GARCH time-series model we use this model for the additive noise modeling in the underwater acoustics applications such as SONAR. At the first of the modeling, we need to the estimation of orders of proposed model, i.e. p and q. In this way, we used both AIC and BIC and so the results of the our simulation almost always reached GARCH(1,1). Therefore,

$$\sigma^2(k) \;=\; \alpha_0^2 + \alpha_1^2 n^2(k-1) + \beta_1^2 \sigma^2(k-1), \tag{3}$$

Generally, the unknown coefficients ($\alpha_0$, $\alpha_1$ and $\beta_0$) are estimated using Maximum Likelihood method[6]. It is well known, kurtosis is an important parameter for analysis of non-Gaussian random processes. It can be shown [6] that if $(\alpha_1^2 + \beta_1^2) < 1$ and $1-(\alpha_1^2 + \beta_1^2)^2 - 2\alpha_1^4 > 0$ then the kurtosis is greater than 3 and GARCH(1,1) can include heavy-tail PDF. Figure (2) show the ability of GARCH modeling for heavy-tail PDF with Excess kurtosis. In the following, we will propose a new direction-finding approach using GARCH noise modeling in the underwater application.

### 3.2. Direction-Finding Approach

It is well known, the performance of the source localization and estimation of DOA in passive array applications such as SONAR are heavily relied upon the particular array signal processing algorithms and additive noise modeling that used in practice. Consequently, we note that in this model, noise is not uniform across $L$ sensors which is a realistic modeling resting on the assumption of non-uniformity [14, 15], and non-stationarity ; i.e., time-varying variance. Thus, the assumed noise model that covers the properties of additive noise is more attractive. Under the above assumption we use the GARCH(1,1) process for the additive noise in direction-finding in array signal processing. Let's assume that a linear array of $L$ omni-directional hydrophones receives $D$ ($D<L$) plane wave impinging from unknown directions of arrivals. The incident plane waves are assume to be narrow-band with a center frequency. Under these conditions, the $k$th snapshot vector of array observation can be expressed as

$$\mathbf{x}(k) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(k) + \mathbf{n}(k), \qquad k = 1, 2, ..., K \tag{4}$$

where $\mathbf{s}(k)$ is the $D\times 1$ vector of the source waveforms, $\mathbf{n}(k)$ is the $L\times 1$ vector of sensor noise, $\mathbf{A}(\boldsymbol{\theta})$ is the $L\times D$ steering matrix, $\boldsymbol{\theta}\triangleq\{\theta_1,...,\theta_D\}^T$ is the $D\times 1$ vector of the unknown signal DOA, $K$ is the number of snapshots, and $(\cdot)^T$ stands for the transpose operation. We make the following assumptions: the signal waveforms are stationary; both temporally and spatially, and the signals and noise are statistically independent of each other. According to the previous noise modeling section, we propose to use the Multivariate GARCH(1,1) for noise modeling in array sensors applications such as SONAR. Thus, using (3) the additive array noise can follow as multivariate GARCH(1,1) with zero mean vector and covariance matrix $\mathbf{Q}(k)$, so:

$$\mathbf{n}(k) \quad \sim \quad \mathcal{M}G(n;\mathbf{0},\mathbf{Q}(k)) \tag{5}$$

where, $\mathcal{M}G$ stands for the multivariate GARCH(1,1), and

$$\mathbf{Q}(k) = diag\{\sigma_1^2(k),\sigma_2^2(k),...,\sigma_L^2(k)\}. \tag{6}$$

In this approach, the additive noise model in every sensors is similar to (3). Consequently, it is well known, one of the efficient methods in the estimation of parameters in array signal processing is the ML approach [9]. In the following, we exploit the Deterministic Maximum Likelihood (DML) approach model so that the signal waveforms are deterministic unknown sequences. Thus, the joint PDF of the observed array snapshots using GARCH(1,1) model is expressed as

$$\begin{aligned}\mathbf{p}_{\boldsymbol{X}|\boldsymbol{\psi}}(X) \quad = \quad & \prod_{k=1}^{K}\frac{1}{\det[\pi\boldsymbol{Q}(\boldsymbol{\psi},k)]}\exp\left\{-[\mathbf{x}(k)-\mathbf{A}(\theta)\mathbf{s}(k)]^H\right.\\ & \left.\mathbf{Q}^{-1}(\boldsymbol{\psi},k)[\mathbf{x}(k)-\mathbf{A}(\theta)\mathbf{s}(k)]\right\}, \end{aligned}\tag{7}$$

where $\boldsymbol{\psi}$ concludes the vectors of the unknown DOAs, signal waveforms and GARCH(1,1) coefficients. Therefore, by using (6) and (7) it can be shown that the following holds for Log-Likelihood[1]:

$$\begin{aligned}L_p(\boldsymbol{\psi}) = & -\sum_{k=1}^{K}\sum_{\ell=1}^{L}\ln(\sigma_\ell^2(k)) - \sum_{k=1}^{K}\{[\mathbf{x}(k)-\mathbf{A}(\theta)\mathbf{s}(k)]^H\\ & \mathbf{Q}^{-1}(\boldsymbol{\psi},k)[\mathbf{x}(k)-\mathbf{A}(\theta)\mathbf{s}(k)]\}\end{aligned}\tag{8}$$

$L_p(\cdot)$ stands for the proposed Log-likelihood function to be maximized over the vector of unknown parameters $\boldsymbol{\psi}$ through ML approach. So, due to the complicated nature of problems, this estimation cannot be found analytically, and $-L_p(\cdot)$ can be minimized through numerical procedures [1] and then unknown parameters are found. For the statistically analysis of proposed method, the CRB is derived in the following section.

## 4. CRAMÉR-RAO BOUND

In order to understand the performance of estimation process using GARCH modeling we develop CRB [1]. If we denote the covariance matrix of the estimation errors by $\mathbf{C}(\boldsymbol{\psi})$, then, the multiple-parameter CRB states that

$$\boldsymbol{C}(\boldsymbol{\psi}) \geq \boldsymbol{CRB}(\boldsymbol{\psi}) \triangleq \boldsymbol{J}^{-1}, \tag{9}$$

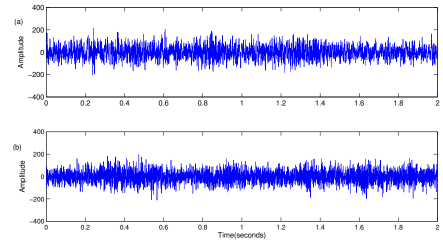


**Fig. 3**. Underwater ambient noise and simulated GARCH(1,1) model (a) measured ambient noise time series and (b) GARCH(1,1) with $\alpha_0^2$=343.6, $\alpha_1^2$=0.84, $\beta_1^2$=0.06

for any unbiased estimate of $\boldsymbol{\psi}$. The $\mathbf{J}$ matrix is commonly referred to as Fisher's information matrix with the following elements

$$J_{ij} \triangleq E\left\{\frac{\partial L(\boldsymbol{\psi})}{\partial\psi_i}\cdot\frac{\partial L(\boldsymbol{\psi})}{\partial\psi_j}\right\} \tag{10}$$

For $k$th single snapshot problem, the $\mathbf{J}$ is obtained from

$$\begin{aligned}J_{ij} = tr\left\{\mathbf{Q}^{-1}(\boldsymbol{\psi})\frac{\partial\mathbf{Q}(\boldsymbol{\psi})}{\partial\psi_i}\mathbf{Q}^{-1}(\boldsymbol{\psi})\frac{\partial\mathbf{Q}(\boldsymbol{\psi})}{\partial\psi_j}\right\} + \\ 2Re\left\{\frac{\partial\mathbf{m}^H(\boldsymbol{\psi})}{\partial\psi_i}\mathbf{Q}^{-1}(\boldsymbol{\psi})\frac{\partial\mathbf{m}(\boldsymbol{\psi})}{\partial\psi_j}\right\}\end{aligned}$$

where,

$$\mathbf{m}(\boldsymbol{\psi}) = \ \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(k). \tag{11}$$

In our proposed method based on GARCH noise model in ML estimation, we have $\mathbf{J}$ as a partitioned matrix,

$$\mathbf{J} = \begin{pmatrix}\mathbf{J}_{\theta\theta} & \mathbf{J}_{\theta s} & \mathbf{J}_{\theta g}\\ \mathbf{J}_{s\theta} & \mathbf{J}_{ss} & \mathbf{J}_{sg}\\ \mathbf{J}_{g\theta} & \mathbf{J}_{gs} & \mathbf{J}_{gg}\end{pmatrix}$$

then, for the DOA estimation, the CRB is computed as

$$\mathbf{CRB}(\boldsymbol{\theta}) = \left\{\mathbf{J}_{\theta\theta} - \left[\mathbf{J}_{\theta s}\mathbf{J}_{\theta g}\right]\mathbf{J}_F^{-1}\left[\mathbf{J}_{s\theta}\mathbf{J}_{g\theta}\right]^T\right\}^{-1} \tag{12}$$

where,

$$\mathbf{J}_F = \begin{pmatrix}\mathbf{J}_{ss} & \mathbf{J}_{sg}\\ \mathbf{J}_{gs} & \mathbf{J}_{gg}\end{pmatrix}.$$

For estimation of CRB, All of the blocks of the above matrixes should be computed. In this general way, we also obtain them to analyze of the statistical performance of the proposed GARCH based method.

## 5. SIMULATION AND RESULTS

In this section, we demonstrate the performance of the proposed approach for modeling of the ambient noise in passive sonar with two major experiments. In the former, we use the recorded ambient noise with one hydrophone in shallow water. In this scenario, order selection and estimation of PDFs of the real and simulated data are considered. Before modeling process, we exploit the available approaches [11, 12] for the estimation of GARCH orders p and q and so find that p=1 and q=1 are sufficient orders for this experiment. Figure 3

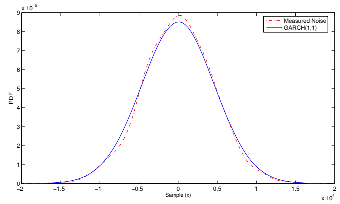


**Fig. 4**. Measured noise and simulated GARCH(1,1) PDF

shows one of the time series of the measured noise and simulated noise with GARCH model. For the statistical comparison of proposed model, the probability of density function(PDF) is estimated for the real and simulated noises and shown in figure 4. In the latter experiment, we use an uniform linear array (ULA) with six half-wavelength inter-element spacing sensors and equally powered narrow-band sources with DOA $= [6.5\,°, 16.5\,°]$ relative to the broadside. In this scenario, the DOA estimation root-mean-square errors (RMSE) of the proposed method (i.e. GML), MUSIC and deterministic ML (DML) results have been compared with derived CRBs, versus different values of SNR and number of snapshots and are shown in figures 5. In this experiment, we considered the collected underwater ambient noise in the shallow waters of Persian Gulf for the performance analysis of the proposed method. In this way, we see that GARCH(1,1) is an appropriate choice for the modeling of the underwater ambient noise and observe that the proposed method has resolved the targets better than the other methods.

## 6. CONCLUSION

In this paper we propose a new method for the underwater ambient noise modeling and the direction-finding using GARCH time series. Measurements of ambient noise in the underwater environments such as shallow waters show that the realistic noise has non-stationary and non-Gaussian nature. As a result, the related methods that utilize the Gaussian assumption, degrade in the real experiments. In this way, a flexible model based on Heteroscedasticity time series is offered that can be used to noise modeling in underwater applications. This model so-called GARCH process accounts for heavy tails PDFs with excess kurtosis and time-varying variances a type of heteroscedasticity. In this way, we utilized GARCH noise modeling in the ML approach to estimate DOAs of sources. For the evaluation of the proposed method, measured underwater ambient noise are used. The results of these simulations verify that the proposed method is suitable for ambient noise modeling and so high-resolution source localization in the realistic underwater environment like shallow waters.

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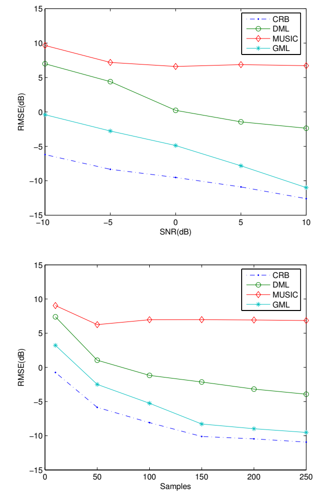


**Fig. 5**. Root-Mean-Square-Error and Cramér-Rao Bound for Deterministic Maximum Likelihood (DML), MUSIC, and proposed method (GML) two targets in underwater ambient noise, (Top) vs. SNR(dB), snapshots=100, (Down) vs. Samples, SNR=0 dB


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