OPTIMAL ARRAY RECEIVER FOR SYNCHRONIZATION OF A BPSK SIGNAL CORRUPTED BY NON CIRCULAR INTERFERENCES

Pascal Chevalier and François Pipon

Thales-Communications, EDS/SPM/SBP, 160 Bd Valmy – BP 82, 92704 Colombes Cedex, France
pascal.chevalier@fr.thalesgroup.com, francois.pipon@fr.thalesgroup.com

ABSTRACT

The synchronization problem in the presence of interferences has been strongly studied these last two decades, mainly to mitigate the multiple access interferences from other users in DS/CDMA systems. Most of the available receivers are very specific of the CDMA context and only some scarce receivers may also be used in other contexts such as F/TDMA systems. However, these receivers assume implicitly or explicitly circular interferences and become sub-optimal for non circular (or improper) interferences, characteristic in particular of radiocommunications networks using rectilinear (or monodimensional) modulations such as BPSK modulation. For this reason, the purpose of this paper is to introduce and to analyze the performance of the optimal array receiver for synchronization of BPSK signals corrupted by non circular interferences. This receiver implements an optimal, in a least square sense, widely linear filtering of the data, mainly considered up to now for demodulation purposes of BPSK signals, followed by a correlation operation. Extensions to MSK and GMSK signals are presented elsewhere.

1. INTRODUCTION

The acquisition or synchronization problem in the presence of interferences has been strongly studied these last two decades, mainly to mitigate the multiple access interferences (MAI) from other users in DS/CDMA systems, from both mono-antenna [1] [6] [12] and multi-antennas [2] [5] [11] receivers. Most of the available receivers are very specific of the CDMA context since they require assumptions such as a spreading sequence which is repeated at each symbol [1] [6] [12], a very large number of MAI [11] or no data on the codes [2] [11]. Only a few scarce receivers may be used in other contexts such as F/TDMA systems, among which we find the maximum likelihood (ML) array receivers presented in [5], which generalize the results of [2]. However, ML receivers presented in [5] assume explicitly circular Gaussian interferences and become sub-optimal for non circular [7] (or improper [9]) interferences, characteristic in particular of radiocommunications networks using rectilinear (or monodimensional) modulations such as BPSK modulation. Note that such a modulation is still of interest for various current wireless systems as explained in [9]. For this reason, the purpose of this paper is to introduce and to analyze the performance of the optimal array receiver for synchronization of BPSK signals corrupted by non circular, and more precisely by rectilinear interferences. This receiver, patented recently [4], implements an optimal, in a least square sense, widely linear (WL) [8] spatial filtering of the data followed by a correlation operation with a training sequence. Extensions to MSK and GMSK signals are still available and are described in [4]. Note that although optimal WL filtering has raised an increasing interest during this last decade for demodulation of BPSK, MSK or GMSK signals corrupted by non circular interferences (see [3] [9] and references herein), it has never been considered, up to now and to our knowledge, for synchronization purposes.

2. HYPOTHESES AND PROBLEM FORMULATION

2.1. Hypotheses

We consider an array of N narrow-band (NB) sensors receiving the contribution of a BPSK useful signal and a total noise composed of rectilinear interferences and background noise. The complex envelope of the useful signal is given by

$$s(t) \;=\; \mu_s \sum_n a_n\, v(t - nT) \qquad (1)$$

where $a_n = \pm 1$ are i.i.d random variables corresponding to the transmitted symbols, T is the symbol duration, $v(t)$ is a raised cosine pulse shaping filter (1/2 Nyquist filter) and μ_s is a real value which controls the instantaneous power of $s(t)$. Noting $\boldsymbol{x}(t)$ the vector of complex amplitudes of the signals at the output of the sensors, the sampled observation vector, $\boldsymbol{x}_v(kT_e) \overset{\Delta}{=} \boldsymbol{x}(t) \otimes v(-t)^*/_{t=kT_e}$, obtained after a matched filtering operation to the pulse shaping filter $v(t)$, is given by

$$\boldsymbol{x}_v(kT_e) \;=\; s_v((k - l_o)T_e)\, \boldsymbol{h}_s + \boldsymbol{b}_{Tv}(kT_e)$$
$$= s_v((k - l_o)T_e)\, \boldsymbol{h}_s + \sum_{p=1}^{P} m_{pv}(kT_e)\, \boldsymbol{h}_p + \boldsymbol{b}_v(kT_e) \qquad (2)$$

where T_e is the sample period such that T/T_e is an integer q, $\otimes$ and * are the convolution and the complex conjugation operations respectively, $s_v(kT_e) \triangleq s(t)\otimes v(-t)^*/_{t=kT_e}$, $m_{pv}(kT_e)$ is the sampled complex envelope of interference p, assumed rectilinear, at the output of the matched filter $v(-t)^*$, $\boldsymbol{b}_{Tv}(kT_e)$ and $\boldsymbol{b}_v(kT_e)$ are the sampled total noise and background noise vectors respectively at the output of the matched filter $v(-t)^*$, $\boldsymbol{h}_s$ and $\boldsymbol{h}_p$ are the channel impulse response vectors of the useful signal and interference p respectively, P is the number of interferences and $l_o T_e$ is the propagation delay of the useful signal. Note that model (2) seems to assume propagation channels with no delay spread (flat fading or free space propagation) but is still valid in the presence of delay spread (selective fading) by considering useful paths associated with delays different from $l_o T_e$ as interferences.

2.2. Second order statistics of the data

The second order (SO) statistics of the data considered in the following correspond to the first, $R_x(k)$, and second, $C_x(k)$, correlation matrix of $\boldsymbol{x}_v(kT_e)$, defined, under the previous assumptions and assuming uncorrelated total noise and useful signal, by

$$R_x(k) \triangleq \mathrm{E}[\boldsymbol{x}_v(kT_e)\, \boldsymbol{x}_v(kT_e)^\dagger] \approx \pi_s(k - l_o)\, \boldsymbol{h}_s\, \boldsymbol{h}_s^\dagger + R(k) \quad (3)$$

$$C_x(k) \triangleq \mathrm{E}[\boldsymbol{x}_v(kT_e)\, \boldsymbol{x}_v(kT_e)^\mathrm{T}] \approx \pi_s(k - l_o)\, \boldsymbol{h}_s\, \boldsymbol{h}_s^\mathrm{T} + C(k) \quad (4)$$

where $^\mathrm{T}$ and † correspond to the transposition and transposition conjugation operation respectively, $\pi_s(k) \triangleq \mathrm{E}[|s_v(kT_e)|^2]$ is the instantaneous power of the useful signal received by an omnidirectional sensor for a free space propagation, $R(k) \triangleq \mathrm{E}[\boldsymbol{b}_{Tv}(kT_e)\ \boldsymbol{b}_{Tv}(kT_e)^\dagger]$ and $C(k) \triangleq \mathrm{E}[\boldsymbol{b}_{Tv}(kT_e)\ \boldsymbol{b}_{Tv}(kT_e)^\mathrm{T}]$ are the first and second correlation function of the total noise vector respectively, given by

$$R(k) \approx \sum_{p=1}^{P} \pi_p(k)\, \boldsymbol{h}_p\, \boldsymbol{h}_p^\dagger + \eta_2\, \mathrm{I} \quad (5)$$

$$C(k) \approx \sum_{p=1}^{P} \pi_p(k)\, \boldsymbol{h}_p\, \boldsymbol{h}_p^\mathrm{T} \quad (6)$$

where $\pi_p(k) \triangleq \mathrm{E}[|m_{pv}(kT_e)|^2]$ is the instantaneous input power of interference p and η_2 is the mean power of the background noise per sensor, assumed spatially white and stationary. Note that the SO statistics of the data are periodic functions of k, with period T, since the useful signal is a cyclostationary BPSK signal with symbol duration T and we assume that the total noise has also this property, which is in particular the case in the presence of interferences generated by the network itself.

2.3. Problem formulation

In a radiocommunication system, training sequences are generally periodically transmitted for synchronization purposes, which means that the useful signal $s_v(kT_e)$ is known by the receiver over some given time intervals of K symbols, where qK is the number of samples of the training sequence. In such a context, assuming in this paper that $R(k)$, $C(k)$ and $\boldsymbol{h}_s$ are unknown, the optimal synchronization problem consists to find the best estimate, $\hat{l}_o$, of l_o from the observed data, $\boldsymbol{x}_v(kT_e)$, and the knowledge of $s_v(kT_e)$, for $0 \le k \le qK - 1$. This best estimate, $\hat{l}_o$, of l_o also corresponds to the delay l for which the known useful samples $s_v(kT_e)$, for $0 \le k \le qK - 1$, are optimally detected from the observation vectors $\boldsymbol{x}_v((k + l)T_e)$, $0 \le k \le qK - 1$. We solve this detection problem in the next section while the problem solved in [5] is an estimation one, done under more restrictive hypotheses.

3. OPTIMAL SYNCHRONIZATION

3.1. Presentation

We consider the optimal synchronization time $l_o T_e$ and the detection problem with two hypotheses H0 and H1, where H0 and H1 correspond to the presence of total noise only and signal plus total noise into $\boldsymbol{x}_v((k + l_o)T_e)$ respectively. Under these two hypotheses and using (2), the observation vector $\boldsymbol{x}_v((k + l_o)T_e)$ can be written as

$$\text{H1:}\ \boldsymbol{x}_v((k + l_o)T_e) = s_v(kT_e)\, \boldsymbol{h}_s + \boldsymbol{b}_{Tv}((k + l_o)T_e) \quad (7)$$

$$\text{H0:}\ \boldsymbol{x}_v((k + l_o)T_e) = \boldsymbol{b}_{Tv}((k + l_o)T_e) \quad (8)$$

Under these assumptions, it is well-known that the receiver which implements the best detection of $s_v(kT_e)$ from $\boldsymbol{x}_v((k + l_o)T_e)$ over the training sequence duration is the likelihood ratio (LR) receiver, which consists to compare to a threshold the function $\mathrm{L}(\boldsymbol{x}_v)(K, l_o)$ defined by

$$\mathrm{L}(\boldsymbol{x}_v)(K, l_o) \triangleq \frac{\mathrm{p}[\boldsymbol{x}_v((k{+}l_o)T_e),\ 0 \le k \le qK - 1,\ /\ \mathrm{H1}]}{\mathrm{p}[\boldsymbol{x}_v((k{+}l_o)T_e),\ 0 \le k \le qK - 1,\ /\ \mathrm{H0}]} \quad (9)$$

where $\mathrm{p}[\boldsymbol{x}_v((k{+}l_o)T_e),\ 0 \le k \le qK - 1,\ /\ \mathrm{H}i]$ ($i = 0, 1$) is the conditional probability density of $[\boldsymbol{x}_v(l_o T_e), \boldsymbol{x}_v((1{+}l_o)T_e),\ldots, \boldsymbol{x}_v((qK{+}l_o{-}1)T_e)]^\mathrm{T}$ under hypothesis Hi. To ensure the stationarity of the total noise vector over the training sequence duration, we only consider one sample of this vector per symbol over this duration, i.e. vectors $\boldsymbol{b}_{Tv}((n + l_o/q)T)$, $0 \le n \le K - 1$. Moreover, we assume that these vectors are Gaussian (in order to exploit the SO statistics of the data only), non circular (since we consider rectilinear interferences) and uncorrelated to each other. Under these assumptions, the probability density of $\boldsymbol{b}_{Tv}((n + l_o/q)T)$ becomes a function of $\boldsymbol{b}_{Tv}((n + l_o/q)T)$ and $\boldsymbol{b}_{Tv}((n + l_o/q)T)^*$ given by [10] :

$$\mathrm{p}[\tilde{\boldsymbol{b}}_{Tv}((l_o/q + n)T)] \triangleq \pi^{-N} \det[R_{\tilde{b}}(l_o)]^{-1/2} \quad (10)$$
$$\exp[-(1/2)\, \tilde{\boldsymbol{b}}_{Tv}((l_o/q + n)T)^\dagger R_{\tilde{b}}(l_o)^{-1}\, \tilde{\boldsymbol{b}}_{Tv}((l_o/q + n)T)]$$

and the LR (9) takes the form

$$L(\boldsymbol{x}_v)(K, l_o) \triangleq \frac{\prod_{n=0}^{K-1} p[\widetilde{\boldsymbol{b}}_{Tv}((l_o/q + n)T) = \widetilde{\boldsymbol{x}}_v((l_o/q + n)T) - s_v(nT)\, \widetilde{\boldsymbol{h}}_s]}{\prod_{n=0}^{K-1} p[\widetilde{\boldsymbol{b}}_{Tv}((l_o/q + n)T) = \widetilde{\boldsymbol{x}}_v((l_o/q + n)T)]} \quad (11)$$

where $\widetilde{\boldsymbol{b}}_{Tv}(kT_e) \triangleq [\boldsymbol{b}_{Tv}(kT_e)^T, \boldsymbol{b}_{Tv}(kT_e)^{\dagger}]^T$, $\widetilde{\boldsymbol{x}}_v(kT_e) \triangleq [\boldsymbol{x}_v(kT_e)^T, \boldsymbol{x}_v(kT_e)^{\dagger}]^T$, $\widetilde{\boldsymbol{h}}_s \triangleq [\boldsymbol{h}_s^T, \boldsymbol{h}_s^{\dagger}]^T$, $s_v(nT) = \mu_s\, r(0)\, a_n$, $r(t) \triangleq v(t) \otimes v(-t)^*$ is a Nyquist filter and where $R_{\widetilde{b}}(l_o) \triangleq E[\widetilde{\boldsymbol{b}}_{Tv}((l_o/q + n)T)\, \widetilde{\boldsymbol{b}}_{Tv}((l_o/q + n)T)^{\dagger}]$ brings together the information contained in both $R(l_o)$ and $C(l_o)$. As $R_{\widetilde{b}}(l_o)$ and $\widetilde{\boldsymbol{h}}_s$ are unknown, they have to be replaced in (11) by their ML estimate. In these conditions, it is possible to show, after some tedious computations (presented elsewhere due to the lack of place), that a sufficient statistic for the optimal detection of the BPSK signal $s_v(nT)$, $0 \le n \le K-1$, from $\widetilde{\boldsymbol{x}}_v((l_o/q + n)T)$, $0 \le n \le K-1$, in a non circular Gaussian total noise is given by

$$\hat{C}_{nc}(l_o) \triangleq \frac{\hat{\boldsymbol{r}}_{\widetilde{x}s}(l_o)^{\dagger}\, \hat{R}_{\widetilde{x}}(l_o)^{-1}\, \hat{\boldsymbol{r}}_{\widetilde{x}s}(l_o)}{(1/K) \sum_{n=0}^{K-1} |s(nT)|^2} \quad (12)$$

where the vector $\hat{\boldsymbol{r}}_{\widetilde{x}s}(l_o)$ and the matrix $\hat{R}_{\widetilde{x}}(l_o)$ are given by

$$\hat{\boldsymbol{r}}_{\widetilde{x}s}(l_o) \triangleq \frac{1}{K} \sum_{n=0}^{K-1} \widetilde{\boldsymbol{x}}_v((l_o/q + n)T)\, s(nT)^* \quad (13)$$

$$\hat{R}_{\widetilde{x}}(l_o) \triangleq \frac{1}{K} \sum_{n=0}^{K-1} \widetilde{\boldsymbol{x}}_v((l_o/q + n)T)\, \widetilde{\boldsymbol{x}}_v((l_o/q + n)T)^{\dagger} \quad (14)$$

and where $0 \le \hat{C}_{nc}(l_o) \le 1$. We deduce from this result that the optimal synchronization consists to compute, at each sample time lT_e, the quantity $\hat{C}_{nc}(l)$ and to compare it to a threshold. Synchronization is then obtained for the sample time $lT_e = \hat{l}_o T_e$, which generates the maximum value of $\hat{C}_{nc}(l)$ among those which are over the threshold. Note that if we a priori assume a circular total noise ($C(l_o) = 0$), we find that the optimal synchronization corresponds, in this case, to the previous one but where $\widetilde{\boldsymbol{x}}_v((l_o/q + n)T)$ has to be replaced by $\boldsymbol{x}_v((l_o/q + n)T)$ in (12), noted in this case $\hat{C}_c(l_o)$. This result has already been obtained in [2] and [5] in the specific context of CDMA networks but for orthogonal and periodic training sequences.

3.2. Interpretation

To give an enlightening interpretation of (12), we introduce the $(2N \times 1)$ WL spatial filter, $\hat{\widetilde{\boldsymbol{w}}}(l) \triangleq [\hat{\boldsymbol{w}}(l)^T, \hat{\boldsymbol{w}}(l)^{\dagger}]^T \triangleq \hat{R}_{\widetilde{x}}(l)^{-1}\, \hat{\boldsymbol{r}}_{\widetilde{x}s}(l)$, whose output, for the observation $\widetilde{\boldsymbol{x}}_v((l/q + n)T)$, is given by $y((l/q + n)T) \triangleq \hat{\widetilde{\boldsymbol{w}}}(l)^{\dagger} \widetilde{\boldsymbol{x}}_v((l/q + n)T)$ and has real values. It is then straightforward to verify that $\hat{C}_{nc}(l)$ can be written as

$$\hat{C}_{nc}(l) \triangleq \frac{(1/K) \sum_{n=0}^{K-1} y((l/q + n)T)\, s(nT)^*}{(1/K) \sum_{n=0}^{K-1} |s(nT)|^2} \quad (15)$$

which corresponds, to within a normalization factor, to the result of the correlation between the training sequence and the output of $\hat{\widetilde{\boldsymbol{w}}}(l)$. The WL filter $\hat{\widetilde{\boldsymbol{w}}}(l)$ corresponds to a least square estimate of the WL filter, $\widetilde{\boldsymbol{w}}(l) \triangleq R_{\widetilde{x}}(l)^{-1} \boldsymbol{r}_{\widetilde{x}s}(l)$, which minimizes the mean square error between $s(nT)$ and the output $\widetilde{\boldsymbol{w}}^{\dagger} \widetilde{\boldsymbol{x}}_v((l/q + n)T)$, where $R_{\widetilde{x}}(l) \triangleq E[\widetilde{\boldsymbol{x}}_v((l/q + n)T)\, \widetilde{\boldsymbol{x}}_v((l/q + n)T)^{\dagger}]$ and $\boldsymbol{r}_{\widetilde{x}s}(l) \triangleq E[\widetilde{\boldsymbol{x}}_v((l/q + n)T)\, s(nT)^*] = \mu_s^2\, r(0)\, r((l - l_o)T_e)\, \widetilde{\boldsymbol{h}}_s$. When l is far from l_o, $r((l - l_o)T_e)$ and then $\widetilde{\boldsymbol{w}}(l)$ are very weak quantities closed to zero, which generate weak values of $\hat{C}_{nc}(l)$. As l approaches l_o, $r((l - l_o)T_e)$ and $\hat{C}_{nc}(l)$ increase and reach their maximum value for $l = l_o$. In this case, $\widetilde{\boldsymbol{w}}(l)$ becomes proportional to the WL spatial matched filter (SMF), $\widetilde{\boldsymbol{w}}_{wlsmf}(l_o) \triangleq R_{\widetilde{b}}(l_o)^{-1} \widetilde{\boldsymbol{h}}_s$, i.e to the WL spatial filter which maximizes the output Signal to Interference plus Noise Ratio (SINR) [3]. This filter discriminates the rectilinear sources by the direction of arrival (DOA) (if $N > 1$) and by the phase [3] and allows in particular single antenna interference cancellation (SAIC) as soon as there exists a phase discrimination between the rectilinear sources, as shown in [3]. On the contrary, under the a priori assumption of a circular total noise, as $\widetilde{\boldsymbol{x}}_v((l_o/q + n)T)$ has to be replaced by $\boldsymbol{x}_v((l_o/q + n)T)$ in (12), the WL SMF previously described has to be replaced by the well-known conventional SMF, $\boldsymbol{w}_{smf}(l_o) \triangleq R(l_o)^{-1} \boldsymbol{h}_s$, which discriminates the sources by the DOA only and which does not allow SAIC.

4. PERFORMANCE OF OPTIMAL RECEIVER

Let us recall that the false alarm rate (FAR) for a given receiver is defined by the probability that the receiver output goes beyond the threshold in the absence of useful signal. Then, to compute and illustrate the performance of both optimal ($\hat{C}_{nc}(l)$) and conventional ($\hat{C}_c(l)$) receivers, we first consider a mono-sensor reception ($N = 1$) and we assume that the useful BPSK signal, received with a Signal to Noise Ratio (SNR) equal to 5 dB, is perturbed by one BPSK interference with an Interference to Noise Ratio (INR) equal to 20 dB. The training sequence is assumed to contain $K = 64$ symbols. Under these assumptions, figure 1 shows the variations of $\hat{C}_c(l_o)$ and $\hat{C}_{nc}(l_o)$ as a function of the phase difference, ψ, between the useful signal and interference, jointly with the thresholds, β_c and β_o, associated with these two receivers for a FAR equal to 0.001. Note the weak value of $\hat{C}_c(l_o)$, almost always below the threshold, whatever this phase difference, preventing from synchronizing the useful signal from the conventional receiver. Note also the values of $\hat{C}_{nc}(l_o)$ beyond the threshold, as soon as this phase difference is not too low, allowing in most cases the synchronization of the useful signal from the optimal receiver even from $N = 1$ sensor. To quantify more precisely the performance of the previous receivers, we consider a burst radiocommunication link for which a training sequence with $K = 64$ symbols is transmitted at each burst. The BPSK useful signal is assumed to be corrupted by one BPSK interference whose INR is equal to 20 dB. The phase and DOA of the sources are random, uniformly distributed between $[0, 2\pi]$ and are assumed to change randomly at each burst. The performance are evaluated over 100000 bursts. Under these assumptions, figure 2 shows the variations of the probability of non synchronization at the output of the conventional (C) and the optimal (NC) receivers as a function of the SNR, for a FAR equal to 0.001 and for several values of the number of sensors. Note that, for $N = 1$, much better performance are achieved by the optimal receiver with respect to the conventional one, especially for low input SINR, due to its capability to reject the interference. Note, for $2 \leq N \leq 4$, the better performance reached by the optimal receiver, due to a better discrimination between the sources and despite of the fact that the conventional receiver spatially rejects the interference.

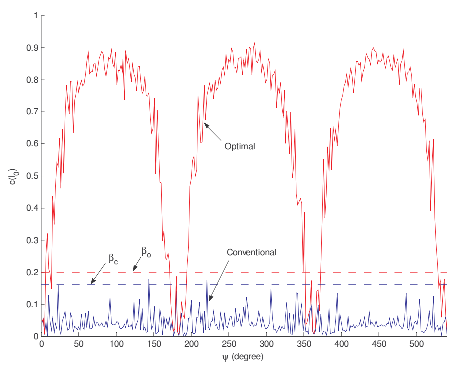

Figure 1 : *$\hat{C}_c(l_o)$ and $\hat{C}_{nc}(l_o)$ as a function of the phase of $h_s{}^*h_1$, $N = 1$, $P = 1$, SNR = 5 dB, INR = 20 dB*

5. CONCLUSION

It has been shown in this paper that taking the non circularity property of rectilinear interferences into account may dramatically improve the performance of both mono and multi-channel receivers for the synchronization of a BPSK signal in a radiocommunication network using this modulation. It allows in particular Single Antenna Interference Cancellation and, for given performances, a reduction of the number of sensors by a factor at least equal to two. These results also hold for MSK and GMSK modulations, as explained in [4], and will be presented in a future paper.

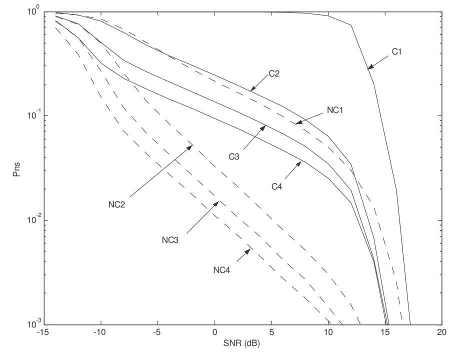

Figure 2 : *Probability of non synchronization as a function of SNR, N = 1, 2, 3, 4, P = 1, INR = 20 dB, FAR = 0.001, Phases and DOA are random, 100000 realisations*

6. REFERENCES

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