WARD: A Weighted Array Data Scheme for Subspace Processing in Impulsive Noise

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ABSTRACT

This paper is concerned with robust array signal processing in impulsive noise environments. A simple weighting signal is defined to weight all sensor data in a snapshot-by-snapshot way, so that the resulting array data have the desired statistical characteristics used in the subspace-based direction-of-arrival (DOA) estimation techniques. Then any traditional subspace-based technique can be used for DOA estimation. In working example, the MUSIC algorithm is employed and a Weighted ARray Data-based MUSIC (WARD-MUSIC) algorithm is detailed. Simulations show the performance advantages of the WARD processing over other related methods in Gaussian mixture noise and symmetric α-stable noise.

1. INTRODUCTION

Direction-of-arrival (DOA) estimation is one of the important research areas of sensor array processing, which is drawn considerable interest for more than three decades (see [1] and references therein). Numerous high-resolution DOA estimation methods have been developed, among which the subspace-based methods received special attention. Typical well known examples of this type of methods include the MUSIC [2] and ESPRIT [3]. A common assumption made in those methods is that the array signals are corrupted by additive, independent and identically distributed (iid), Gaussian noise. Then it is possible to separate signal- and noise- subspaces and estimate the source DOAs using the second-order statistics of the array measurements.

Although the Gaussian distributions and processes can model large types of signals and noises, in practical applications, the Gaussian noise assumption proves inappropriate [4]-[6]. The noise processes in wireless communication channels, underwater acoustics and other applications can not be well described by the Gaussian model [5]. These processes have been shown to be non-Gaussian and/or impulsive, or contain outliers in nature. The finite Gaussian-mixture noise model [11] and the symmetric α-stable ($S\alpha S$) distribution [7] are often used to describe those impulsive processes.

Since the conventional methods perform poorly in impulsive noise environment, the development of robust DOA estimation methods in such noise environment received much attention. Some methods are claimed to work in different non-Gaussian noise, while some methods are specifically developed for certain non-Gaussian noise. The data-adaptive zero-memory nonlinearity (DA-ZMNL) pre-processing method in [8] and the nonlinearly weighted least squares methods in [12] are general and can perform the DOA estimations for both noise models. The methods in [9] and [10] using FLOMs can be applied in the $S\alpha S$ noise. In [13], with the assumption of spherically symmetric array noise [14], a subspace-based algorithm using spatial sign covariance matrix (SCM-MUSIC) is derived from the Euclidian norm and is studied to exhibit high-resolution performance. Subspace-based method by the optimal maximum likelihood (ML) formulation and interpolation for Gaussian mixture noise is developed in [11].

In this paper, we presented a novel scheme for subspace-based DOA estimation. Under the assumption that the noise has symmetric probability density function[1] (pdf), we find that the array output can be weighted by a single weighting signal, so that the resulting data have the statistical characteristics required by the traditional subspace-based methods. Then the subspace-based methods can be used to perform DOA estimations as usual. In working example, the weighting scheme is used with the MUSIC algorithm and a Weighted ARray Data-based MUSIC (WARD-MUSIC) algorithm is presented. Simulations show that the WARD-MUSIC exhibits performance superior to other related methods in Gaussian mixture noise and $S\alpha S$ noise.

In this paper, we use some notational conventions shown as follows

A^T = the transpose of maxtix A

A^* = the conjugate of matrix A

A^H = the conjugate transpose of matrix A

$E\{\cdot\}$ = the expectation operator

2. PROBLEM STATEMENT

[1] In this paper, we assume that a complex random variable has symmetric pdf if its real and imaginary parts have independent, identically symmetric pdf.

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We assume a uniformly spaced linear array of L sensors receiving P narrow-band plane wave signals from far-field emitters with the same known center frequency and different directions. Then the array output vector, at the tth observation instant, called the tth snapshot, can be expressed as

$$\mathbf{x}(t)=\mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t)+\mathbf{n}(t) \qquad (1)$$

where $\boldsymbol{\theta}$ is the $P\times 1$ direction vector representing the DOAs of the P received signals, $\mathbf{s}(t)$ is the $P\times 1$ waveform vector of incident signals, $\mathbf{n}(t)$ is the $L\times 1$ vector of additive noises, and $\mathbf{A}(\boldsymbol{\theta})$ is the direction matrix given by $\mathbf{A}(\boldsymbol{\theta})=\left[\mathbf{a}(\theta_1),\mathbf{a}(\theta_2),\ldots,\mathbf{a}(\theta_P)\right]$ with the steering vector, $\mathbf{a}(\theta)$, corresponding to the DOA θ, as

$$\mathbf{a}(\theta)=\left[1,e^{-j2\pi\frac{d}{\lambda}\sin\theta},\ldots,e^{-j2\pi(L-1)\frac{d}{\lambda}\sin\theta}\right]^T \qquad (2)$$

where d denotes the sensor spacing and λ denotes the wavelength.

Our objective is to estimate the DOAs, $\theta_1,\theta_2,\ldots,\theta_P$, of the incident signals from the N snapshots of the array $\mathbf{x}(1),\mathbf{x}(2),\ldots,\mathbf{x}(N)$.

Toward this aim, we are going to make the following assumptions regarding the array, the signals and the noise:

AS1 The number of signals is known and is less than the number of sensors, i.e.,$P < L$.

AS2 The set of any P steering vectors is linearly independent.

AS3 The incident signals are uncorrelated zero mean stationary complex processes having symmetric pdfs.

AS4 The additive noise signals are iid complex processes with symmetric pdfs.

AS5 The noise and incident signals are uncorrelated.

Note that except the assumption AS4, other assumptions are common in DOA estimation problems. Assumption AS4 is general, including large types of non-Gaussian noises. The Gaussian noise is a special case of the assumption. For the Gaussian noise, the assumptions AS3 and AS4 together imply that the observed array data are zero-mean processes with the covariance matrix given by

$$\mathbf{R}=E\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\}=\mathbf{A}\mathbf{R}_s\mathbf{A}^H+\mathbf{R}_n \qquad (3)$$

where

$$\mathbf{R}_s=\operatorname{diag}\left\{\sigma_1^2,\sigma_2^2,\cdots,\sigma_P^2\right\} \qquad (4)$$

$$\mathbf{R}_n=\sigma_n^2\mathbf{I} \qquad (5)$$

$\sigma_p^2=E\left\{\left|s_p(t)\right|^2\right\}$ denotes the variance of the pth signal, and σ_n^2 is the noise power at each sensor. By exploiting the eigen-structure of (3), a number of subspace-based methods can be developed [1] .

3. THE SUBSPACE BASED DOA ESTIMATION USING WEIGHTED ARRAY DATA

As mentioned above, the traditional subspace-based methods are not suitable when the additive noise is impulsive. To realize the DOA estimation using the subspace-based techniques, we weight array data by a weighting function. For array data $\mathbf{x}(t)$, we define a weighting signal as

$$w(t)=\frac{1}{\max\left(\left|x_1(t)\right|,\left|x_2(t)\right|,\cdots,\left|x_L(t)\right|\right)} \qquad (6)$$

Then the weighted array data is given by

$$\begin{aligned}\mathbf{y}(t)&=w(t)\mathbf{x}(t)\\&=\mathbf{A}(\boldsymbol{\theta})w(t)\mathbf{s}(t)+w(t)\mathbf{n}(t)\\&=\mathbf{A}(\boldsymbol{\theta})\mathbf{r}(t)+\mathbf{m}(t)\end{aligned} \qquad (7)$$

where $\mathbf{r}(t)=\left[r_1(t),r_2(t),\cdots,r_P(t)\right]^T$ and $\mathbf{m}(t)=\left[m_1(t),m_2(t),\cdots,m_L(t)\right]^T$, with $r_p(t)=w(t)s_p(t)$ and $m_i(t)=w(t)n_i(t)$. $r_p(t)$ and $m_i(t)$ are called as weighted signal and weighted noise, respectively. It is proved theoretically in [15] that the weighted array data for the array noise with symmetric pdfs have the following properties.

P1 The weighted array signal and weighted array noise are of zero mean, i.e.,

$$E\left\{\mathbf{r}(t)\right\}=\mathbf{0} \text{ and } E\left\{\mathbf{m}(t)\right\}=\mathbf{0} \qquad (8)$$

P2 The weighted array noises of different sensors are uncorrelated and have identical variances, i.e.,

$$E\left\{\mathbf{m}(t)\mathbf{m}^H(t)\right\}=\eta_m^2\mathbf{I} \qquad (9)$$

where $\eta_m^2<\infty$ is the weighted noise power at each sensor.

P3 The weighted array signals of different sensors are uncorrelated and have finite variances, i.e.,

$$E\left\{\mathbf{r}(t)\mathbf{r}^H(t)\right\}=\operatorname{diag}\left\{\eta_1^2,\eta_2^2,\cdots,\eta_P^2\right\} \qquad (10)$$

where $\eta_p^2=E\left\{\left|r_p(t)\right|^2\right\}<\infty$ denotes the variance of the pth weighted signal.

P4 The weighted array noises and signals are uncorrelated, i.e.,

$$E\left\{r_p(t)m_i^*(t)\right\}=0,\quad \begin{matrix}p=1,2,\cdots,P\\ i=1,2,\cdots,L\end{matrix} \qquad (11)$$

Based on those properties, we have the following theorem.

Theorem: The covariance matrix defined from the weighted array data (7)

$$\mathbf{C}=E\left\{\mathbf{y}(t)\mathbf{y}^H(t)\right\} \qquad (12)$$

is bounded and can be expressed as

$$\begin{aligned}\mathbf{C} &= E\left\{\mathbf{y}(t)\mathbf{y}^H(t)\right\} \\ &= E\left\{w(t)\mathbf{x}(t)\mathbf{x}^H(t)w^*(t)\right\} \\ &= \mathbf{A}\mathbf{\Gamma}_r\mathbf{A}^H + \eta_m^2\mathbf{I}\end{aligned} \tag{13}$$

where $\mathbf{\Gamma}_r = E\left\{\mathbf{r}(t)\mathbf{r}^H(t)\right\} = \mathrm{diag}\left\{\eta_1^2, \eta_2^2, \cdots, \eta_P^2\right\}$.

It is noted that (13) is similar to (3). A major advantage of the weighting processing is to suppress of the impulsive noise, and hence to enhance signal-to-noise ratio (SNR). A simulation examination of the property is given in next Section for Gaussian mixture noise. From the theorem, we can do subspace-based processing on the weighted data as usual. Here, we present a Weighted ARray Data based MUSIC (WARD-MUSIC) implementation of the proposed scheme. Given array data $\mathbf{x}(1), \mathbf{x}(2), \ldots, \mathbf{x}(N)$ and the number of signals, the WARD-MUSIC algorithm is comprised of the following steps:

S1: For each time t ($t = 1, 2, \cdots, N$), compute $w(t)$ and the corresponding $\mathbf{y}(t)$ using (6) and (7) .

S2: Estimate the weighted array covariance matrix by

$$\hat{\mathbf{C}} = \frac{1}{N}\sum_{t=1}^{N}\mathbf{y}(t)\mathbf{y}^H(t) \tag{14}$$

S3: Perform eigenvalue decomposition to the matrix $\hat{\mathbf{C}}$, and construct $L \times (L-P)$ matrix $\hat{\mathbf{E}}_n = \left[\hat{\mathbf{e}}_{P+1}, \ldots, \hat{\mathbf{e}}_L\right]$, where $\hat{\mathbf{e}}_{P+1}, \ldots, \hat{\mathbf{e}}_L$ are the eigenvectors associated with the smallest $L-P$ eigenvalues of $\hat{\mathbf{C}}$.

S4: Compute the MUSIC spectrum as

$$V(\theta) = \frac{1}{\mathbf{a}^H(\theta)\hat{\mathbf{E}}_n\hat{\mathbf{E}}_n^H\mathbf{a}(\theta)} \tag{15}$$

S5: Select the angles corresponding to P peaks, which are the estimates of DOAs.

4. SIMULATION

In this section, we present simulation results to illustrate the effectiveness of the WARD scheme. We study the DOA estimation errors in Gaussian mixture noise and *SαS* noise. We use a linear array with eight sensors spaced at a half wavelength apart. The number of signals is assumed to be known. In each experiment, we perform 500 Monte Carlo simulations.

A. Peformace of MUSIC, SCM-MUSIC, WARD-MUSIC, ML and CRB in Gaussian mixture noise

In the first simulation, we compare the RMSEs of the MUSIC, SCM-MUSIC, WARD-MUSIC, ML algorithms and CRB developed in [11] in the Gaussian mixture noise [11], with pdf

$$f(z) = \sum_{l=1}^{L}\frac{\varepsilon_l}{\pi\sigma_l^2}\exp\left\{-\frac{zz^*}{\sigma_l^2}\right\}, \sum_{l=1}^{L}\varepsilon_l = 1 \tag{16}$$

where ε_l is the probability that the complex random variable z is chosen from the lth term in the mixture pdf. In simulation, We assume two equal-strength signals coming from directions 10° and 15°. Fig.1(a) gives the RMSEs of the algorithms versus the snapshot number, the noise parameters selected are $L = 2$, $\sigma_1^2 = 1$, $\sigma_2^2 = 1000$ and $\varepsilon_2 = 0.05$. Fig1(b) gives the RMSEs of the algorithms versus the signal power, the selected noise parameters are $L = 2$, $\sigma_1^2 = 1$, $\sigma_2^2 = 100$ and $\varepsilon_2 = 0.1$, and the number of snapshot is 200. From Fig.1, we can see the performance of WARD-MUSIC is superior to the MUSIC and the SCM-MUSIC and is close to the ML. It should be noted that the ML algorithm obtains the optimal performance at the cost of computational complexity due to the involvement of the expectation maximization (EM) iterations. The comparable performance of WARD processing to ML is due to the SNR enhancement. To study the signal-to-noise ratio variations of the array data before and after weighting, we define a weighted SNR (WSNR) using the weighted signals and noises as

$$\mathrm{WSNR} = 10\log\frac{\sum_{t=1}^{N}\left\{\left|w(t)s(t)\right|^2\right\}}{\sum_{t=1}^{N}\left\{\left|w(t)n(t)\right|^2\right\}} \tag{17}$$

For SCM-MUSIC, $w(t) = \left(\sum_{i=1}^{L}\left|x_i(t)\right|^2\right)^{-1/2}$, and for WARD-MUSIC, $w(t)$ is defined as in (6). Here, we assume one signal coming at direction 10° . Fig.2 shows the SNR variations of the first three algorithms versus the signal power and ε_2, respectively. It can be seen that the WARD processing enhances the SNR for the non-Gaussian noise.

B. Performance of FLOM-MUSIC, ZMNL-MUSIC, SCM-MUSIC, WARD-MUSIC, ML and CRB in SαS noise

We now compare the performance of FLOM-MUSIC, ZMNL-MUSIC, SCM-MUSIC, WARD-MUSIC, ML and CRB in *SαS* noise, which is usually described by its characteristic function given by

$$\varphi(t) = e^{\left(-\gamma|t|^\alpha\right)} \tag{18}$$

where characteristic exponent α takes values $0 < \alpha \le 2$, and γ ($\gamma > 0$) is the dispersion of the distribution. The Gaussian process is the special case of the α-stable processes for $\alpha = 2$. Fig.3(a) shows the RMSEs of the algorithms for different characteristic exponents α. Two equal-strength signals are coming from directions 10° and 16°. The signal power is assumed to be 15dB, and the snapshot number is 200. Fig.3(b) shows the performance of the algorithms as a function of snapshot number at $\alpha = 0.8$. From Fig.3, we can find that the WARD-MUSIC algorithm outperforms SCM-MUSIC and ZMNL-MUSIC algorithms and approaches to

ML and CRB in terms of RMSEs. We approximate the CRB for the characteristic exponents $1 < \alpha < 2$ by linear interpolation of the values at $\alpha = 1$ and $\alpha = 2$.

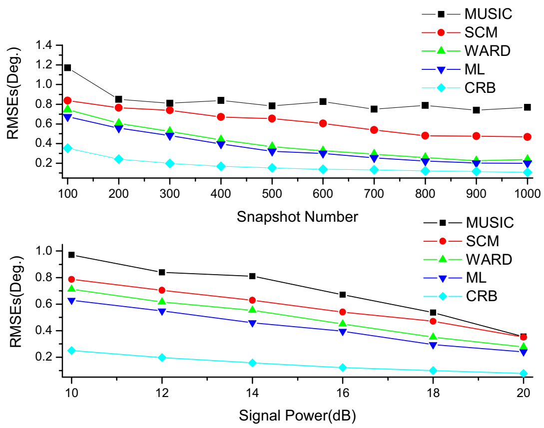

Fig.1 RMSEs of DOA estimation in Gaussian mixture noise

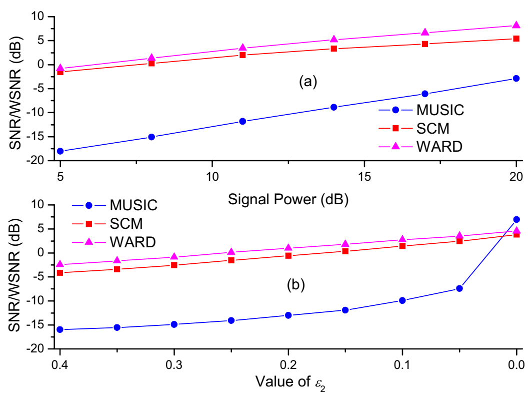

Fig.2 SNR variations in Gaussian mixture noise

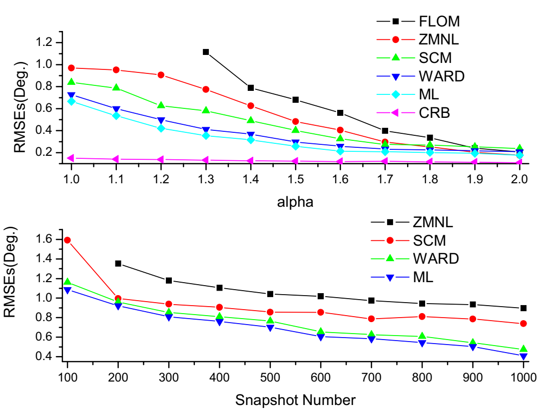

Fig.3 RMSEs of DOA estimation in *SαS* noise

5. CONCLUSION

In this paper, we have presented a weighted array data-based scheme for DOA estimation in additive impulsive noise environments. A simple strategy for the selection of weighting signal is suggested. It is found that the WARD has the desired statistical characteristics used in the subspace-based DOA estimation techniques. In working example, a WARD-MUSIC is detailed and performance analyses are simulated. It is found that the WARD-MUSIC enhances the SNR and improves the quality of DOA estimates, in comparison with other related algorithms.

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