POSITIONING FOR NLOS PROPAGATION: ALGORITHM DERIVATIONS AND CRAMER-RAO BOUNDS

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ABSTRACT

Mobile positioning has drawn significant attention in recent years. In dealing with the non–line–of–sight (NLOS) propagation error, the dominant error source in the mobile positioning, most previous research in this area has focused on the NLOS identification and mitigation. In this paper, we investigate new positioning algorithms to take advantage of the NLOS propagation paths rather than cancelling them. Based on the prior information about the NLOS path, a least squares based position estimation algorithm is developed and its performance in terms of root mean square error (RMSE) is also analyzed. Furthermore, the maximum likelihood based algorithm is presented to jointly estimate the mobile's and scatterers' positions. The Cramer–Rao lower bound on the RMSE is derived for the benchmark of the performance comparison. Finally, the performances of the proposed algorithms are evaluated analytically and via computer simulations. Numerical results demonstrate that the simulated results closely match the derived analytical results.

I. INTRODUCTION

Wireless positioning has received increasing attention over the past decade [1]. All the positioning algorithms in the literature assume that the line–of–sight (LOS) propagation paths exist between the mobile station (MS) and the fixed stations (FSs). In the presence of nonline–of–sight (NLOS) propagation, the major positioning errors result from the measurement noise and the NLOS propagation error which is the dominant factor [2].

Thus far, most of research on NLOS errors focuses on the NLOS error mitigation techniques, i.e., how to detect the errors and remove their impacts [3]–[5]. However, up to the authors' knowledge, positioning techniques which take *benefit* from the NLOS propagation paths have not been considered in the open literature. In a rich scattering environment, most of the propagation paths between the MS and FSs are NLOS, occasionally the first path between the MS and the home FS can be assumed as LOS. In this paper, we present a novel positioning algorithm which takes advantage of the NLOS paths by assuming prior knowledge about parameters of each NLOS path. Each path is characterized by a triple (α, β, d) where α stands for the angle of departure (AOD), β denotes the angle of arrival (AOA), and d defines the distance of the propagation path. To limit the scope of this paper, we do not consider how to estimate the parameters (α, β, d) at the FS. To summarize, the main contributions of this paper are two-fold:

This research was supported by Centre for Wireless Communications, University of Oulu, EU project PULSERS and Nokia Foundation. The full version of the paper has been submitted to IEEE Transactions on Vehicular Technology.

1) Possible region and position of the MS: Based on the measurement (α, β, d) of a NLOS path, we derive the possible region of the MS. It turns out to be a line segment. The least squares (LS) algorithm is proposed to estimate the position of the MS in the presence of multiple NLOS paths. We assume that all the measurements are corrupted by Gaussian noise, the root mean square error (RMSE) of the each coordinate of the position estimate is analyzed and compared to computer simulations.

2) Maximum likelihood algorithm and Cramer-Rao lower bound for joint MS And scatterers position estimation: Based on the knowledge of the parameters of multiple NLOS paths, we derive the maximum likelihood (ML) algorithm for joint the MS's and scatterers' position estimation. The Cramer-Rao lower bound (CRLB) for the variance of the MS position estimation is derived as well for the benchmark of the performance comparison. The performance of the algorithm in terms of RMSE are studied by analysis and computer simulations.

II. SYSTEM MODEL

Upper- and lower-case boldface letters denote matrices and vectors, respectively, $(.)^{\mathrm{T}}$ denotes the transpose. Let N_{f} denote the number of FSs which perceive the transmitted signal from the MS. No LOS paths exist between the MS and any connected FS. Each propagation path is parameterized by a triple (α, β, d), i.e., the AOD α, the AOA β and the distance d of the propagation path from the MS to the corresponding FS. We assume each FS has the knowledge of the number of paths corresponding to the MS, and parameters associated with each path. Depending on the requirement of the positioning accuracy and the positioning algorithm to be performed, the proposed algorithm can be employed in either the FS or the information processing center (IPC) which controls several FSs, such as the radio network controller in a cellular network. If the positioning algorithm is performed in the home FS, only the propagation paths corresponding to that particular FS can be utilized. On the other hand, if the positioning of the MS is performed in the IPC, the proposed algorithm would take advantage of the information about either all the propagation paths or only the strongest path related to each connected FS. For simplicity, we will focus on the situation where the positioning algorithm is run in the IPC taking only the strongest paths corresponding to each controlled FS into account.

III. LEAST SQUARES ALGORITHM

III-A. Algorithm Derivation

To set up the stage for the proposed algorithm, we need to find out the possible region of the MS when the FS has the information on the AOD α, AOA β and distance d of the strongest path.

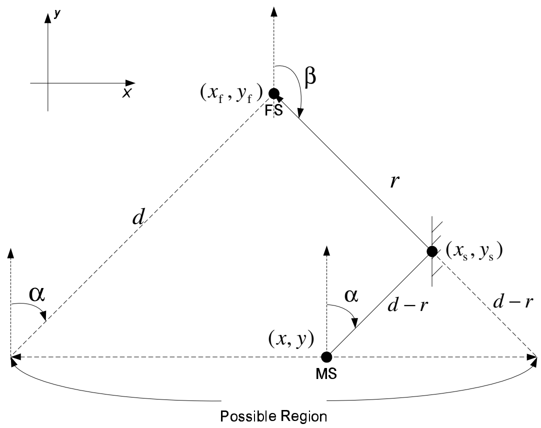

Fig. 1. Possible region of the MS.

However, it needs to have knowledge about the position of the scatterer $(x_{\text{s}}, y_{\text{s}})$. As shown in Fig. 1, $(x_{\text{f}}, y_{\text{f}})$ defines the position of the FS, r stands for the distance between the FS and the scatterer. Therefore, the possible coordinates of the scatterer are

$$\begin{aligned} x_{\text{s}} &= x_{\text{f}} + r\sin\beta, r \in (0, d), \\ y_{\text{s}} &= y_{\text{f}} + r\cos\beta, r \in (0, d), \end{aligned} \tag{1}$$

and the coordinates of the MS are expressed as

$$\begin{aligned} x &= x_{\text{s}} - (d-r)\sin\alpha, r \in (0, d), \\ y &= y_{\text{s}} - (d-r)\cos\alpha, r \in (0, d). \end{aligned} \tag{2}$$

It follows by substituting (1) into (2) that the possible position of the MS can be described as the following straight–line equation[1] of the slope–intercept form

$$y = k(\alpha, \beta)x + b(\alpha, \beta, d), \tag{3}$$

where

$$\begin{aligned} k(\alpha, \beta) &= \frac{\cos\alpha + \cos\beta}{\sin\alpha + \sin\beta}, \qquad (4) \\ b(\alpha, \beta, d) &= -k(\alpha, \beta)(x_{\text{f}} - d\sin\alpha) + y_{\text{f}} - d\cos\alpha. \qquad (5) \end{aligned}$$

This implies that if we have the knowledge about two propagation paths originating from the MS, the position of the MS can be estimated as the intersection of two lines which are derived from (3). We assume that N_{f} FSs located at $(x_{\text{f,i}}, y_{\text{f,i}}), i \in \{1, 2, \ldots, N_{\text{f}}\}$ can receive the signal from the MS. The parameter vector associated with the ith FS is denoted by $\boldsymbol{\theta}_i = (\alpha_i, \beta_i, d_i)^{\text{T}}$. Let $\boldsymbol{\theta} = \left(\boldsymbol{\theta}_1{}^{\text{T}}, \ldots, \boldsymbol{\theta}_{N_{\text{f}}}{}^{\text{T}}\right)^{\text{T}}$ denote the overall parameter vector estimates. Let $b_i = b(\alpha_i, \beta_i, d_i)$ and $k_i = k(\alpha_i, \beta_i)$ denote the x-intercept and the slope of the linear equation associated with the strongest path received by the ith FS, respectively. To achieve the minimum equation–error norm, the LS coordinate estimate $(\hat{x}_{\text{LS}}, \hat{y}_{\text{LS}})$ of the MS can be obtained by

$$(\hat{x}_{\text{LS}}, \hat{y}_{\text{LS}}) = \arg\min_{(x,y)} \sum_{i=1}^{N_{\text{f}}} (k_i x + b_i - y)^2. \tag{6}$$

[1]Precisely speaking, the possible region of the MS should be a line segment instead of the line of infinite length, however the line expression simplifies the derivation of the proposed algorithm without the loss of the accuracy of the positioning algorithm.

It follows straightforwardly that

$$\begin{aligned} \hat{x}_{\text{LS}}(\boldsymbol{\theta}) &= \frac{\sum_{i=1}^{N_{\text{f}}} b_i \sum_{i=1}^{N_{\text{f}}} k_i - N_{\text{f}} \sum_{i=1}^{N_{\text{f}}} b_i k_i}{N_{\text{f}} \sum_{i=1}^{N_{\text{f}}} k_i^2 - \left(\sum_{i=1}^{N_{\text{f}}} k_i\right)^2}, \\ \hat{y}_{\text{LS}}(\boldsymbol{\theta}) &= \frac{\sum_{i=1}^{N_{\text{f}}} b_i + \hat{x}_{\text{LS}} \sum_{i=1}^{N_{\text{f}}} k_i}{N_{\text{f}}}, \\ &= \frac{\sum_{i=1}^{N_{\text{f}}} b_i \sum_{i=1}^{N_{\text{f}}} k_i^2 - \sum_{i=1}^{N_{\text{f}}} k_i \sum_{i=1}^{N_{\text{f}}} b_i k_i}{N_{\text{f}} \sum_{i=1}^{N_{\text{f}}} k_i^2 - \left(\sum_{i=1}^{N_{\text{f}}} k_i\right)^2}. \end{aligned} \tag{7}$$

III-B. Root Mean Square Error Analysis

Let $\alpha_i^o, \beta_i^o, d_i^o$ denote the actual AODs, AOAs and distances of the propagation paths of interest. We assume that the estimated parameters α_i, β_i and d_i are independently Gaussian distributed random variables [6][2], i.e., $\alpha_i \sim \mathcal{N}(\alpha_i^o, \sigma_{\alpha_i}^2)$, $\beta_i \sim \mathcal{N}(\beta_i^o, \sigma_{\beta_i}^2)$ and $d_i \sim \mathcal{N}(d_i^o, \sigma_{d_i}^2)$. We define $\boldsymbol{\theta}_i^o = (\alpha_i^o, \beta_i^o, d_i^o)^{\text{T}}$ as the parameter vector of actual values. Let $\boldsymbol{\theta}^o = \left(\boldsymbol{\theta}_1^{o\text{T}}, \ldots, \boldsymbol{\theta}_{N_{\text{f}}}^{o}{}^{\text{T}}\right)^{\text{T}}$ denote the actual parameter vector of all paths. When the variances of the estimated parameters $\boldsymbol{\theta}$ are small, we define $\boldsymbol{\sigma}_i^2 = (\sigma_{\alpha_i}^2, \sigma_{\beta_i}^2, \sigma_{d_i}^2)^{\text{T}}, i = 1, 2, \ldots, N_{\text{f}}$, the estimated coordinates (7) of the MS are described as

$$\begin{aligned} \hat{x}_{\text{LS}}(\boldsymbol{\theta}) &\approx \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o) + \nabla \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o)(\boldsymbol{\theta} - \boldsymbol{\theta}^o), \\ \hat{y}_{\text{LS}}(\boldsymbol{\theta}) &\approx \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o) + \nabla \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)(\boldsymbol{\theta} - \boldsymbol{\theta}^o), \end{aligned} \tag{8}$$

where

$$\begin{aligned} \nabla \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o) &= \frac{\partial \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial \boldsymbol{\theta}^{\text{T}}} \in \mathbb{R}^{1 \times 3N_{\text{f}}}, \\ \nabla \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o) &= \frac{\partial \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial \boldsymbol{\theta}^{\text{T}}} \in \mathbb{R}^{1 \times 3N_{\text{f}}}. \end{aligned} \tag{9}$$

The gradient vector $\nabla \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o)$ in (8) is calculated as

$$\begin{aligned} \frac{\partial \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial \alpha_i} &= \frac{\partial \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial k_i} \frac{\partial k(\boldsymbol{\theta}_i^o)}{\partial \alpha} + \frac{\partial \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial b_i} \frac{\partial b(\boldsymbol{\theta}_i^o)}{\partial \alpha}, \\ \frac{\partial \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial \beta_i} &= \frac{\partial \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial k_i} \frac{\partial k(\boldsymbol{\theta}_i^o)}{\partial \beta} + \frac{\partial \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial b_i} \frac{\partial b(\boldsymbol{\theta}_i^o)}{\partial \beta}, \\ \frac{\partial \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial d_i} &= \frac{\partial \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial b_i} \frac{\partial b(\boldsymbol{\theta}_i^o)}{\partial d}, i = 1, 2, \ldots, N_{\text{f}}. \end{aligned} \tag{10}$$

As in (10), the gradient vector $\nabla \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)$ in (8) becomes

$$\begin{aligned} \frac{\partial \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial \alpha_i} &= \frac{\partial \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial k_i} \frac{\partial k(\boldsymbol{\theta}_i^o)}{\partial \alpha} + \frac{\partial \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial b_i} \frac{\partial b(\boldsymbol{\theta}_i^o)}{\partial \alpha}, \\ \frac{\partial \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial \beta_i} &= \frac{\partial \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial k_i} \frac{\partial k(\boldsymbol{\theta}_i^o)}{\partial \beta} + \frac{\partial \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial b_i} \frac{\partial b(\boldsymbol{\theta}_i^o)}{\partial \beta}, \\ \frac{\partial \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial d_i} &= \frac{\partial \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial b_i} \frac{\partial b(\boldsymbol{\theta}_i^o)}{\partial d}, i = 1, 2, \ldots, N_{\text{f}}, \end{aligned} \tag{11}$$

where $\frac{\partial \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial k_i}$ and $\frac{\partial \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial b_i}, i = 1, 2, \ldots, N_{\text{f}}$ are expressed as

$$\begin{aligned} \frac{\partial \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial k_i} &= \frac{\hat{x}_{\text{LS}}(\boldsymbol{\theta}^o) + P \frac{\partial \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial k_i}}{N_{\text{f}}}, \\ \frac{\partial \hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial b_i} &= \frac{1 + P \frac{\partial \hat{x}_{\text{LS}}(\boldsymbol{\theta}^o)}{\partial b_i}}{N_{\text{f}}}. \end{aligned} \tag{12}$$

Let $\boldsymbol{\varphi} = (x^o, y^o)^{\text{T}}$ stand for the actual position of the MS. Therefore, the variance of the estimation error of the coordinates

[2]The MUSIC estimator was shown to be Gaussian distributed for sufficiently large measurements.

$(\hat{x}_{\text{LS}}, \hat{y}_{\text{LS}})$ is

$$\begin{aligned} \mathrm{E}(\hat{x}_{\text{LS}} - x^o)^2 &\approx \nabla\hat{x}_{\text{LS}}(\boldsymbol{\theta}^o)\text{diag}(\boldsymbol{\sigma}_1^2, \ldots, \boldsymbol{\sigma}_{N_\text{f}}^2)\nabla\hat{x}_{\text{LS}}^{\text{T}}(\boldsymbol{\theta}^o), \\ \mathrm{E}(\hat{y}_{\text{LS}} - y^o)^2 &\approx \nabla\hat{y}_{\text{LS}}(\boldsymbol{\theta}^o)\text{diag}(\boldsymbol{\sigma}_1^2, \ldots, \boldsymbol{\sigma}_{N_\text{f}}^2)\nabla\hat{y}_{\text{LS}}^{\text{T}}(\boldsymbol{\theta}^o) \quad (13) \end{aligned}$$

We denote the RMSE $\sigma_{\text{LS}} = \sqrt{(\mathrm{E}(\hat{x}_{\text{LS}} - x^o)^2 + \mathrm{E}(\hat{y}_{\text{LS}} - y^o)^2)/2}$ as the average RMSE of each estimated coordinate of the MS.

IV. MAXIMUM LIKELIHOOD POSITIONING ESTIMATION AND CRAMÉR-RAO LOWER BOUND

IV-A. Joint MS and Scatterers Position Estimation

Sect. III addressed using LS algorithm to estimate the MS' position. Indeed, all the observed parameters $\boldsymbol{\theta}$ provide information about not only the position of the MS but also the positions of the associated scatterers. We define $(x_{\text{s}i}^o, y_{\text{s}i}^o), i = 1, 2, \ldots, N_\text{f}$ as the actual coordinates of the scatterers. According to Fig. 1, the real parameter $\boldsymbol{\theta}_i^o$ depends on the coordinates $(x_{\text{s}i}^o, y_{\text{s}i}^o)$ and (x^o, y^o) as follows

$$\begin{aligned} \alpha_i^o &= \begin{cases} \text{arc tan}\frac{x_{\text{s}i}^o - x^o}{y_{\text{s}i}^o - y^o} & x_{\text{s}i}^o \geq x^o, y_{\text{s}i}^o \geq y^o \\ \pi + \text{arc tan}\frac{x_{\text{s}i}^o - x^o}{y_{\text{s}i}^o - y^o} & x_{\text{s}i}^o \geq x^o, y_{\text{s}i}^o < y^o \\ \pi + \text{arc tan}\frac{x_{\text{s}i}^o - x^o}{y_{\text{s}i}^o - y^o} & x_{\text{s}i}^o < x^o, y_{\text{s}i}^o < y^o \\ 2\pi + \text{arc tan}\frac{x_{\text{s}i}^o - x^o}{y_{\text{s}i}^o - y^o} & x_{\text{s}i}^o \geq x^o, y_{\text{s}i}^o \geq y^o \end{cases} \\ \beta_i^o &= \begin{cases} \text{arc tan}\frac{x_{\text{s}i}^o - x_{\text{f}i}^o}{y_{\text{s}i}^o - y_{\text{f}i}^o} & x_{\text{s}i}^o \geq x_{\text{f}i}^o, y_{\text{s}i}^o \geq y_{\text{f}i}^o \\ \pi + \text{arc tan}\frac{x_{\text{s}i}^o - x_{\text{f}i}^o}{y_{\text{s}i}^o - y_{\text{f}i}^o} & x_{\text{s}i}^o \geq x_{\text{f}i}^o, y_{\text{s}i}^o < y_{\text{f}i}^o \\ \pi + \text{arc tan}\frac{x_{\text{s}i}^o - x_{\text{f}i}^o}{y_{\text{s}i}^o - y_{\text{f}i}^o} & x_{\text{s}i}^o < x_{\text{f}i}^o, y_{\text{s}i}^o < y_{\text{f}i}^o \\ 2\pi + \text{arc tan}\frac{x_{\text{s}i}^o - x_{\text{f}i}^o}{y_{\text{s}i}^o - y_{\text{f}i}^o} & x_{\text{s}i}^o \geq x_{\text{f}i}^o, y_{\text{s}i}^o \geq y_{\text{f}i}^o \end{cases} \\ d_i^o &= \sqrt{(x_{\text{s}i}^o - x^o)^2 + (y_{\text{s}i}^o - y^o)^2} \\ &\quad + \sqrt{(x_{\text{s}i}^o - x_{\text{f}i}^o)^2 + (y_{\text{s}i}^o - y_{\text{f}i}^o)^2} \qquad (14) \end{aligned}$$

As in Sect. III-B, all the estimated parameters are assumed to be independently Gaussian distributed random variables. We define $p\left(\boldsymbol{\theta}; x^o, y^o, (x_{\text{s}i}^o, y_{\text{s}i}^o)_{i=1,2,\ldots,N_\text{f}}\right)$ as the parameterized joint probability density function (PDF) of all the observed parameters. Due to the assumption that all the observed parameters are independently Gaussian distributed random variables, the likelihood function $\mathcal{L}\left(\boldsymbol{\theta}; x^o, y^o, (x_{\text{s}i}^o, y_{\text{s}i}^o)_{i=1,2,\ldots,N_\text{f}}\right) = \ln p\left(\boldsymbol{\theta}; x^o, y^o, (x_{\text{s}i}^o, y_{\text{s}i}^o)_{i=1,2,\ldots,N_\text{f}}\right)$ can be expressed as (15), By ignoring the constant term in (15), we define the objective function as (16). The ML based joint MS and scatterers position estimator becomes (17).

IV-B. Cramér-Rao Lower Bound

The CRLB is the lower bound on the variance of any unbiased estimator for unknown parameters [7]. Let us define $\boldsymbol{x}_\text{s}^o = (x_{\text{s}1}^o, x_{\text{s}2}^o, \ldots, x_{\text{s}N_\text{f}}^o)^{\text{T}}$, $\boldsymbol{y}_\text{s}^o = (y_{\text{s}1}^o, y_{\text{s}2}^o, \ldots, y_{\text{s}N_\text{f}}^o)^{\text{T}}$, $\boldsymbol{\psi} = \left(\boldsymbol{x}_\text{s}^{o\text{T}}, \boldsymbol{y}_\text{s}^{o\text{T}}\right)$ and $\boldsymbol{\rho} = \left(\boldsymbol{\varphi}^{\text{T}}, \boldsymbol{\psi}^{\text{T}}\right)^{\text{T}}$. The Fisher information matrix (FIM) $\boldsymbol{I}(\boldsymbol{\rho}) \in \mathbb{R}^{(2N_\text{f}+2)\times(2N_\text{f}+2)}$ can be expressed as

$$\boldsymbol{I}(\boldsymbol{\rho}) = \mathrm{E}\left(\frac{\partial\mathcal{L}(\boldsymbol{\theta};\boldsymbol{\rho})}{\partial\boldsymbol{\rho}}\frac{\partial\mathcal{L}(\boldsymbol{\theta};\boldsymbol{\rho})}{\partial\boldsymbol{\rho}^{\text{T}}}\right). \qquad (18)$$

Therefore, the RMSE $\sigma_{\text{ML}} = \sqrt{(\mathrm{E}(\hat{x}_{\text{ML}} - x^o)^2 + \mathrm{E}(\hat{y}_{\text{ML}} - y^o)^2)/2}$ of ML based positioning estimator has the lower bound

$$\sigma_{\text{ML}} \geq \text{CRLB} = \sqrt{\frac{\left(\boldsymbol{I}^{-1}(\boldsymbol{\rho})\right)_{11} + \left(\boldsymbol{I}^{-1}(\boldsymbol{\rho})\right)_{22}}{2}}. \qquad (19)$$

We partition the FIM $\boldsymbol{I}(\boldsymbol{\rho})$ as follows

$$\boldsymbol{I}(\boldsymbol{\rho}) = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B}^{\text{T}} & \boldsymbol{C} \end{pmatrix}, \qquad (20)$$

where

$$\begin{aligned} \boldsymbol{A} &= \mathrm{E}\left(\frac{\partial\mathcal{L}(\boldsymbol{\theta};\boldsymbol{\rho})}{\partial\boldsymbol{\varphi}}\frac{\partial\mathcal{L}(\boldsymbol{\theta};\boldsymbol{\rho})}{\partial\boldsymbol{\varphi}^{\text{T}}}\right) \in \mathbb{R}^{2\times 2}, \\ \boldsymbol{B} &= \mathrm{E}\left(\frac{\partial\mathcal{L}(\boldsymbol{\theta};\boldsymbol{\rho})}{\partial\boldsymbol{\varphi}}\frac{\partial\mathcal{L}(\boldsymbol{\theta};\boldsymbol{\rho})}{\partial\boldsymbol{\psi}^{\text{T}}}\right) \in \mathbb{R}^{2\times 2N_\text{f}}, \\ \boldsymbol{C} &= \mathrm{E}\left(\frac{\partial\mathcal{L}(\boldsymbol{\theta};\boldsymbol{\rho})}{\partial\boldsymbol{\psi}}\frac{\partial\mathcal{L}(\boldsymbol{\theta};\boldsymbol{\rho})}{\partial\boldsymbol{\psi}^{\text{T}}}\right) \in \mathbb{R}^{2N_\text{f}\times 2N_\text{f}}. \qquad (21) \end{aligned}$$

V. NUMERICAL EXAMPLES

This section presents analytical and simulation results to illustrate the performance of the proposed LS algorithm and the ML algorithm. The positions of the MS, FSs and scatterers are given in Table I. We assume the parameters associated to different propagation paths have the same normalized estimation error variance, i.e., $\sigma_{\alpha_i}^2 = \sigma_\alpha^2, \sigma_{\beta_i}^2 = \sigma_\beta^2, \sigma_{d_i}^2 = \sigma_d^2$, for all $i = 1, 2, 3, 4$.

We examine the effects of the standard deviations of estimated parameters and the number of FSs employed on the RMSE of the LS based positioning algorithm. When we only employ two out of the four NLOS paths shown in Table I, there are $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ combinations of two NLOS paths. Different combinations may lead to different RMSE performances, e.g., the performance yielded by the FS pair $(1, 2)$ could be different from that of the FS pair $(3, 4)$. To obtain the average performance of employing the same number of NLOS paths, all RMSE performances under different FS combinations are averaged out.

Fig. 2 shows the analytical RMSE performance with respect to (σ_d, σ_β) when $\sigma_\alpha = \sigma_\beta$. Fig. 3 shows the analytical and simulated RMSE performance of the LS positioning algorithm when 2, 3 or 4 NLOS paths are exploited. It shows the effect of the number of FSs employed along with σ_d when $\sigma_\alpha = \sigma_\beta$. All the simulations are performed with 1000 independent runs.

Fig. 4 shows the RMSE performance of the ML based algorithm and the corresponding CRLBs against σ_d. Also shown is the performance of the LS algorithm to highlight the performance gain of the ML algorithm over the LS algorithm. The MATLAB function *fmincon.m* is employed to find the ML solution of (17). All the simulations of the ML algorithm employ the solution yielded by the LS algorithm as the initial searching point. It shows that the ML algorithm does improve the performance significantly at the cost of the increased computational complexity. It is interesting to note that the performance of the ML algorithm is close to the CRLB. This demonstrates that 4 NLOS paths furnished by 4 FSs are enough to make the ML algorithm be an efficient estimator which is able to achieve the optimal performance, i.e., the CRLB.

VI. CONCLUSIONS

In this paper, we considered the position estimation of the MS in the NLOS scenario. With the knowledge about the AOA, the AOD and the distance associated with each NLOS path, the LS algorithm

$$\mathcal{L}\left(\boldsymbol{\theta}; x^o, y^o, (x^o_{\mathrm{s}i}, y^o_{\mathrm{s}i})_{i=1,2,\ldots,N_{\mathrm{f}}}\right) = \sum_{i=1}^{N_{\mathrm{f}}} \left(\ln \frac{1}{(2\pi)^{3/2}\sigma_{\alpha_i}\sigma_{\beta_i}\sigma_{d_i}} - \frac{1}{2}\left(\frac{(\alpha_i - \alpha_i^o)^2}{\sigma_{\alpha_i}^2} + \frac{(\beta_i - \beta_i^o)^2}{\sigma_{\beta_i}^2} + \frac{(d_i - d_i^o)^2}{\sigma_{d_i}^2} \right) \right). \tag{15}$$

$$\mathcal{O}\left(\boldsymbol{\theta}; x^o, y^o, (x^o_{\mathrm{s}i}, y^o_{\mathrm{s}i})_{i=1,2,\ldots,N_{\mathrm{f}}}\right) = \frac{1}{2}\sum_{i=1}^{N_{\mathrm{f}}} \left(\frac{(\alpha_i - \alpha_i^o)^2}{\sigma_{\alpha_i}^2} + \frac{(\beta_i - \beta_i^o)^2}{\sigma_{\beta_i}^2} + \frac{(d_i - d_i^o)^2}{\sigma_{d_i}^2} \right). \tag{16}$$

$$(\hat{x}_{\mathrm{ML}}, \hat{y}_{\mathrm{ML}}, (\hat{x}_{\mathrm{MLS}i}, \hat{y}_{\mathrm{MLS}i})_{i=1,2,\ldots,N_{\mathrm{f}}}) = \arg \min_{x^o, y^o, (x^o_{\mathrm{s}i}, y^o_{\mathrm{s}i})_{i=1,2,\ldots,N_{\mathrm{f}}}} \mathcal{O}\left(\boldsymbol{\theta}; x^o, y^o, (x^o_{\mathrm{s}i}, y^o_{\mathrm{s}i})_{i=1,2,\ldots,N_{\mathrm{f}}}\right). \tag{17}$$

Table I. Coordinates[Meter] of the MS, FSs and scatterers

MS	FS / Scatterer	FS / Scatterer	FS / Scatterer	FS / Scatterer
(25,25)	(0,50)/(40,10)	(-50,0)/(-25,30)	(0,-50)/(-30,-30)	(50,0)/(35,-40)

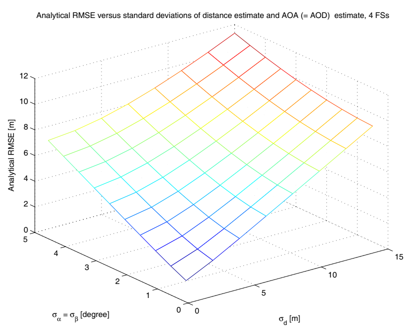

Fig. 2. Analytical RMSE versus standard deviations of distance estimate and AOA (= AOD) estimate, 4 FSs.

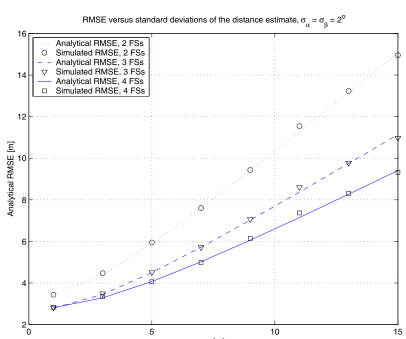

Fig. 3. RMSE versus standard deviations of the distance estimate, $\sigma_\alpha = \sigma_\beta = 2^o$.

was proposed to exploit multiple NLOS paths to estimate the MS' position. The ML algorithm was further developed to improve the performance of the LS algorithm, and it was able to estimate the positions of both the MS and scatterers. The CRLB was also derived to benchmark the performance of the positioning algorithms. It was shown both analytically and through computer simulations that the proposed algorithms are able to estimate the MS position only by employing the NLOS paths, and the ML algorithm can achieve the optimal RMSE performance, i.e., the CRLB.

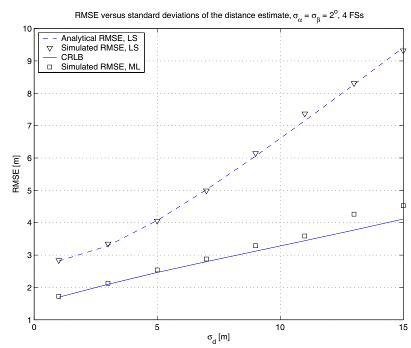

Fig. 4. RMSE of the ML algorithm and the CRLB, $\sigma_\alpha = \sigma_\beta = 2^o$.

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