AN IMPROVED PARTIAL ADAPTIVE NARROW-BAND BEAMFORMER USING CONCENTRIC RING ARRAY

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ABSTRACT

Partial adaptation is often used to reduce the computation and improve tracking ability of an adaptive array. In some practical situations, the received signal to be processed contains some interferences whose characteristics are known. The previously proposed partially adaptive concentric ring array is not able to utilize the prior information of known interferences without sacrificing the number of degrees of freedom, which will cause higher steady state error and smaller number of interferences that can be cancelled. We propose in this paper an improved partially adaptive concentric ring array that can utilize the prior knowledge to improve performance and maintain the same number of degrees of freedom. The proposed method designs the non-adaptive weights to remove the known interferences, and is shown to provide much faster convergence speed and lower steady state error than the original method.

1. INTRODUCTION

Array signal processing is a popular technique in acoustic and radar applications because it adds the spatial dimension in addition to time, and hence improves the performance in signal acquisition and interference rejection. Perhaps the most popular array is the Uniform Linear Array (ULA) because of its simple structure that allows very efficient processing in Direction of Arrival (DOA) estimation and signal enhancement. ULA is very effective for 2-D beamforming. In 3-D space, ULA creates ambiguity with its beampattern warping around the array [1] and is not appropriate for 3-D beamforming. Concentric Ring Array (CRA) [2], [3] has been proposed for 3-D beamforming which is able to eliminate DOA ambiguity and provides frequency invariant design for broadband beamformer.

In practice, the direction of all the interferences may not be known a priori and the weights in the beamformer are made adaptive to minimize the array output subject to a set of constraints including a unity gain in the look direction of the signal. 3-D beamforming typically requires a huge number of array elements (over one hundred) to achieve good performance. As a result, the convergence speed and tracking performance could be poor. Reducing the number of adaptive coefficients is necessary to improve performance and achieve real time processing.

Partially adaptive approach has been proposed to reduce the number of adaptive coefficients. Partial adaptive array can be beamspace or element space. Recently, Li & Ho [4] proposed an element space partially adaptive array in which each ring is considered as a sub-array that performs fixed beamforming using the delay-and-sum weights [5], and the outputs from the concentric rings are combined using a set of adaptive weights. The partially adaptive array improves the convergence speed significantly compared to fully adaptive array, and the steady state residual interference and noise power is only increased slightly.

One drawback of the previously proposed partially adaptive circular array is that the number of interferences that can be removed is limited by the number of adaptive weights which is equal to the number of rings. Furthermore, it does not make use of any prior knowledge about any interference whose directions may be known. Although constraints on the adaptive weights can be used to eliminate the interferences quickly from the known directions; however, this will unnecessarily reduce the number of degrees of freedom (DOF's) in the adaptive weights which will increase the steady state residual error level and limits the extra number of interferences that can be removed.

In some scenarios, the characteristics and DOA of some interferences are known. In this paper, we propose an enhanced partially adaptive array that can make use of this prior knowledge to improve convergence speed, whilst at the same time maintaining the same number of DOF's in the adaptive weights to assure good steady state behavior.

The idea of the proposed method is not to restrict the intra-ring weights to be delay-and-sum, but rather select the weights so that it minimizes the output in each ring through the knowledge of the signal's DOA and the known DOA's and characteristics of some of the interferences. The output from every ring is then fed to a General Sidelobe Canceller (GSC) based adaptive structure [4] to perform a second level of optimization to remove the interferences whose characteristics are not known. The proposed array reduces interferences much faster as compared to the original method and achieves smaller steady state residual error level.

This paper is organized as follows. Section 2 reviews briefly the fully and partially adaptive concentric ring arrays. Section 3 presents the proposed improved partially adaptive array. Section 4 contains the experimental results and analysis, the paper is summarized in Section 5.

2. PREVIOUS ADAPTIVE CRA DESIGN

2.1. Fully Adaptive CRA

We shall consider a concentric Circular multi Ring Array that is composed of M concentric rings as shown in Fig. 1. The number of receiving elements in ring i is N_i and the total number of elements

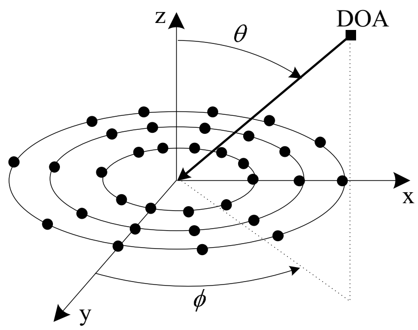

Fig. 1. Concentric Ring Array

is K=N_1+....+N_M. The output of the array at time t is:

$$z(t) = \sum_{i=1}^{M}\sum_{k=1}^{N_i} v_{ik}^{*} x_{ik}(t) = \mathbf{v}^H \mathbf{x}(t) \qquad (1)$$

where $x_{ik}(t)$ is the received signal at the element k of ring i. v_{ik} is the corresponding array weight, and (*) represents complex conjugate. In vector form $\mathbf{v}$ is the weight vector and $\mathbf{x}(t)$ is the input vector.

In fully adaptive array the weighting coefficients are found adaptively by minimizing the output power $E[|z(t)|^2]$ subject to a set of constraints (Linearly Constrained Minimum Variance criteria, or LCMV). Usually, it includes one constraint to maintain unity gain at the DOA of the desired signal. This approach has the advantage of reducing the interferences from any DOA's (different than the desired signal DOA) and achieving a low steady state residual error. The disadvantage of this approach is its slow convergence, poor tracking ability and high computational cost.

2.1. Partially Adaptive CRA

The partially adaptive CRA [4], [6] overcomes these deficiencies by adapting only one weight per ring, thereby reducing greatly the number of weights that need to be adapted.

The technique considers each ring as a subarray in which fixed beamforming using delay-and-sum weights is applied. The outputs from different rings are then combined adaptively to form the final output. To be specific, let $\mathbf{h}_i$ be the vector containing the delay-and-sum weights for ring i, whose k^{th} element is:

$$h_{ik} = \frac{1}{N_i} e^{j\frac{2\pi}{\lambda} R_i[\sin\theta_0 \cos(\phi_0 - v_k)]} \qquad k = 1,\ldots,N_i. \qquad (2)$$

where λ is the wavelength, R_i the radius of the ring, θ_0 and ϕ_0 are the desired signal's elevation and azimuth angles respectively, and $v_k = 2\pi k / N_i$ is the azimuth angle of the k^{th} element in the ring. Denote the received signal vector of ring i as $\mathbf{x}_i(t)$. Then, the output of the ring is:

$$y_i(t) = \mathbf{h}_i^{H} \mathbf{x}_i(t) \qquad i = 1,\ldots,M. \qquad (3)$$

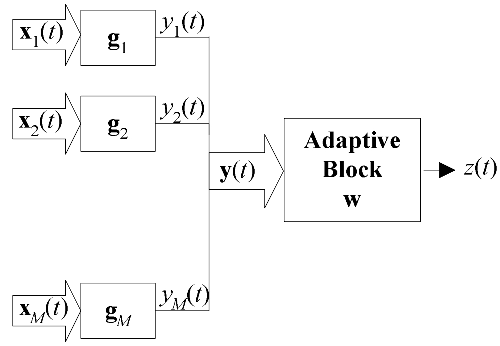

Fig. 2. Block Diagram of the proposed partially adaptive array

Each ring output is then multiplied by an adaptive inter-ring weight to find the final output as:

$$z(t) = \mathbf{w}^H \mathbf{y}(t) \qquad (4)$$

where $\mathbf{y}(t) = [y_1(t), \ldots, y_M(t)]^T$ and $\mathbf{w}$ the adaptive inter-ring weight vector. The inter-ring weight vector $\mathbf{w}$ is found adaptively [4] subject to the same conditions as in the fully adaptive approach; i.e. by minimizing the output power $E[|z(t)|^2]$ subject to a set of constraints.

This partially adaptive array has better tracking ability and faster convergence due to the small number of adaptive weights; however, the steady state error is somewhat higher than that in the fully adaptive array. Also, the number of interferences that can be cancelled is limited by the number of adaptive weights minus the number of constraints.

3. IMPROVED PARTIALLY ADAPTIVE CONCENTRIC RING ARRAY

Fig. 2 shows the proposed design block diagram. In practice, the characteristics and DOA's of some interferences may be available. Instead of using simply delay-and-sum weights within each ring as in [2], the proposed method selects the intra-ring weights $\mathbf{g}_i$ to minimize the known interferences in each ring. They are found by minimizing the output power of each ring $E[|y_i^e(t)|^2]$ subject to the unity gain constraint at the desired DOA, where $y_i^e(t)$ is the essential output of the ring when only the desired signal, the known interferences and the noise are present

Since the intra-ring weights $\mathbf{g}_i$ are found by minimizing the output power of the i^{th} ring over the known interferences only, the resultant beampattern from $\mathbf{g}_i$ may create higher gain in some other directions in which the unknown interferences may come in. The consequence is putting extra burden on the adaptive weights and could lead to a higher steady state residual error level in the final array output. To keep this effect minimal, we modify the cost function to find $\mathbf{g}_i$ as follow by adding a penalty term that contains the distance between $\mathbf{g}_i$ and the delay and sum weights:

$$\mathbf{g}_i = \arg\min\left\{(1-\alpha)E[|y_i^e(t)|^2] + \alpha\|\mathbf{g}_i - \mathbf{h}_i\|^2\right\} \qquad (5)$$

subject to $\mathbf{h}_i^H \mathbf{g}_i = 1$; where α is a penalty factor. Equation (5) provides a tradeoff optimization for the improved intra-ring weight vector. A smaller value of α allows larger deviation from the delay-and-sum vector and a larger reduction amount of known interferences. The solution reduces back to delay and sum if α is equal to unity.

The minimization problem can be solved by the use of Lagrange multipliers. This method finds the minimum of a function subject to one or more constraints by introducing a Lagrange multiplier η, and creating an auxiliary function as:

$$J = (1-\alpha) E[\left|y_i^e(t)\right|^2] + \alpha \left\|\mathbf{g}_i - \mathbf{h}_i\right\|^2 + \eta\left(1 - \mathbf{g}_i^H \mathbf{h}_i\right) \tag{6}$$

where $E[\left|y_i^e(t)\right|^2] = \mathbf{g}_i^H \mathbf{R}_i^e \mathbf{g}_i$, $\mathbf{R}_i^e = E\left[\mathbf{x}_i^e(t)\mathbf{x}_i^e(t)^H\right]$ and $\left\|\mathbf{g}_i - \mathbf{h}_i\right\|^2 = (\mathbf{h}_i - \mathbf{g}_i)^H(\mathbf{h}_i - \mathbf{g}_i)$.

Taking the derivative of J with respect to $\mathbf{g}^*_i$ yields:

$$\frac{\partial J}{\partial \mathbf{g}^*_i} = (1-\alpha)\mathbf{R}_i^e \mathbf{g}_i - \alpha(\mathbf{h}_i - \mathbf{g}_i) - \eta\,\mathbf{h}_i \tag{7}$$

Setting the gradient to zero and solving for $\mathbf{g}_i$ gives:

$$\mathbf{g}_i = (\alpha + \eta)\left[(1-\alpha)\mathbf{R}_i^e + \alpha\mathbf{I}\right]^{-1}\mathbf{h}_i \tag{8}$$

Since $1 = \mathbf{h}_i^H \mathbf{g}_i$ from the constraint, pre-multiplying (8) by $\mathbf{h}_i^H$ forms:

$$1 = (\alpha + \eta)\mathbf{h}_i^H\left[(1-\alpha)\mathbf{R}_i^e + \alpha\mathbf{I}\right]^{-1}\mathbf{h}_i \tag{9}$$

Finally combining (8) and (9) by eliminating $\alpha + \eta$ yields:

$$\mathbf{g}_i = \frac{\left[(1-\alpha)\mathbf{R}_i^e + \alpha\mathbf{I}\right]^{-1}\mathbf{h}_i}{\mathbf{h}_i^H\left[(1-\alpha)\mathbf{R}_i^e + \alpha\mathbf{I}\right]^{-1}\mathbf{h}_i} \qquad i = 1,\ldots,M. \tag{10}$$

It is important to note that $\mathbf{R}_i^e$ is the covariance matrix from the essential input and can be theoretically found to include the information of the known interference characteristics and noise as:

$$\mathbf{R}_i^e = \mathbf{s}_{i0}\mathbf{s}_{i0}^H + \sum_{l=1}^{I} E_l \mathbf{s}_{il}\mathbf{s}_{il}^H + E_N\mathbf{I} \qquad i = 1,\ldots,M. \tag{11}$$

where E_l is the power of the l^{th} interference from a total of I known interferences, E_N is the noise power, and $\mathbf{s}_{i0}$ is the delay vector for a signal coming from the desired DOA onto ring i and has the elements:

$$s_{i0k} = e^{j\frac{2\pi}{\lambda}R_i[\sin\theta_0 \cos(\phi_0 - v_k)]} \qquad k = 1,\ldots,N_i. \tag{12}$$

$\mathbf{s}_{il}$ is the delay vector for the l^{th} interference coming from its known DOA$_l$, and is calculated as in (12) by replacing (θ_0, ϕ_0) by (θ_l, ϕ_l). These delay vectors are only dependent on the ring's geometry and on the DOA's of the incoming signal and interferences.

The penalty factor has to be carefully selected so that the array is still tuned to the desired DOA and at the same time rejects the known interferences implicit in the essential covariance matrix. If the penalty term is too close to zero, a negative side effect could appear in which a possible higher gain than the one from the delay-and-sum weights may occur in the directions of the unknown interferences. Once the intra-ring weights are obtained from (10), the output of each ring is calculated as:

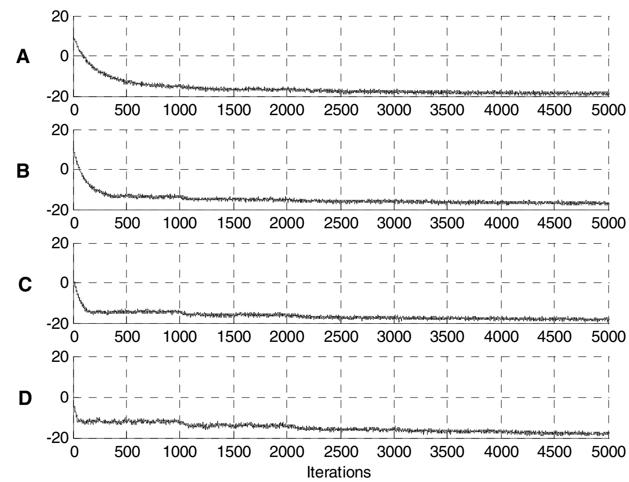

Fig. 3. Residual interference and noise power in log scale with respect to iterations. **A**: fully adaptive. **B**: original partially adaptive as in [4]. **C**: improved partially, one interference known, two unknown. **D**: improved partially, two interferences known, one unknown.

$$y_i(t) = \mathbf{g}_i^H \mathbf{x}_i(t), \qquad i = 1,\ldots,M. \tag{13}$$

Each ring output is then multiplied by an adaptive inter-ring weight to find the form output as:

$$z(t) = \mathbf{w}^H \mathbf{y}(t) \tag{14}$$

The adaptive inter-ring weights $\mathbf{w}$ are found using a GSC configuration as in [4]. It decomposes the adaptive weights in to constrained and unconstrained components. The constrained part is not adaptive and the associated array response component is called the quiescent response. The unconstrained part consists of a blocking matrix which eliminates the desired signal, followed by the adaptive weights. The adaptive algorithm used is the NLMS [7] that minimizes the instantaneous output squared magnitude $\left|z(t)\right|^2$ through iteration.

4. EXPERIMENTAL RESULTS

To demonstrate the performance of the improved partially adaptive array, we present a design example as shown in Fig. 2 for the processing of a narrowband signal at 1 kHz. The concentric ring array has 68 elements arranged in 4 rings. The elements are equally spaced in each ring, and the number of elements in the rings, from the innermost are 12, 12, 20 and 24. Each ring is treated independently to calculate the $\mathbf{g}_i$ weights. The output of each ring is then multiplied by the adaptive weights $\mathbf{w}$ to obtain the final output.

The received array signal is simulated by a computer, which contains the desired signal coming from the DOA

($\phi = 0^0$, $\theta = 90^0$). The interference is composed of three narrowband signals of 1kHz coming from DOA's: ($\phi = 120^0$, $\theta = 75^0$), ($\phi = 150^0$, $\theta = 90^0$) and ($\phi = 220^0$, $\theta = 80^0$). The background noise is Gaussian and omni-directional. The signal to interference ratio (SIR) is -25dB, -35dB, and -30dB respectively, and the signal to background noise ratio is 0dB. The number of ensemble averages is 100. The penalty term α is set to 0.1. The processing results are presented in Fig. 3 and in Table I.

Fig. 3 shows the convergence result of the residual interference and noise power in four situations. The first graph shows the results for fully adaptive array. The convergence speed is slower than that in the other situations; however it achieves the smallest steady state error. The second graph shows the original partially adaptive array results [4]. The convergence rate is faster; however, the steady state error is slightly higher that in the fully adaptive array situation. The lower two graphs show the convergence results with the new improved array for two cases. In case I, the interference coming at, DOA ($\phi = 220^0$, $\theta = 80^0$) is known. Case II is when two interferences coming at DOA's ($\phi = 150^0$, $\theta = 90^0$) and ($\phi = 220^0$, $\theta = 80^0$) are known. In both cases the initial residual interference and noise power is much smaller than those in the fully adaptive and the original partially adaptive method. This is due to the elimination of the known interferences at the intra-ring level from the better intra-ring weights design given in (10). Also, convergence rates are faster. Their steady state errors are smaller than in the original partially adaptive array and closer to the values from the fully adaptive array. Case I initially converges slower than case II, but achieves smaller steady state error. Case II has the fastest initial convergence; however, after this initial stage, the values become larger than that in case I. One possible explanation is that the reshaping effect on the beampattern formed from $\mathbf{g}_i$ to cancel the two known interferences can force higher gain values at other spatial locations, thereby increasing the burden to the adaptive weights that now have to cancel the unknown interference and noise with higher power.

Iterations	Fully	Org. Partial	Case I	Case II
2	11.5665	11.9432	2.0878	0.7087
400	0.0943	0.0604	0.0489	0.0923
1200	0.0324	0.0456	0.0359	0.0629
2000	0.0267	0.0385	0.0306	0.0505
4000	0.0183	0.0283	0.0210	0.0256
8000	0.0163	0.0257	0.0181	0.0210
12000	0.0160	0.0254	0.0177	0.0203
20000	0.0163	0.0252	0.0183	0.0198

TABLE I

Comparison of residual interference and noise power, $\alpha = 0.1$

Table I shows the interference and noise power at different iterations. The second column is the results for fully adaptive array. The third column is the results for the original partially adaptive array. Columns four and five are the results for case I and II of the new improved partially adaptive array. The starting values for case I and case II are much lower than that in the original approaches. The effectiveness of the new intra-ring weights design is evident, and some of the interferences have been eliminated at this stage. In case II the starting value is smaller than that in case I because one more interference is known. The convergence in case I is faster than the convergence in the original partially adaptive array and at all times it maintains smaller values. Case II starts with a smaller value than all other cases; however, it cannot achieve smaller steady state error than the original partially adaptive array until the 4000th iteration. Then, the residual interference and noise error becomes smaller.

5. SUMMARY

In this paper, we have proposed an improved partially adaptive concentric ring array for the beamforming of a narrow-band signal in which the knowledge of some interferences are available. The partial adaptive array partitions the array weight vector into two components: intra-ring weights and inter-ring weights. The intra-ring weights are designed through LCMV over the interferences whose characteristics are known. The inter-ring weights are adaptive to remove the unknown interferences. The intra-ring weight design also includes a penalty term to limit the deviation from delay-and-sum solution to avoid the possibility of having higher gain over DOA's of the unknown interferences. The good performance of the proposed partially adaptive array is corroborated through simulations. The future plan is to investigate the choice of the penalty factor with respect to the number of known interferences and the signal-to-noise ratio. We also plan to compare our technique to the low rank dimension reduction approach for improving the adaptation speed.

6. ACKNOWLEDGMENTS

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