IDMR BEAMFORMING UNDER DIRECTION-INDEPENDENT STEERING VECTOR MISMATCH

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ABSTRACT

The output SINR achieved via Indirect Dominant Mode Rejection (IDMR) beamforming is substantially higher than the achieved with other beamforming algorithms. IDMR is based on a parametric estimate of the covariance matrix which is obtained using an estimate of the directions of the dominant sources and assuming that the array manifold is available. In most applications the array manifold is not known precisely and performance of IDMR can be deteriorated. The focus of this paper is to enable the IDMR beamformer to operate in a scenario of direction-independent steering vector mismatch. A modified version of an algorithm introduced by Friedlander is employed to estimate the direction-independent mismatch. Thereafter IDMR is implemented. Simulation analysis reveals that this technique enables IDMR to operate in a scenario of direction-independent manifold mismatch.

1. INTRODUCTION

It is well know that under low sample support the Minimum Variance Distortionless Response (MVDR) beamformer is non optimal [1] and low-rank MVDR based beamformers such as Principal Component Inverse (PCI), Dominant Mode Rejection (DMR), Conjugate Gradients (CG) can yield a higher output SINR than the full-rank MVDR beamformer. However, the output SINR obtained with these low-rank beamformers depends on the rank of operation and there is not yet an effective rule to select the optimal rank [2]. In [3] we introduced the IDMR beamformer which uses a parametric estimate of the covariance matrix to cancel the correlation between the desired signal and the interference. Due to finite sample averaging, residual correlations between sources are present in the sample covariance matrix, causing a degradation in the performance of MVDR based beamformers. Simulation analysis revealed that the output SINR obtained with the IDMR beamformer can be substantially higher than the obtained with PCI, DMR and CG.

The IDMR beamformer is based on a parametric estimate of the covariance matrix which is obtained assuming knowledge of the array manifold and using MUSIC to identify the location of the dominant sources. Since knowledge of the array manifold is required to form the covariance matrix, the IDMR beamformer has its performance deteriorated if there is steering vector mismatch. Steering vector mismatch is classified in two different types: direction-independent mismatch which equally affects signals arriving from different directions and direction-dependent mismatch which affects signals arriving from different directions in different ways. The work presented in this paper is focused on a technique to enable the IDMR beamformer to operate in a scenario of direction-independent array manifold mismatch.

It is proposed in this paper to use an algorithm introduced by Friedlander in [4] to estimate the direction-independent array manifold mismatch. Friedlander's algorithm proposes to estimate the mismatch vector with the objective of making the signal steering vectors orthogonal to the noise subspace. In this paper the researchers show that only the steering vector associated with the dominant signals are to be used and that the algorithm fails to estimate the mismatch vector when non dominant signal steering vectors are used.

Once an estimate of the steering vector mismatch is available IDMR is then implemented accounting for the steering vector mismatch. Simulation analysis reveal that this technique enables the IDMR beamformer to operate in a scenario of direction-independent manifold mismatch. In this scenario the output SINR obtained with IDMR is substantially higher than the output SINR obtained with either DMR or CG.

2. REVIEW OF IDMR

Consider an array of m sensors receiving signals from k sources of emission at directions θ_i. $i = 1, \ldots, k$ with respect to the array. It is assumed that the desired source is narrow-band and that narrow-band filtering about the center frequency of the desired source, f_o, occurs at the front end of the receiver such that the $k-1$ interfering signals are narrow-band and co-located in frequency with the desired signal at the beamformer input.

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The true covariance matrix $\mathbf{R}$ has the form

$$\mathbf{R} = \mathbf{A}\mathbf{P}\mathbf{A}^H + \sigma_n^2\mathbf{I}, \tag{1}$$

where $\mathbf{A}$ is called the signal direction matrix (SDM) and is given by $\mathbf{A} = [\mathbf{a}(\theta_1) \vdots \mathbf{a}(\theta_2) \vdots \ldots \vdots \mathbf{a}(\theta_K)]$, where $\mathbf{a}(\theta_i)$ $i = 1, ..., k$ are the steering vectors associated with the directions θ_i of the k signal sources. The matrix $\mathbf{P}$ is the source-signal correlation matrix, where σ_i^2 is the ith element along the main diagonal and is equal to the power of the signal arriving from the direction θ_i. σ_n^2 is the power of the spatially and temporally white Gaussian noise. $\mathbf{I}$ is the $m \times m$ Identity matrix.

Eqn. (1) can be rewritten as:

$$\mathbf{A}\mathbf{P}\mathbf{A}^H = \mathbf{R} - \sigma_n^2\mathbf{I}. \tag{2}$$

Multiplying the left hand side of the equation above by the pseudo-inverse $\mathbf{A}^\dagger = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$, and multiplying the right hand side by $\mathbf{A}^{\dagger H}$ yields:

$$\mathbf{P} = \mathbf{A}^\dagger(\mathbf{R} - \sigma_n^2\mathbf{I})\mathbf{A}^{\dagger H}. \tag{3}$$

MUSIC or other direction estimation algorithm is applied to the sample covariance matrix $\hat{\mathbf{R}}$ to estimate the signal directions. Subsequently, the SDM denoted by $\hat{\mathbf{A}}$ is formed from the estimated signal directions, given the known form of the array manifold. $\hat{\mathbf{A}}$ along with the sample covariance matrix $\hat{\mathbf{R}}$ and an estimate of the noise power $\hat{\sigma}_n^2$, is used to estimate the source-signal correlation matrix $\hat{\mathbf{P}}$ according to Eqn. (3). The elements not along the main diagonal of $\hat{\mathbf{P}}$, arising from residual correlations between sources due to finite sample averaging (or even due to true correlation between sources) are discarded, giving rise to a diagonalized source-signal correlation matrix estimate denoted $\hat{\mathbf{P}}_d$. Substituting $\hat{\mathbf{A}}$, $\hat{\mathbf{P}}_d$, $\hat{\sigma}_n^2$ into Eqn. (1) yields a parametric estimate of the covariance matrix denoted by $\mathbf{R}_{idmr}$, where the contributions due to correlations among sources are removed. The MVDR beamformer associated to the direction θ is then formed using $\mathbf{R}_{idmr}$ as

$$\mathbf{w}(\theta) = \frac{\mathbf{R}_{idmr}^{-1}\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\mathbf{R}_{idmr}^{-1}\mathbf{a}(\theta)}. \tag{4}$$

It has been observed that when refining the estimate of the correlation matrix, it is better to discard the signal that is to be extracted. This occurs because when we are estimating the signal direction there is an embedded mismatch to the true direction of the signal. Therefore, if the desired signal is used in the estimate the parametric covariance matrix the resulting beamformer may have a null in the direction of the desired signal. In this case the desired signal would be taken as an interferer. A simple way to avoid the desired signal in the refined correlation matrix is to disregard any signal within a certain distance of the look-direction θ.

The underlining idea of the Indirect-DMR technique is to form a parametric estimate of the correlation matrix with accurate information on the location and power of the dominant interferers. With this parametric estimate of the correlation matrix the MVDR beamformer allocates nulls at the positions of the interferes and the resulting beamformer is then used to extract the desired signal. It is important to note that low power interferers can be discarded when forming the parametric correlation matrix $\mathbf{R}_{idmr}$. This because low power interferers do not play a critical role in the total interference and, in most situations, it is difficult to detect and accurately locate their positions.

3. ESTIMATION OF DIRECTION-INDEPENDENT STEERING VECTOR MISMATCH

Let $\mathbf{a}(\theta)$ be the $m \times 1$ nominal steering vector for a signal arriving at the array from the direction θ, where m is the number of sensors. Assuming the presence of a steering vector mismatch which is independent of the direction θ the true steering vector $\tilde{\mathbf{a}}(\theta)$ is given by

$$\begin{aligned} \tilde{\mathbf{a}}(\theta) &= \boldsymbol{\gamma} \odot \mathbf{a}(\theta) \\ &= \boldsymbol{\Gamma}\mathbf{a}(\theta), \end{aligned} \tag{5}$$

where $\odot$ is the Hadamard product, $\boldsymbol{\gamma}$ is the $m \times 1$ array mismatch vector and $\boldsymbol{\Gamma}$ is a diagonal matrix where the diagonal is $\boldsymbol{\gamma}$.

Each component of $\boldsymbol{\gamma}$ represents a complex gain for each array element. The gain at each sensor needs to be estimated in relation to the gain of a reference sensor. Therefore, without loss of generality the reference sensor is assumed to be the first array element. Consider $\mathbf{U}$ to be the matrix which columns $\mathbf{u}_i$ are the noise eigenvectors of the sample covariance matrix $\hat{\mathbf{R}}$.

A direction finding algorithm such as MUSIC is employed to obtain a first estimate of the directions of the n dominant signals arriving at the array. The corresponding nominal steering vector $\mathbf{a}_j$, $j = 1 \ldots n$ are computed.

The algorithm consists of minimizing the projection of the steering vectors into the noise subspace:

$$\begin{aligned} &\min_{\boldsymbol{\Gamma}} \sum_{i=1}^{p}\sum_{j=1}^{n} |\mathbf{u}_i(\boldsymbol{\Gamma}\mathbf{a}_j)|^2 \\ &\text{s.t. } \boldsymbol{\gamma}^H\boldsymbol{\delta} = 1 \end{aligned} \tag{6}$$

where $\boldsymbol{\Gamma} = \text{diag}(\boldsymbol{\gamma})$ is a $m \times m$ diagonal matrix which the diagonal is the array mismatch vector $\boldsymbol{\gamma}$, p is the estimated dimension of the noise subspace, and $\boldsymbol{\delta} = [1\ 0\ 0\ ...\ 0]^T$. The constraint $\boldsymbol{\gamma}^H\boldsymbol{\delta} = 1$ guarantees that the first sensor is the reference sensor and thus has a unity gain. We note that Eqn. (6) is valid when s is underestimated, however it is not valid when p is overestimated. Therefore we observe that it is better to underestimate rather than to overestimate p. We can write the objective function of Eqn. (6) as:

$$\min_{\mathbf{\Gamma}} \sum_{i=1}^{p} \sum_{j=1}^{n} |\mathbf{u}_i^H (\mathbf{\Gamma} \mathbf{a}_j)|^2 =$$

$$\min_{\mathbf{\Gamma}} \sum_{i=1}^{p} \sum_{j=1}^{n} (\mathbf{\Gamma} \mathbf{a}_j)^H \mathbf{u}_i \mathbf{u}_i^H (\mathbf{\Gamma} \mathbf{a}_j) =$$

$$\min_{\mathbf{\Gamma}} \sum_{j=1}^{n} (\mathbf{\Gamma} \mathbf{a}_j)^H \left[\sum_{i=1}^{p} \mathbf{u}_i \mathbf{u}_i^H \right] (\mathbf{\Gamma} \mathbf{a}_j) =$$

$$\min_{\mathbf{\Gamma}} \sum_{j=1}^{n} (\mathbf{\Gamma} \mathbf{a}_j)^H \mathbf{U}\mathbf{U}^H (\mathbf{\Gamma} \mathbf{a}_j) =$$

$$\min_{\boldsymbol{\gamma}} \sum_{j=1}^{n} (diag(\mathbf{a}_j)\boldsymbol{\gamma})^H \mathbf{U}\mathbf{U}^H (diag(\mathbf{a}_j)\boldsymbol{\gamma}) =$$

$$\min_{\boldsymbol{\gamma}} \sum_{j=1}^{n} \boldsymbol{\gamma}^H diag(\mathbf{a}_j)^* \mathbf{U}\mathbf{U}^H diag(\mathbf{a}_j) \boldsymbol{\gamma} =$$

$$\min_{\boldsymbol{\gamma}} \boldsymbol{\gamma}^H \left[\sum_{j} diag(\mathbf{a}_j)^* \mathbf{U}\mathbf{U}^H diag(\mathbf{a}_j) \right] \boldsymbol{\gamma} \qquad (7)$$

Defining the matrix

$$\mathbf{Z} = \left[\sum_{j=1}^{n} diag(\mathbf{a}_j)^* \mathbf{U}\mathbf{U}^H diag(\mathbf{a}_j) \right] \qquad (8)$$

the problem can be re-stated as:

$$\min_{\boldsymbol{\gamma}} \boldsymbol{\gamma}^H \mathbf{Z} \boldsymbol{\gamma} \qquad (9)$$
$$\text{s.t. } \boldsymbol{\gamma}^H \boldsymbol{\delta} = 1$$

The solution of the above minimization problem is well known and is given by:

$$\boldsymbol{\gamma} = \frac{\mathbf{Z}^{-1}\boldsymbol{\delta}}{\boldsymbol{\delta}^H \mathbf{Z}^{-1} \delta} \qquad (10)$$

The algorithm to estimate the sensors gain vector can be summarized as:

i. Using the sample covariance matrix and the nominal values of the steering vectors, MUSIC is applied to estimate the directions of the dominant signals.

ii. The sensors' gain vector $\boldsymbol{\gamma}$ is estimated with the objective to minimize the projection of the dominant signal steering vectors into the noise subspace.

iii. Go back to step *i replacing the nominal steering vector by the corrected steering vector. Stop the algorithm when $\boldsymbol{\gamma}$ converges.*

4. SIMULATIONS

Simulations were conducted employing a uniform and linear array of $m = 24$ elements with half-wavelength spacing receiving plane-wave signals. The noise at each array element is spatially and temporally white Gaussian. The incident signals are modeled as narrowband with amplitudes modeled as complex Gaussian random processes. The scenario is composed of 12 uncorrelated incident signals with arrival directions (in degrees) and respective SNRs (in dB) at each array element as shown in Table 1. The array mismatch gain at each array element was modeled as a complex Gaussian random variable with a standard deviation of $\sigma_n = 0.5$ centered at $1 + j0$. That is, the real and imaginary parts are independent and have a standard deviation of $1/\sqrt{8}$. To estimate the covariance matrix 24, snapshots were used.

Angle	**SNR**
6.8	30.0
-34.7	29.4
47.9	29.0
11.8	28.0
-69.7	22.7
19.8	21.5
-41.5	20.0
-46.6	19.6
-50.9	11.9
24.2	11.2
86.9	10.7
0	10.0

Table 1. Direction and SNR of incident signals.

It can be observed that different values of the number of dominant signals, n, may be used in Eqn. (6). This is the total number of dominant steering vectors that will be made orthogonal to the noise subspace by multiplying these steering vectors by the mismatch gain vector $\boldsymbol{\gamma}$. Simulations were performed with either 5 or 10 dominant signals. Simulations reveal that using 5 dominant signals yields better estimates of $\boldsymbol{\gamma}$ than when using 10 dominant signals.

Fig. 1 shows the real and imaginary parts of each component of the estimated mismatch gain vector $\boldsymbol{\gamma}$. The black circle indicates the region within one standard deviation of the mean $1 + j0$. The vector has 24 components; the first component is deterministic with a value of $1 + j0$. The imaginary part of each component is plotted on the Y axis, and the real part on the X axis. The true values of each component are marked with an "X"; the first estimate is marked with an "o" and the final estimate after 6 iterations is marked with a "*".

After estimating the sensor mismatch vector $\boldsymbol{\gamma}$ the IDMR beamformer was employed using the estimates of $\boldsymbol{\gamma}$ to correct the nominal value of the steering vectors. Figs. 2 and 3, for $n = 5$ and $n = 10$, respectively plot the average output SINR over 200 simulation runs obtained with IDMR when γ is estimated. Curves obtained when the true value of the mismatch is employed and when the mismatch is not estimated are also plotted. It can be observed that performance of IDMR is severally degraded if the mismatch is not accounted for. For comparison reasons the output SINR obtained with CG and DMR are also plotted. Two curves are plotted for CG and two for DMR. In one curve the mismatch is ignored and in the other the true value of the mismatch is taken into account. It can be observed that both CG and DMR are less sensitive to the mismatch than IDMR. However, when the mismatch is estimated IDMR yields a significantly higher output SINR. From Fig. 3 it can be observed that if 10 dominant sources are used in Eqn. (6) the estimates of the mismatch are less accurate and performance is degraded. The degradations is caused by the fact that there are higher errors made in the estimated directions when less dominant sources are used in Eqn. (6).

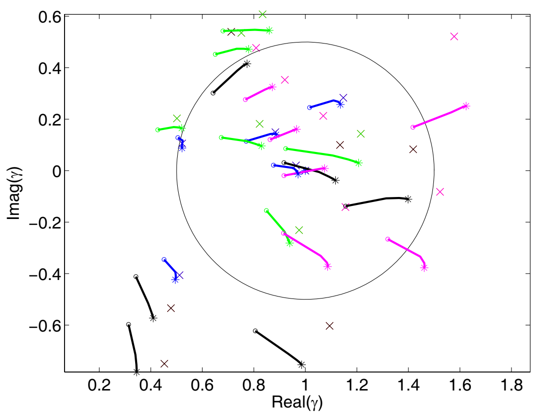

Fig. 1. $\hat{\gamma}$.

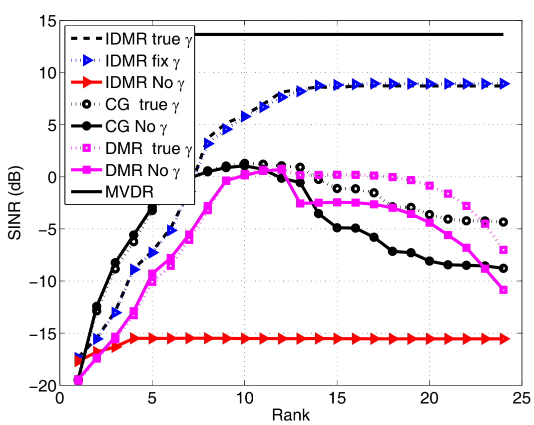

Fig. 2. Output SINR using 5 dominant signals.

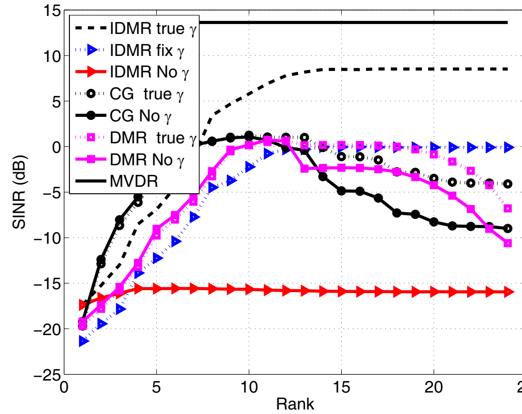

Fig. 3. Output SINR using 10 dominant signals.

5. CONCLUSION

To enable the IDMR beamformer to operate in a scenario of direction-independent steering vector mismatch an iterative algorithm to estimate the referred mismatch was proposed. This algorithm is a modified version of an algorithm proposed by Friedlander. It was shown that in order to properly estimate the mismatch vector, accurate estimates of the angles of the dominant sources must be available. Simulation analysis revealed that IDMR may be applied in conjunction with the algorithm proposed herein to circumvent steering vector mismatch. Simulations also revealed that in a scenario of direction-independent steering vector mismatch the output SINR obtained with IDMR is significantly higher than the obtained with either DMR or CG.

6. REFERENCES

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