LOCALIZATION OF DIFFUSIVE SOURCES USING DISTRIBUTED SEQUENTIAL BAYESIAN METHODS IN WIRELESS SENSOR NETWORKS

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ABSTRACT

We develop an efficient distributed sequential Bayesian estimation method to localize a diffusive source in wireless sensor networks. Potential applications include security, environmental monitoring, pollution control, and explosives detection. We first derive the physical model of the substance dispersion by solving the diffusion equations under different environmental scenarios. We then integrate the derived dispersion models into the distributed processing technologies, and propose a distributed sequential Bayesian localization technique, in which the state belief is transmitted in the wireless sensor networks and updated using the measurements from the new sensor node. In order to decrease the required communication burden we propose two parameterizable belief approximations: a Gaussian approximation and a new linear combination of polynomial Gaussian approximation. We also apply the idea of information-driven sensor scheduling and select the next sensor node according to certain criterions to reduce the response time and save energy consumption of the sensor network.

1. INTRODUCTION

In this paper we address the problem of developing efficient distributed parameter estimation methods to localize a diffusive source using wireless sensor networks. The proposed methods can be easily extended to other applications related to monitoring diffusion phenomena. Potential applications include security, environmental monitoring, pollution control, and explosives detection.

Recently, wireless sensor networks have been the object of intensive interest in the research community [1]. Inexpensive, smart nodes with multiple on-board sensors, networked through wireless lines as well as Internet and deployed in large numbers, interact intelligently with the physical world. In a typical wireless sensor network, each node operates unattended with limited battery power and limited signal-processing capability; it communicates wirelessly with the other nodes in its radio communication range. These wireless communications consume a significant part of the available energy, and hence dominate the life of the wireless sensor network. Therefore, to solve these problems, the essential point of this paper is to propose a distributed parameter estimation method based on a sequential Bayesian approach and information-driven dynamic collaborative information processing [2]. The proposed methods are suitable for the strict constraints on the power and computational capabilities of the sensor nodes, and hence can be implemented efficiently in wireless sensor networks. In our previous work, we proposed model-based integrated biochemical sensor array processing methods for detecting and estimating dispersions in realistic environments [3]. However, these methods use centralized processing approaches, and cannot be directly used in wireless sensor networks.

We first derive the physical models for the spatial and temporal concentration distribution of the dispersed substance from a diffusive source by solving the diffusion equations under different initial conditions and environmental scenarios. We then transform the dispersion models to parametric statistical measurement models, and develop a distributed parameter estimation method using a sequential Bayesian approach. The main idea here is that the posterior density function, also known as the belief, of the parameters of interest is updated incrementally when new measurements are obtained, until a desired performance threshold is satisfied. The estimates of the parameters are calculated from the obtained belief according to some criteria. In contrast to the ordinary sequential Bayesian methods in which the belief is updated sequentially in the time domain [4], in our method the belief is transmitted in the sensor network and updated incrementally in the space domain. Since the information is transmitted only between sensor nodes and their neighbors, a fusion center is not needed; hence, we realize a fully distributed estimation.

The derived dispersion models yield statistical measurement models that are highly nonlinear and non-Gaussian, especially when nuisance parameters appear. Therefore, existing recursive Bayesian estimation methods [5] cannot be directly applied. Some possible alternative methods, such as sequential Monte Carlo methods [4], are also inapplicable because they require large data transmissions between the sensor nodes. Since decreasing the power consumption is always an important issue in wireless sensor networks, in our methods we propose to approximate the belief by a family of parameterizable probability distributions and transmit the belief parametrically; therefore, we decrease the communication requirements dramatically. We propose two parametric belief representations: a Gaussian approximation and a new linear combination of polynomial Gaussian (LPG) function approximation.

In Section 2 we derive the physical and statistical measurement models of the diffused substance. In Section 3 we present the proposed distributed sequential Bayesian estimation method. Numerical examples are given in Section 4 to illustrate the performance of proposed method.

2. PHYSICAL AND MEASUREMENT MODELS

In this section we first derive computational physical models describing the space-time substance dispersion mechanisms under various environmental scenarios; then we transform the dispersion model to a statistical measurement model.

This work was supported by National Science Foundation Grant CCR-0330342.

2.1. Physical Models for Dispersion

We model the transport of a substance from a diffusion source by solving the diffusion equation. Various initial and boundary conditions and environmental effects are considered according to different scenarios. We first compute the concentration for a stationary impulse point source to find the Green function; then we extend the results to a continuous source by integrating the source release rate with the Green function.

(i) Stationary impulse source: Let $c(\boldsymbol{r}, t)$ denote the concentration of the diffused substance at a position $\boldsymbol{r} = (x, y, z)$ and time t. For a source-free volume and space-invariant diffusivity κ, the concentration of a dispersed substance follows the diffusion equation

$$\frac{\partial c(\boldsymbol{r},t)}{\partial t} = \kappa \nabla^2 c(\boldsymbol{r},t) = \kappa \left[\frac{\partial^2 c(\boldsymbol{r},t)}{\partial x^2} + \frac{\partial^2 c(\boldsymbol{r},t)}{\partial y^2} + \frac{\partial^2 c(\boldsymbol{r},t)}{\partial z^2} \right]. \tag{1}$$

The appropriate boundary and initial conditions are applied to solve this differential equation. For simplicity of the presentation, we consider the diffusion equation only in the z dimension. It can be easily extended to the three-dimensional cases.

Initial and boundary conditions: In order to calculate the Green function, we assume that there is an impulse point source of a unit mass release rate stationary at $z = z_0$, i.e., the initial condition is $c_z(z,t) = \delta(z - z_0)\delta(t - t_{\mathrm{I}})$, where t_{I} is the initial release time. We assume a homogeneous semi-infinite medium in the region $z > 0$ that can be used to model the diffusion in air above the ground. For a generally permeable boundary condition, a reasonable assumption is the rate of loss of the diffusing substance is proportional to the actual concentration in the surface at any time [3], i.e.,

$$-\kappa \frac{\partial c_z}{\partial z} = -\alpha c_z \quad \text{at} \quad z = 0, \tag{2}$$

where α is a constant of proportionality.

Solutions: Solving the diffusion equation under an impermeable boundary condition and the above initial conditions, we obtain the solution

$$c_z(z,t) = \frac{1}{2\sqrt{\pi\kappa(t - t_{\mathrm{I}})}} \left\{ \exp\left[-\frac{(z - z_0)^2}{4\kappa(t - t_{\mathrm{I}})} \right] + \exp\left[-\frac{(z + z_0)^2}{4\kappa(t - t_{\mathrm{I}})} \right] \right\}. \tag{3}$$

This solution can also be interpreted as a superposition of contributions from the actual source and a mirror-image source of equal magnitude at $z = -z_0$.

External forces: In the presence of external forces such as wind, flow, and gravity, the diffusion equation (1) takes the form

$$\frac{\partial c_z}{\partial t} = \kappa \frac{\partial^2 c_z}{\partial z^2} - v_z \frac{\partial c_z}{\partial z}, \tag{4}$$

where v_z represents the wind speed in the z direction. The second term on the right-hand side is called *advection*. We can reduce the above problem to the ordinary diffusion equation without the advection term by applying the following transformation

$$c_z = c_z^* \exp\left\{ \frac{v_z}{2\kappa}(z - z_0) - \frac{v_z^2 (t - t_{\mathrm{I}})}{4\kappa} \right\} \tag{5}$$

to the differential equation (4); then, we can implement the same procedures in the previous sections to solve this type of problem.

Turbulence: We also note that in most problems, air turbulence can have significant impact on the concentration distribution. However, the above solution still holds as a reasonable approximation in many cases, with only a larger diffusivity, typically from 10^2 to 10^{10} times larger [3].

(ii) Continuous sources: A solution for a continuous source is deduced from the corresponding Green function by integrating it with the source release rate function. Suppose that we have a stationary point source continuously releasing a substance at a mass rate $\mu(t)$. Let $c_{\mathrm{Green}}(\boldsymbol{r}, t)$ denote the Green's function of the impulse source case. Then the concentration of a continuous point source is obtained using the integral:

$$c(\boldsymbol{r}, t) = \int_{t_{\mathrm{I}}}^{t} \mu(\tau) c_{\mathrm{Green}}(\boldsymbol{r}, t - \tau)\, \mathrm{d}\tau. \tag{6}$$

2.2. Measurement Models

To model the measurements, we consider a spatially distributed wireless sensor network. Each sensor node in this network is located at a known position and can measure the substance concentration from the diffusive sources. Assuming the physical models derived above represent the underlying dispersion mechanism, we obtain a measurement model for a sensor node at a position $\{\boldsymbol{r}_i\}$ and taking measurements at time $\{t_j\}$ as

$$y(\boldsymbol{r}_i, t_j) = c(\boldsymbol{r}_i, t_j) + b + e(\boldsymbol{r}_i, t_j), \quad e(\boldsymbol{r}_i, t_j) \sim \mathcal{N}(0, \sigma^2), \tag{7}$$

where $c(\boldsymbol{r}_i, t_j)$ is the concentration of interest; b is a bias term, representing the sensor's response to foreign substances and assumed to be a unknown constant; and $e(\boldsymbol{r}_i, t_j)$ is the sensor's noise, assumed to be Gaussian distributed, independent in time and space. For the simplicity of presentation we assume the source's substance-releasing rate is time invariant, and we denote $y_{ij} = y(\boldsymbol{r}_i, t_j)$, $e_{ij} = e(\boldsymbol{r}_i, t_j)$, and $\mu a_{ij}(\boldsymbol{\theta}) = c(\boldsymbol{r}_i, t_j)$ where $\boldsymbol{\theta}$ represents the unknown source and medium parameter vector.

Nuisance parameters: For a 2D localization problem, only the source position parameters $(x_0,\ y_0)$ are of interest; others are nuisance parameters. For these nuisance parameters, some of them, e.g., κ, b, and σ^2 can be measured during the calibration phase. For the remaining nuisance parameters, e.g., source release rate μ, we can remove them by integration as follows. We first assume an *a priori* probability density function (PDF) for μ, which is a uniform distribution between μ_{low} and μ_{high}. Denote the unknown parameter vector as $\boldsymbol{\theta} = [x_0,\ y_0]^{\mathrm{T}}$. Consequently, according to the measurement model in (6), the marginal PDF of y_{ij} given $\boldsymbol{\theta}$ is

$$p(y_{ij} \mid \boldsymbol{\theta}) \quad = \quad \int p(y_{ij} \mid \boldsymbol{\theta},\ \mu) p(\mu)\, \mathrm{d}\mu. \tag{8}$$

3. DISTRIBUTED SEQUENTIAL BAYESIAN ESTIMATION

In this section, we first develop the proposed distributed sequential Bayesian estimation algorithm to localize a diffusive source in wireless sensor networks. We then propose two parametric belief representations: Gaussian approximation and LPG function approximation. Finally, we discuss the sensor node selection strategies.

3.1. Distributed Sequential Bayesian Estimation Algorithm

The proposed scheme of localizing a diffusive source has two phases: the measuring phase and the estimation phase. In the measuring phase, the activated sensor nodes for such a task measure the substance concentration at specific time samples $t_j,\ j = 1, \ldots, N$, and then return to sleeping status. In the estimation phase, we process a sequential Bayesian algorithm activated by an initial sensor node, to localize the diffusion source. Here, we denote the collection of all measurements at the ith sensor node by $\boldsymbol{y}_i$, and the measurement sequence up to the ith sensor node by $\boldsymbol{y}_{1:i}$.

In wireless sensor networks, because of their energy limitation and failure tolerance properties, the usually centralized Bayesian estimation methods cannot be practically directly applied. Hence, we propose a distributed sequential Bayesian estimation, in which the state belief is transmitted in the sensor network and updated using the previous information and the measurements at the current node. Suppose the current ith sensor node obtains the belief passed from the $(i-1)$th sensor node as $p(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i-1})$, and the likelihood function of the observation at the ith sensor node is $p(\boldsymbol{y}_i \mid \boldsymbol{\theta})$. We make the following conditional independence assumption:

A1: Conditioned on $\boldsymbol{\theta}$, the measurements at the current sensor node $\boldsymbol{y}_i$ are independent of the measurements at the previous sensor node $\boldsymbol{y}_{1:i-1}$, i.e.,

$$p(\boldsymbol{y}_i \mid \boldsymbol{\theta}, \boldsymbol{y}_{1:i-1}) = p(\boldsymbol{y}_i \mid \boldsymbol{\theta}). \tag{9}$$

Then, we can prove that using $\boldsymbol{y}_i$ we can update the belief to $p(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i})$ by applying the Bayesian rule as

$$p(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i}) = \frac{p(\boldsymbol{y}_i \mid \boldsymbol{\theta}) p(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i-1})}{\int p(\boldsymbol{y}_i \mid \boldsymbol{\theta}) p(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i-1}) \, \mathrm{d}\boldsymbol{\theta}}. \tag{10}$$

At the first sensor node where $i = 1$, we have

$$p(\boldsymbol{\theta} \mid \boldsymbol{y}_1) = \frac{p(\boldsymbol{y}_1 \mid \boldsymbol{\theta}) \pi(\boldsymbol{\theta})}{\int p(\boldsymbol{y}_1 \mid \boldsymbol{\theta}) \pi(\boldsymbol{\theta}) \, \mathrm{d}\boldsymbol{\theta}}, \tag{11}$$

where $\pi(\boldsymbol{\theta})$ is the prior PDF for the location $\boldsymbol{\theta}$. Therefore, the current minimum-mean-squared error (MMSE) estimate at sensor node i can be calculated as

$$\begin{aligned} \widehat{\boldsymbol{\theta}} &= \mathrm{E}[\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i}] \\ &= \frac{\int \boldsymbol{\theta} p(\boldsymbol{y}_i \mid \boldsymbol{\theta}) p(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i-1}) \, \mathrm{d}\boldsymbol{\theta}}{\int p(\boldsymbol{y}_i \mid \boldsymbol{\theta}) p(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i-1}) \, \mathrm{d}\boldsymbol{\theta}}. \end{aligned} \tag{12}$$

In Equation (10) we observe that the current belief is a product of the previous belief and the current likelihood function (up to a normalized coefficient), which provides us with a sequential algorithm to update the belief. In the algorithm, we need to transmit the current belief to the next sensor node. Considering the strict power and resource constraints in wireless sensor networks, we propose to approximate the state belief by a family of parameterizable distributions, through which we decrease the communication burden significantly.

3.2. Belief Approximation

We propose two parametric distribution approximations: a Gaussian approximation and a new approach — a linear combination of polynomial Gaussian (LPG) approximation. In both of these methods, we consider the accuracy of the approximation as well as decreasing the computation complexity to fit the limited processing capability of the sensor nodes.

(i) Gaussian approximation: For the continuous posterior distribution function, there exists an asymptotic posterior normality property [6] such that for a large number of observations, the posterior function becomes highly peaked and behaves like a multivariate normal density. Hence, we first propose to approximate the belief $p(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i})$ by a multivariate Gaussian distribution with mean $\mathrm{E}[\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i}]$ and covariance $\mathrm{Cov}[\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i}]$ as

$$\mathrm{E}[\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i}] = \frac{\int \boldsymbol{\theta} \left[\prod_{j=1}^{N} p(y_{ij} \mid \boldsymbol{\theta})\right] p(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i-1}) \, \mathrm{d}\boldsymbol{\theta}}{\int \left[\prod_{j=1}^{N} p(y_{ij} \mid \boldsymbol{\theta})\right] p(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i-1}) \, \mathrm{d}\boldsymbol{\theta}} \tag{13}$$

$$\mathrm{Cov}[\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i}] = \mathrm{E}\left[(\boldsymbol{\theta} - \mathrm{E}[\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i}])(\boldsymbol{\theta} - \mathrm{E}[\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i}])^{\mathrm{T}} \mid \boldsymbol{y}_{1:i}\right]. \tag{14}$$

Thus, in order to update the belief, we need to transmit only the above mean and covariance, with total of 5 scalars, to the next sensor node for our current localization problem, which decreases the communications required between sensor nodes dramatically. To calculate the mean and covariance efficiently, we apply the Laplace approximation to calculate the integration in (13) and (14) (see [7] for details). Even though the Gaussian approximation method leads to low wireless transmission and low computational complexity, its approximation accuracy is not very high; hence it is suitable for cases when the belief density is Gaussian-like, or the processing capacity at the sensor node is very limited.

(ii) LPG function approximation: We propose to use a family of LPG functions to approximate the belief. The PG functions represent Gaussian density functions multiplied by polynomials, and LPG denotes a linear combination of PG functions. Using LPG functions to approximate the belief in wireless sensor networks has the advantages that (i) this approximation family is a natural generalization of the Gaussian approximation, thus it provides higher approximation accuracy that leads to lower total data transmission; (ii) any moments of LPG functions can be evaluated analytically, which decreases the computation complexity at each sensor node. In order to process the approximation efficiently, we propose such an algorithm in which we separate the whole approximation procedure into two phases: local approximation and global approximation. That is, we first approximate the belief at several important peaks (peaks whose contribution to the density cannot be ignored) using PG functions, and then we create an optimal linear combination of these local expansions to represent the global belief function.

(a) Local approximations: Here, we first localize the important peaks of the belief, then apply an orthogonal family of PG functions to approximate the density around each peak. For such an orthogonal family we use the Hermite functions, which (up to normalization factors) are given by a Gaussian density multiplied by Hermite polynomials. Denote $f_l(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i})$ as the local approximation of the belief around the lth peak, we have the orthogonal expansions in terms of multivariate Hermite polynomials as

$$f_l(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i}) = \phi(\boldsymbol{\theta}; \boldsymbol{\mu}, \Sigma) \sum_{0 \le \nu_1 + \nu_2 \le k} C_{\boldsymbol{\nu}} \cdot \mathrm{He}_{\boldsymbol{\nu}}(\boldsymbol{\theta}; \boldsymbol{\mu}, \Sigma) \tag{15}$$

where k is the bounded degree of the polynomials; $\phi(\boldsymbol{x}; \boldsymbol{\mu}, \Sigma)$ is the multinormal probability density function; $\mathrm{He}_{\boldsymbol{\nu}}(\boldsymbol{x}; \boldsymbol{\mu}, \Sigma)$ denotes the multivariate Hermite polynomials

$$\mathrm{He}_{\boldsymbol{\nu}}(\boldsymbol{x}; \boldsymbol{\mu}, \Sigma) = \frac{(-1)^{\Sigma \nu_i}}{\phi(\boldsymbol{x}; \boldsymbol{\mu}, \Sigma)} \frac{\partial^{\Sigma \nu_i}}{\partial \boldsymbol{x}^{\boldsymbol{\nu}}} \phi(\boldsymbol{x}; \boldsymbol{\mu}, \Sigma), \tag{16}$$

here $\boldsymbol{x}^{\boldsymbol{\nu}} = x_1^{\nu_1} \cdots x_n^{\nu_n}$; $C_{\boldsymbol{\nu}}$ are the expansion coefficients that need to be determined. In our algorithm, for the multivariate Gaussian density, we use the position of the peak as its mean $\boldsymbol{\mu}_l$, and let its associated covariance matrix Σ_l be equal to the negative inverse of the Hessian (at the peak) of the belief logarithm, i.e.,

$$\Sigma_l = (-H_l)^{-1} = [-\nabla^2 L_i(\boldsymbol{\mu}_l)]^{-1} \tag{17}$$

where $L_i(\boldsymbol{\theta}) = \log p(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i})$; we calculate the expansion coefficients C_ν using the least square (LS) method.

(b) Global approximations: After we calculate the local expansions for all the peaks, we merge them to obtain a global approximation of the belief based on an optimal linear combinations. Then the final global approximation is

$$f(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i}) = \sum_l \lambda_l f_l(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i}) \tag{18}$$

where the combination coefficients λ_l are determined using linear LS methods on a new global mesh of measurement points.

3.3. Sensor Node Scheduling

In the proposed distributed sequential Bayesian estimation method, we update the belief incrementally by incorporating the measurements of other nearby sensor nodes. However, not all available sensor nodes in the network provide useful information that improves the estimate; furthermore, some information may be redundant. Therefore, we need to select an optimal subset and an optimal order of incorporating these measurements into our belief update, which provides a faster reduction in estimation uncertainty and incurs a lower communication burden. We propose to use the following two information utility measures: Mahalanobis distance and covariance-based measure.

(i) Mahalanobis distance: At the current ith sensor node, the utility function for selecting the next sensor node, with respect to the current source position estimate characterized by the posterior mean $\widehat{\boldsymbol{\theta}}_i$ and covariance Σ_i, is defined as the negative of the Mahalanobis distance, i.e.,

$$-(\boldsymbol{r}_{i+1} - \widehat{\boldsymbol{\theta}}_i)^{\mathrm{T}} \Sigma_i^{-1} (\boldsymbol{r}_{i+1} - \widehat{\boldsymbol{\theta}}_i), \tag{19}$$

where $\boldsymbol{r}_{i+1}$ is the position of the next sensor node. This utility function is easy to calculate and works well under simple environmental scenarios.

(ii) Covariance-based measure: In this case we derive utility measures based on the covariance Σ of the posterior distribution $p(\boldsymbol{\theta} \mid \boldsymbol{y}_{1:i+1})$. In order to calculate this covariance matrix, we need to obtain measurements from the nearby sensor nodes. In order to avoid requiring the measurements from each neighboring node, we propose to predict the covariance matrix according to the current belief and the prior knowledge about the sensor positions and sensing model.

4. NUMERICAL EXAMPLES

In this section, we present numerical examples to illustrate the performance of our proposed distributed sequential Bayesian localization method. We will compare the efficiency of the different belief representation methods, and consider the influence of the diffusion models and sensor selection criteria on the performance of the localization algorithms. In these examples, $K = 300$ wireless sensor nodes are randomly deployed in a square area of 100×100 length-units. For the diffusion model, we illustrate the environment as a homogeneous semi-infinite medium with impermeable boundary. We use the scenario of a stationary source located at the boundary surface, i.e., $z_0 = 0$. The other parameters are taken to be $\mu = 1$ Kg/s, $\kappa = 40$ unit^2/s, and $t_{\mathrm{I}} = 0$ s.

We first compare the performance of the proposed estimation algorithm using the LPG approximation and the Gaussian approximation. The estimation bias and covariance with respect to the number of used sensor nodes are illustrated in Fig. 1. We observe that the distributed estimation using the LPG approximation converges to the true value much faster than the Gaussian approximation, since the former provides us a more accurate belief representation. In Fig. 1 we can see that when we use the LPG approximation, we need to use only 7 sensor nodes to reach the performance threshold $\det(\Sigma) = 10^{-5}$, whereas for the Gaussian approximation, we need 20 nodes. In the next example, we assume there exists wind in the medium with the speed $\boldsymbol{v} = (-20, -10)$ length-unit/s. We implement the proposed distributed localization algorithm with two different sensor selection criteria: the Mahalanobis distance and the covariance-based measure. Their performance comparison is shown in Fig. 2. We observe that the covariance-based measure performs better than the Mahalanobis distance, which means the covariance-based measure is more suitable for a complex scenario.

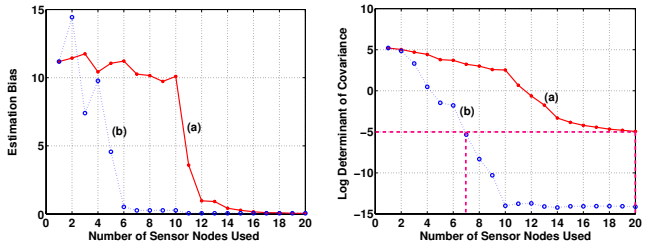

Fig. 1. Estimation bias and log determinant of the estimation covariance vs. the number of sensor nodes used under different belief representations: (a) Gaussian; (b) LPG.

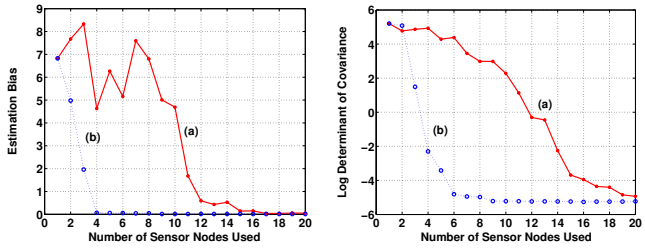

Fig. 2. As in Fig. 1, but for different sensor selection criteria: (a) Mahalanobis distance; (b) covariance-based measure.

5. CONCLUSIONS

We addressed the problem of deriving efficient distributed estimation methods to localize a diffusive source in wireless sensor networks. We integrated the underlying dispersion model into the signal processing technologies and proposed a distributed sequential Bayesian estimation. We propose two parameterizable belief approximations: a Gaussian approximation and a new LPG approximation. We also apply the idea of information-driven sensor scheduling and select the next sensor node optimally to reduce the response time and save energy consumption of the sensor network.

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