DECENTRALIZED DETECTION AND LOCALIZATION THROUGH SENSOR NETWORKS DESIGNED AS A POPULATION OF SELF-SYNCHRONIZING OSCILLATORS

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ABSTRACT

The detection and localization of an event through a sensor network is a topic that has attracted considerable attention recently because of many potential applications. Typically, these decisions are taken by conveying the sensor measurements to a sink node that processes the data and provides an estimate. However, the presence of a sink node creates a bottleneck that is the cause of potential congestions and it poses problems of scalability. In this work, we propose a decentralized decision scheme that is capable to achieve optimal decisions without requiring a fusion center. The network is composed of a set of mutually coupled oscillators, where each node is coupled only to the nearest nodes. We show how to achieve optimal detection for both deterministic and random signals by properly selecting the parameters of the coupling mechanism. Furthermore, if the nodes know their own positions and the network is connected, we show how to make each node able to perform a totally distributed energybased source localization.

1. INTRODUCTION

The fundamental challenge in the research on sensor networks focuses on the possible strategies to design networks composed of a multitude of cheap, lightweight components that are possibly individually inaccurate but, as a whole, are capable of providing overall reliable decisions. Many efforts have been devoted to the optimization of the mechanisms to convey local estimates or decisions to a fusion center [1], [2]. However, it is precisely this intrinsic need of gathering all relevant information at the sink nodes that makes the whole process critical, as there may be congestions around the sink node that may prevent important information to reach the control centers. Most of the current research on sensor networks today aims at exporting part of the huge background of knowledge accumulated in telecommunication networks in the sensor field, with the specific task of designing energy-efficient communication systems. However, in most applications, the requirements and constraints present in sensor networks are so different from the equivalent values typically occurring in telecommunication networks that it may be more advisable to shift the basic paradigm and devise totally innovative decision and communication strategies. An interesting alternative approach was proposed by Rabbat and Nowak in [3], where *in-network*, or distributed, data processing is performed on each node so that a parameter estimate is made circulate through the network and, along the way, small adjustments are implemented based on local data, in order to, possibly, reach an optimal estimate. Along a conceptually similar framework, Hong, Cheow and Scaglione [5] suggested the use of mutually coupled oscillators as the basic mechanism to reach network consensus without the need for sending the data to a fusion center. The principle ensuring the self-synchronization capability of the system proposed in [5], [4] relied on a theorem proved by Mirollo and Strogatz in [6], where the network was supposed to be fully connected. This assumption was later removed by Lucarelli and Wang in [7], who proved that the only really needed property is global connectivity, that is the property that there is a path between each pair of nodes. This was a significant step, as it relaxes the need for global coupling, as local coupling is sufficient, provided that the global connectivity is guaranteed. The oscillator and coupling model proposed in [5], [4], and [7] associates the local estimate to the time shift of a pulse oscillator. However, especially for large scale network, this may create a problem, as the information bearing time shift may become indistinguishable from the propagation delay. In [9] it was proposed a different model able to remove this potential ambiguity and to reach decentralized maximum likelihood estimates through mutual coupling among nearby nodes. The aim of this work is to show how to exploit the model propose in [9] to achieve optimal detection and localization of spatially distributed events through local coupling.

2. SELF-SYNCHRONIZATION OF LOCALLY COUPLED OSCILLATORS

The proposed sensor network is composed of N nodes and each node is equipped with four basic components: i) a *transducer* that senses the physical parameters of interest (e.g., temperature, concentration of contaminants, radiation, etc.); ii) a *local detector* or *estimator* that, based on the sensed quantities, takes an initial decision; iii) a *dynamical system* (termed oscillator, for simplicity) whose state evolves in time according to a differential equation which is periodically initialized with the local decision and it is coupled with the states of nearby sensors; iv) a *radio interface* that transmits the state of the associated dynamical system and receives the state of nearby nodes.

Denoting by ω_i a function of the initial local measurement taken by node *i*, the dynamical system (oscillator) present in node *i* evolves according to the following differential equation [8]

$$\dot{\theta}_i(t) = \omega_i + \frac{K}{c_i} \sum_{j=1}^N a_{ij} f[\theta_j(t) - \theta_i(t)], \ i = 1, \dots, N, \quad (1)$$

where $\theta_i(t)$ is the state function of the *i*th sensor ($\theta_i(0)$ may be initialized as a random number); $f(\cdot)$ is a monotonically increasing nonlinear odd function of its argument that takes into account the mutual coupling between the sensors; K is a control loop gain; c_i is a coefficient that quantifies the attitude of the *i*-th sensor to adapt its state as a function of the signals received from the other nodes: the

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higher is c_i , the less is the attitude of the *i*-th node to change its original decision ω_i . The running decision, or estimate, of each sensor is encoded in its pulsation $\dot{\theta}_i(t)$. The coefficients a_{ij} take into account the local coupling between oscillators. We assume that two oscillators are coupled (i.e., $a_{ij} \neq 0$), only if their distance is smaller than the coverage radius of each sensor¹. The global decision is taken as the value assumed by the pulsation $\dot{\theta}_i(T)$ after a time T from the beginning of the mutual interaction among the nodes. The model (1), in the case of $f(x) = \sin(x)$, $c_i = 1$, and $a_{ij} = 1, \forall i, j$, was extensively studied by Kuramoto in the fascinating world of chemical waves [10].

In the rest of the paper, we will denote the parameters ω_i and the functions $\theta_i(t)$ as the natural pulsations and the instantaneous phases of the *i*-th oscillator, in accordance to Kuramoto's terminology. However, it is important to emphasize that in our model neither ω_i nor $\theta_i(t)$ are necessarily the pulsation and instantaneous phase of a sinusoidal carrier. They are, in general, physical parameters whose choice is dictated by implementation constraints. For example, the oscillators may be pulsed oscillators, as in ultra-wideband systems, where $\theta_i(t)$ is the instant in which the *i*-th node emits a pulse. In this case, the information is carried by the rate with which the pulse emission time varies with time, somehow mimicking the neurons activity in the brain.

One of the properties that play a fundamental role in the overall system synchronization is the network connectivity, that is the property that each node is linked to each other node through a proper path. To make explicit the network connectivity properties, it is better to rewrite (1) introducing the so called *incidence* matrix B, defined as follows. Given an oriented graph G composed by N vertices and E edges, B is the $N \times E$ matrix with elements

$$B_{ij} = \begin{cases} 1, & \text{if the edge } j \text{ is incoming to vertex } i; \\ -1, & \text{if the edge } j \text{ is outcoming from vertex } i; \\ 0, & \text{otherwise.} \end{cases}$$
(2)

Given the $N \times 1$ vector $\mathbf{1}_N$, composed of all ones, it is easy to check that the incidence matrix satisfies the following property:

$$\mathbf{1}^T \boldsymbol{B} = \mathbf{0}^T. \tag{3}$$

Given B, the symmetric $N \times N$ matrix L, defined as $L := BB^T$, is called the *Laplacian* of G. The Laplacian has several important properties: L is always semi-definite positive with the smallest eigenvalue always equal to 0; the algebraic multiplicity of the null eigenvalue is equal to the number n_c of connected components of the graph. Hence, if the graph is connected, $n_c = 1$ and rank(L) = N - 1.

Using the incidence matrix \boldsymbol{B} , we can rewrite system (1) in compact form as

$$\dot{\boldsymbol{\theta}}(t) = \boldsymbol{\omega} - K \boldsymbol{D}_c^{-1} \boldsymbol{B} \boldsymbol{D}_a f[\boldsymbol{B}^T \boldsymbol{\theta}(t)], \qquad (4)$$

where $D_c := \text{diag} \{c_1, \ldots, c_N\}^2$, and D_a is an $E \times E$ diagonal matrix, whose diagonal entries are all the weights a_{ij} , indexed from 1 to E; the symbol f(x) has to be intended as the vector whose k-th component is $f(x_k)$. We say that the overall population of oscillators synchronizes if all sensors end up oscillating with the same

pulsation, i.e. $\dot{\theta}_i(t) = \dot{\theta}^*(t), \forall i$, after a proper transient. Multiplying (4) by the row vector $\boldsymbol{c}^T := \boldsymbol{1}_N^T \boldsymbol{D}_c$ from the left side, we obtain

$$\boldsymbol{c}^{T}\dot{\boldsymbol{\theta}}(t) = \boldsymbol{c}^{T}\boldsymbol{\omega} - K \mathbf{1}_{N}^{T}\boldsymbol{B}\boldsymbol{D}_{a}f[\boldsymbol{B}^{T}\boldsymbol{\theta}] = \boldsymbol{c}^{T}\boldsymbol{\omega}, \qquad (5)$$

where, in the second row of (5), we have used (3). Hence, if the system is capable to synchronize, in the sense defined before, then the common pulsation must be equal to

$$\dot{\theta}^*(t) = \omega^* = \frac{c^T \omega}{\mathbf{1}_N^T c} = \frac{\sum_{i=1}^N c_i \,\omega_i}{\sum_{i=1}^N c_i}.$$
(6)

In a companion paper [12], we showed that there exist two critical values of K, namely K_L and K_U , such that for $K > K_U$ the network converges to the unique equilibrium point (6), which is proved to be globally asymptotically stable, and for $K < K_L$, the network does not converge. It is difficult to provide the exact value of K_U , but we showed that an upper bound is

$$K_U \le \frac{2 \| \boldsymbol{D}_c \boldsymbol{\Delta} \boldsymbol{\omega} \|}{f_{max} \lambda_2(\boldsymbol{L}_a)},\tag{7}$$

where $\lambda_2(L_a)$ is the second-smallest eigenvalue³ of the weighted Laplacian $L_a := BD_a^2 B^T$, $\Delta \omega$ is the vector whose *i*-th entry is $\omega_i - \omega^*$ and $f_{max} = lim_{x\to\infty} f(x)$. Let us consider now some applications of this self-synchronization model.

3. DECENTRALIZED DETECTION

We denote with $r_i[n]$ the signal observed by sensor *i*, at time *n*. The detection problem can be cast as a binary hypothesis test, where the two hypotheses are

$$\mathcal{H}_0: r_i[n] = v_i[n]$$

$$\mathcal{H}_1: r_i[n] = s_i[n] + v_i[n], \qquad (8)$$

with i = 1, ..., N and n = 1, ..., L, where $s_i[n]$ is the useful signal and $v_i[n]$ is the additive noise, modeled as a set of Gaussian random variables with zero mean and variance σ_i^2 (the subscript *i* indicates the sensor and the index *n* is the time index). We consider two typical detection scenarios to show how to design the coupling mechanism in order to make the network converge to the optimal detector.

3.1. Deterministic signal

In this case, we consider only one snapshot, i.e. L = 1. The useful signal $s_i[1] = s_i$ is deterministic and assumed to be known to the *i*-th sensor. This model is relevant, for example, in all applications where the network is aimed at detecting specific, a-priori known, spatial patterns described by the set of values s_i , with i = 1, ..., N. The optimal detector for this observation model consists in comparing the log-likelihood ratio with a threshold $\log(\gamma)$ and the decision rule is [11]:

$$T(\mathbf{r}) := \sum_{i=1}^{N} \frac{r_i \, s_i}{\sigma_i^2} - \sum_{i=1}^{N} \frac{s_i^2}{2\sigma_i^2} \overset{H_1}{\underset{H_0}{\gtrless}} \log(\gamma), \tag{9}$$

where, following the Neyman-Pearson criterion, the threshold γ is chosen in order to maximize the detection probability, for a given

¹The coverage radius is assumed to be the same for all sensors, even though this could be changed to accommodate for different network topological models, like small worlds or scale-free networks.

²We assume $c_i \neq 0, \forall i$.

³The smallest eigenvalue of L_a is zero by construction.

false alarm probability. In a conventional sensor network, the formation of this sufficient statistic requires all nodes to transmit their observations r_i , noise variances σ_i^2 and the structure of the useful signals s_i to a sink node. Conversely, in our network, if we set

$$\omega_i = \frac{r_i s_i - s_i^2/2}{\sigma_i^2} \quad \text{and} \quad c_i = 1 \tag{10}$$

and take K greater than the bound (7), by virtue of (6), *each node* converges to $T(\mathbf{r})$. This requires only local coupling among nearby nodes, provided that the network connectivity is guaranteed. To get the estimate (9), it is only necessary to wait for the time necessary to the network to converge. This time is inversely proportional to $K\lambda_2(\mathbf{L}_a)$. Waiting for a time T sufficiently large, from (6) and (10) each oscillator ends up with a value $\dot{\theta}^*(T) = T(\mathbf{r})/N$. Hence, it is sufficient for each sensor to compare its own $\dot{\theta}^*(T)$ with a threshold $\log(\gamma)/N$. What is important to emphasize is that this optimal detector is obtained without requiring any node to know all coefficients s_i, r_i , and σ_i^2 , for $i = 1, \ldots, N$; each node is only required to exchange its evolving state with the nearby nodes.

The previous approach can be also generalized to the case where the network has to recognize one out of, let us say M, spatial patterns encoded through the sets $S_m := \{s_i^{(m)}\}, m = 1, \ldots, M$. In such a case, we have a multiple hypothesis testing which can be implemented in a decentralized fashion, by letting the previous mechanism run for M times, each time using the initialization

$$\omega_i^{(m)} = \frac{r_i^{(m)} s_i^{(m)} - s_i^{(m)^2}/2}{\sigma_i^2} \quad \text{and} \quad c_i = 1.$$
(11)

At the end of the procedure, each sensor ends up with a set of values $\dot{\theta}^*_{(m)}(T)$ and it decides for the pattern that gives rise to the largest value of $\dot{\theta}^*(m)(T)$.

3.2. Random signal

In many applications, the useful signal is not known a priori, but it can be reasonably modeled as a random process. We consider now the case where the random variables describing both useful signals and noise are spatially uncorrelated. Assuming, in particular, that both useful signal and noise are modeled as independent Gaussian random processes, and that the signal variance σ_{si}^2 at each node is a priori known, the optimal detector is the weighted energy detector for which the decision rule is [11]

$$T(\mathbf{r}) := \sum_{i=1}^{N} \frac{\sigma_{si}^{2} r_{i}^{2}}{\sigma_{i}^{2} (\sigma_{si}^{2} + \sigma_{i}^{2})} \overset{H_{1}}{\underset{H_{0}}{\gtrless}} \gamma',$$
(12)

where γ' is the threshold that provides the desired false alarm rate. Proceeding as before, this detector can be implemented in a decentralized manner, by initializing each sensor with the values

$$\omega_i = \frac{\sigma_{si}^2 r_i^2}{\sigma_i^2 (\sigma_{si}^2 + \sigma_i^2)} \quad \text{and} \quad c_i = 1 \tag{13}$$

and comparing the equilibrium value $\dot{\theta}^*(T)$ with a threshold γ'/N .

In the case where the variances σ_{si}^2 are not a priori known, we need to use the Generalized Likelihood Ratio Test (GRLT) [11], in which case, the decision rule consists in comparing the following statistic

$$T(\mathbf{r}) := \sum_{i=1}^{N} \left[\frac{1}{L} \sum_{k=1}^{L} r_i^2[k] \left(\frac{1}{\sigma_i^2} - \frac{1}{\hat{P}_i + \sigma_i^2} \right) - \log \left(\frac{\hat{P}_i + \sigma_i^2}{\sigma_i^2} \right) \right]$$
(14)

with a suitable threshold. In (14), \hat{P}_i denotes the ML estimate of the useful signal power, under hypothesis \mathcal{H}_1 , obtained as

$$\hat{P}_{i} = \left(\frac{1}{L}\sum_{k=1}^{L}r_{i}^{2}[k] - \sigma_{i}^{2}\right)^{+}.$$
(15)

4. DECENTRALIZED LOCALIZATION

Let us consider now a planar network composed of nodes uniformly distributed over a square centered on the origin of the plane and let us assume that each node knows its own coordinates, let us say (x_i, y_i) . We assume that when the event of interest occurs, it generates a power that radiates isotropically from the point where the event originated, and it attenuates with a power law. More specifically, denoting with (x_c, y_c) the coordinates of the target event, we assume that the power impinging on the *i*-th sensor depends on the distance as

$$P_i = \frac{\mathcal{P}_0}{1 + [(x_i - x_c)^2 + (y_i - y_c)^2]^{\eta/2}/\lambda^2}$$
(16)

where η is the path loss exponent and λ is a parameter that quantifies the power spreading. Within this setup, we wish to estimate the coordinates of the event of interest. Assuming a large density of sensors, a meaningful candidate is the estimator of the center of gravity of the powers received from all the sensors. We show now how to achieve this estimate in a decentralized manner, through a proper choice of the oscillators coefficients. The proposed strategy consists in the following steps: i) each node estimates the received power $P_i = P(x_i, y_i)$ and sets $\omega_i = x_i$ and $c_i = P_i$; ii) the network is let to evolve until an equilibrium is reached. Let us call T the duration of the evolution. From (6), if T is sufficiently large, the final value of $\dot{\theta}^*(t)$ is then

$$\hat{x}_c := \dot{\theta}^*(T) = \frac{\sum_{i=1}^N x_i P_i}{\sum_{i=1}^N P_i}.$$
(17)

Similarly, initializing each node with the values $\omega_i = y_i$ and $c_i = P_i$, the final equilibrium will be

$$\hat{y}_c := \dot{\theta}^*(T) = \frac{\sum_{i=1}^N y_i P_i}{\sum_{i=1}^N P_i}.$$
(18)

Equations (17) and (18) show that each node is able to achieve, in principle, an estimate of the center of gravity of the event, without the need of sending its measurements to a sink node. In some cases, where some P_i are null (or very small), we cannot use $c_i = 0$ in (1). This problem can be circumvented by letting the network to evolve twice to get each estimate: once with $\omega_i = x_i P_i$ and $c_i = 1$ to get the numerator of (17) and then with $\omega_i = P_i$ and $c_i = 1$ to get the denominator of (17).

In general, the power P_i is only estimated within some error due to both additive noise and finite size samples. We have derived a small perturbation analysis of the estimator in the presence of estimation errors, modeled as additive Gaussian random variables with variance σ_v^2 , and the result is that, at a first order approximation (i.e., for small errors), the estimation is unbiased and the estimation variance is approximately

$$\sigma_{x_c}^2 \approx \frac{\sigma_v^2}{\left(\sum_{i=1}^N P_i\right)^2} \left[\sum_{i=1}^N x_i^2 + \frac{N}{\left(\sum_{i=1}^N P_i\right)^2} \left(\sum_{i=1}^N x_i P_i \right) \right].$$
(19)

5. RESULTS

As an example of decentralized detection, in Fig. 1 we report the detection probability as a function of the peak SNR, defined as $SNR := \mathcal{P}_0/\sigma_n^2$, where σ_n^2 is the noise variance, assumed to be same for all sensors, for a $P_{fa} = 10^{-3}$, for different numbers of sensors. The path loss exponent is $\eta = 1$ and $\lambda = 10^{0.5}$. We can clearly see how the performance improves as N increases.



Fig. 1. Detection probability vs. peak SNR, for different number of sensors N.

As an example of localization, in Fig. 2 we report the variance of the decentralized center of gravity estimator, as a function of the average SNR, defined as $SNR := \sum_{i=1}^{N} P_i^2 / N \sigma_v^2$, for different values of η and λ . The figure shows both simulations and the theoretical derivations given by (19). We can verify that, at high SNR, the theoretical expressions allow us to predict the simulation results very well.



Fig. 2. Estimation variance vs. SNR, for different values of η and λ .

6. CONCLUSION

In summary, we have shown that if the whole sensor network observes one event, designing the network nodes as mutually coupled estimators with a proper coupling mechanism, it is possible to achieve optimal decentralized detection and localization with the only necessary assumption that the network is connected and the coupling strength is sufficiently large. The phenomenon considered in this paper is stationary, but the generalization to a slowly changing environment can be made provided that the evolution of the the observed phenomenon is slower than the network adaptation time, which is roughly inversely proportional to the product $K\lambda_2(L)$. Interestingly, this time constant depends on the network topology through the coefficient $\lambda_2(L)$, which means that different topologies give rise to different adaptation capabilities.

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