CENTRALIZED AND DISTRIBUTED SOURCE LOCALIZATION BY A NETWORK OF SENSORS USING GUARANTEED SET ESTIMATION

Michel Kieffer and Éric Walter

LSS – CNRS – Supélec – Université Paris-Sud
Plateau de Moulon, 91192 Gif-sur-Yvette, France

ABSTRACT

This paper is about source localization in a network of sensors from readings of signal strength. Contrary to previously published results, the source signal strength and path loss exponent are not assumed known *a priori*. The measurement errors are assumed bounded, with known bounds. The problem is then solved both in a centralized and in a distributed context using bounded-error parameter estimation techniques and interval analysis. Simulation results with realistic measurements are provided.

1. INTRODUCTION

The problem of localizing a source that emits an acoustic or electromagnetic wave using a set of distributed sensors has received a growing interest in recent years, see, *e.g.*, [1, 2]. The localization technique used depends on the type of information available to the sensor nodes. Time of arrival (TOA), time difference of arrival (TDOA) and angle of arrival (AOA) usually provide the best results [3], however, these quantities are most difficult to obtain, as they require, a good synchronization between timers (for TOA), exchanges between sensors (for TDOA) or multiple antennas (for AOA). Contrary to TOA, TDOA or AOA data, readings of signal strength (RSS) at a given sensor are easily obtained, as they only require low-cost sensors or are already available, as in IEEE 802.11 wireless networks, where these data are provided by the MAC layer [2].

This paper focuses on source localization from RSS data. Centralized approaches have been proposed to solve this problem for acoustic sources [4] and for sources emitting electromagnetic waves, see, *e.g.*, [5–7]. In the first case, some knowledge of the decay rate of the RSS (*path loss exponent*) is needed for efficient nonlinear least squares estimation. In the second case, an off-line training phase is required to allow maximum *a posteriori* localization. In both cases, a good initial guess of the location of the source facilitates convergence to the global minimum of the cost function. Distributed approaches have also been employed, *e.g.*, in [8], where a distributed version of nonlinear least squares has been presented. When badly initialized, it suffers from the same convergence problems as the centralized approach, as illustrated in [9], which advocates projection on convex sets. However, this requires an accurate knowledge of the source signal strength and of the path loss exponent.

The new approach presented in this paper does not need such a knowledge. A model of the RSS data will be used to achieve localization without prior training. All measurement errors will be assumed bounded, with known bounds. Under this assumption, localization translates into the characterization of the *set* of all parameters of the source that are consistent with the measurements, error bounds and measurement model. Various techniques are available to characterize this set in an approximate, but *guaranteed* way. Here, set inversion and interval constraint propagation (ICP) [10] will be put at work, respectively in a centralized and in a distributed context. This paper extends results presented in [11], where a less realistic RSS model is used.

Section 2 presents localization in a bounded-error context. Required notions on interval analysis are recalled in Section 3. Section 4 is dedicated to centralized localization while Section 5 focuses on distributed localization. The techniques proposed are compared in Section 6 in realistic simulations.

2. BOUNDED-ERROR SOURCE LOCALIZATION

Consider a network of L sensors, located on a plane in a limited region of space. The location of the ℓ-th sensor is assumed perfectly known and denoted by $\mathbf{r}_\ell \in \mathbb{R}^2$, $\ell = 1 \dots L$. A source has to be localized in this sensor network from RSS data at the sensors. The source is characterized by its unknown location $\boldsymbol{\theta}$ and emitted power P_0 (in dBm) at a short reference distance d_0.

The mean power $\overline{P}(d)$ (in dBm) received by the ℓ-th sensor is described the Okumura-Hata model [12]

$$\overline{P}_{\mathrm{dB}}(d) = P_0 - 10 n_{\mathrm{p}} \log \frac{d}{d_0}, \tag{1}$$

where n_{p} is the path-loss exponent and $d = |\mathbf{r}_\ell - \boldsymbol{\theta}|$. Except in open fields, n_{p} depends on the relative position of the source and sensor. Here, n_{p} is assumed constant for all sensors of the field. Usually, the variance σ_{p}^2 of the received power is modeled as log-normal, *i.e.*, normal in dB [6]. Here, the measurement noise is assumed to remain within some known bounds, *i.e.*, the RSS satisfies

$$P_{\mathrm{dB}}(d) \in \left[P_0 - 10 n_{\mathrm{p}} \log \frac{d}{d_0} - e, P_0 - 10 n_{\mathrm{p}} \log \frac{d}{d_0} + e \right] \tag{2}$$

where e is known. Using this model, it is possible to write the RSS by sensor ℓ as $y_\ell = y_{\mathrm{m}}(\boldsymbol{\theta}, A, n_{\mathrm{p}}, \ell)\, w_\ell$, $\ell = 1 \ldots L$, with

$$y_{\mathrm{m}}(\boldsymbol{\theta}, A, n_{\mathrm{p}}, \ell) = \frac{A}{|\mathbf{r}_\ell - \boldsymbol{\theta}|^{n_{\mathrm{p}}}},\ A = 10^{P_0/10} d_0^{n_{\mathrm{p}}}, \qquad (3)$$

and $w_\ell \in [w] = \left[10^{-e/10}, 10^{e/10}\right]$. The parameter vector to be estimated is thus $\mathbf{p} = (\boldsymbol{\theta}^{\mathrm{T}}, A, n_{\mathrm{p}})^{\mathrm{T}}$ and $y_{\mathrm{m}}(\boldsymbol{\theta}, A, n_{\mathrm{p}}, \ell)$ will be written as $y_{\mathrm{m}}(\mathbf{p}, \ell)$.

The aim of this paper is to characterize the *set* $\mathbb{P} \subset [\mathbf{p}]_0$ of all parameter vectors that are *consistent* with the measurements, the RSS model (3), and the noise bounds. The initial search box $[\mathbf{p}]_0$ is assumed to contain the actual parameter value $\mathbf{p}^*$. $\mathbb{P}$ may then be defined as follows

$$\mathbb{P} = \left\{\mathbf{p} \in [\mathbf{p}]_0 \mid y_{\mathrm{m}}(\mathbf{p}, \ell) \in [y_\ell], \ell = 1 \ldots L\right\},$$

where $[y_\ell] = y_\ell / [w]$ is assumed to contain the actual noise-free RSS by sensor ℓ.

Characterizing $\mathbb{P}$ is a classical set-inversion problem that may be solved using interval analysis [10, 13], presented in Section 3. When centralized localization is considered, the computing capacity is not a problem and one may want to describe $\mathbb{P}$ as accurately as possible, see Section 4. With distributed localization, computing capacity becomes critical, and interval constraint propagation will be used, see Section 5.

3. INTERVAL ANALYSIS

Interval analysis provides a set of tools to compute with intervals $[x] = [\underline{x}, \overline{x}] \subset \mathbb{R}$ and vector of intervals (or *boxes*) $[\mathbf{x}] = [\underline{\mathbf{x}}, \overline{\mathbf{x}}] \subset \mathbb{R}^n$. For any function $f : \mathcal{D} \subset \mathbb{R} \longrightarrow \mathbb{R}$ defined as a combination of arithmetical operators and elementary functions, interval analysis makes it possible to build *inclusion functions* $[f]$ satisfying

$$\forall [x] \subset \mathcal{D},\ f([x]) \subset [f]([x]), \qquad (4)$$

where $[f]([x])$ is an interval. For example, the *natural inclusion function* is obtained by replacing all occurrences of a real variable by its interval counterpart. It thus becomes possible to enclose the set of all values taken by a function over a given interval into a computable image interval. For more details, see [10, 13].

4. CENTRALIZED LOCALIZATION

In this case, all measurements provided by the L sensors are transmitted to a central processing unit. Let

$$\mathbf{y}_{\mathrm{m}}(\mathbf{p}) = (y_{\mathrm{m}}(\mathbf{p}, 1), \ldots, y_{\mathrm{m}}(\mathbf{p}, L))^{\mathrm{T}} \qquad (5)$$

be the RSS model for the L sensors and

$$[\mathbf{y}] = ([y_1], \ldots, [y_L])^{\mathrm{T}}$$

denote the box formed by all RSS data with their uncertainty interval.

The SIVIA algorithm [14] provides a union $\overline{\mathbb{P}}$ of non-overlapping boxes (*subpaving*) *guaranteed* to contain $\mathbb{P}$. The boxes of $\overline{\mathbb{P}}$ are obtained from successive bisections and selections starting from $[\mathbf{p}]_0$. The only requirement is the availability of an inclusion function for (5). The bisections and selections are performed as follows. SIVIA recursively applies the next three tests, starting from $[\mathbf{p}]_0$:

– If $[\mathbf{y}_{\mathrm{m}}]([\mathbf{p}]) \subset [\mathbf{y}]$, then according to (4), for all $\mathbf{p} \in [\mathbf{p}]$, $\mathbf{y}_{\mathrm{m}}(\mathbf{p}) \in [\mathbf{y}]$. Any $\mathbf{p} \in [\mathbf{p}]$ is thus consistent with the measurements, model structure, and noise bounds. Thus $[\mathbf{p}]$ is proved to be included in $\mathbb{P}$ and is stored in $\overline{\mathbb{P}}$.

– If $[\mathbf{y}_{\mathrm{m}}]([\mathbf{p}]) \cap [\mathbf{y}] = \emptyset$, then, again according to (4), for all $\mathbf{p} \in [\mathbf{p}]$, $\mathbf{y}_{\mathrm{m}}(\mathbf{p}) \notin [\mathbf{y}]$. Therefore, there is no $\mathbf{p}$ in $[\mathbf{p}]$ that is consistent with the measurements, model structure and noise bounds and $[\mathbf{p}]$ can be discarded.

– In all other cases, if the width $w([\mathbf{p}])$ of $[\mathbf{p}]$ is larger than some specified precision parameter ε, it is bisected to get two subboxes on which the same tests are applied. If $w([\mathbf{p}]) < \varepsilon$, $[\mathbf{p}]$ is stored in $\overline{\mathbb{P}}$.

The parameter ε is used to tune the trade-off between complexity and accuracy of description of $\mathbb{P}$ by $\overline{\mathbb{P}}$. For more details about convergence and complexity issues, see [14]. It is possible to build from SIVIA an algorithm that is robust to *outliers*, *i.e.*, data for which (2) does not hold, see [10].

5. DISTRIBUTED LOCALIZATION

Implementing SIVIA on each node would be too complex for sensors with low computing capabilities. Moreover, transmitting subpavings would require too much wireless resource. On the other hand, evaluating a *box* containing $\mathbb{P}$ in a distributed fashion is still possible using ICP [15, 16].

The main idea is as follows. Assume that sensor ℓ has obtained an estimate $[\mathbf{p}]$ for $\mathbb{P}$ from one of its neighbors. Using its RSS, this sensor may be able to evaluate a new box $[\mathbf{p}']$ such that $\mathbb{P} \subset [\mathbf{p}'] \subset [\mathbf{p}]$. This is performed by discarding from $[\mathbf{p}]$ parts that are not consistent with $[y_\ell]$ using ICP. Sensor ℓ then transmits the updated estimate $[\mathbf{p}']$ to its neighbors. More sophisticated strategies for exchanging information between sensors may be considered [17].

5.1. Interval constraint propagation

A constraint satisfaction problem consists of a set of *variables* $\{x_1, x_2 \ldots x_M\}$ associated to *domains*, here intervals, $\{[x_1], [x_2] \ldots [x_M]\}$ to which these variables have to belong. The variables are linked by a vector constraint

$$\mathbf{f}(x_1, x_2 \ldots x_M) = \mathbf{0}. \qquad (6)$$

The purpose of ICP is to *contract* $([x_1] \ldots [x_M])$, *i.e.*, find a smaller domain $([x_1'] \ldots [x_M']) \subset ([x_1] \ldots [x_M])$ still containing all $\{x_1, x_2 \ldots x_M\}$ such that (6) is satisfied. Many techniques have been proposed to solve this problem, see, *e.g.*, [15, 16]. Due to lack of space, the basic idea of ICP will be presented on a very simple example.

Consider three variables x, y, and z, their domains $[x] = [-10, 1]$, $[y] = [-1, 8]$, and $[z] = [1, 20]$ and a constraint

$x + y = z$. This constraint implies that $z \in [x] + [y] = [-11, 9]$. Thus, $z \in [z'] = [-11, 9] \cap [1, 20] = [1, 9]$. Now the constraint implies that $x \in [z'] - [y] = [-7, 10]$, thus $x \in [x'] = [-7, 10] \cap [-10, 1] = [-7, 1]$. Similarly, $y \in [y'] = [0, 8]$.

When the reduced domain turns out to be empty, one has proved that (6) admits no solution in the given domains. In [18], an algorithm is presented that provides the most efficient contraction for variables belonging to intervals and constraints that may be described by a tree, *i.e.*, allowing each variable to be written as a function of the others.

5.2. Application to distributed localization

Here, contrary to classical constraint satisfaction problems, the variables and constraints are *distributed* on the sensor network, [19, 20]. However, ICP may still be used at each individual sensor. At sensor ℓ, the variables are y_ℓ, $\boldsymbol{\theta}$, A, and n_p, their domains are $[y_\ell]$, measured at the sensor, and $[\boldsymbol{\theta}]$, $[A]$ and $[n_\mathrm{p}]$, obtained from its neighbors. The variables must satisfy the constraint provided by the RSS model

$$y_\ell - \frac{A}{|\mathbf{r}_\ell - \boldsymbol{\theta}|^{n_\mathrm{p}}} = 0. \tag{7}$$

From (7), the contracted domains may be written as

$$[y'_\ell] = [y_\ell] \cap \frac{[A]}{|\mathbf{r}_\ell - [\boldsymbol{\theta}]|^{[n_\mathrm{p}]}},$$
$$[A'] = [A] \cap [y'_\ell]\,|\mathbf{r}_\ell - [\boldsymbol{\theta}]|^{[n_\mathrm{p}]},$$
$$[n'_\mathrm{p}] = [n_\mathrm{p}] \cap \left(\log([A']) - \log([y'_\ell])\right) / \log(|\mathbf{r}_\ell - [\boldsymbol{\theta}]|),$$
$$[\theta'_1] = [\theta_1] \cap \left(r_{\ell,1} \pm \sqrt{\left([A'] / [y'_\ell]\right)^{2/[n'_\mathrm{p}]} - (r_{\ell,2} - [\theta_2])^2} \right),$$
$$[\theta'_2] = [\theta_2] \cap \left(r_{\ell,2} \pm \sqrt{\left([A'] / [y'_\ell]\right)^{2/[n'_\mathrm{p}]} - (r_{\ell,1} - [\theta_1])^2} \right).$$

In the last two update equations, the set intersecting $[\theta_1]$ and $[\theta_2]$ may consist of two disconnected intervals. In this case, the smallest interval containing the result is evaluated.

Using [18], it can be shown that the contraction is optimal with respect to the information available at the ℓ-th sensor. However, when considering all constraints simultaneously, the optimality conditions no longer hold. Cycling through the sensor network, as in [8,9] improves the estimation. More details about the optimization of sensor communications may be found in [19].

6. EXAMPLE

Consider networks of $L = 5000$ sensors randomly distributed over a field of 100 m$\times$100 m. A source has been placed at $\boldsymbol{\theta}^* = (50\,\mathrm{m}, 50\,\mathrm{m})^\mathrm{T}$, such that $P_0 = 20$ dBm, $d_0 = 1$ m and $n_\mathrm{p} = 2$ (n_p is assumed constant over the field). The measurement noise is such that $e = 4$ dBm. This corresponds to $A = 100$.

For 100 realizations of the sensor field, data have been simulated with (2). To limit computational load, only sensors such that $y_\ell > 10$ participate to localization. The initial search box for $\mathbf{p}$ is taken as $[0, 100] \times [0, 100] \times [50, 200] \times [2, 4]$ in a first scenario, where A (or P_0) is assumed unknown. In a second scenario, A is assumed perfectly known. For the distributed approach, five cycles in the sensor network are performed.

The two proposed techniques are compared to localization by a closest point approach (CPA), which searches for the index of the sensor with the largest RSS $\ell_\mathrm{CPA} = \arg\max_\ell y_\ell$ and uses the location of this sensor $\widehat{\boldsymbol{\theta}}_\mathrm{CPA} = \mathbf{r}_{\ell_\mathrm{CPA}}$ as an estimate for $\boldsymbol{\theta}^*$. This technique, albeit it is not the most efficient [4], performs well for dense sensor networks, as here. Point estimates for $\boldsymbol{\theta}^*$ are evaluated as $\widehat{\boldsymbol{\theta}}_\mathrm{C} = \mathrm{mid}\left(\left[\mathrm{proj}_{\boldsymbol{\theta}}\overline{\mathbb{P}}\right]\right)$, the midpoint of the smallest box containing the projection of $\overline{\mathbb{P}}$ onto the $\boldsymbol{\theta}$-plane in the centralized approach and as the center of the projection onto the $\boldsymbol{\theta}$-plane of the solution box $[\mathbf{p}]$, $\widehat{\boldsymbol{\theta}}_\mathrm{D} = \mathrm{mid}(\mathrm{proj}_{\boldsymbol{\theta}}\,[\mathbf{p}])$, in the distributed approach.

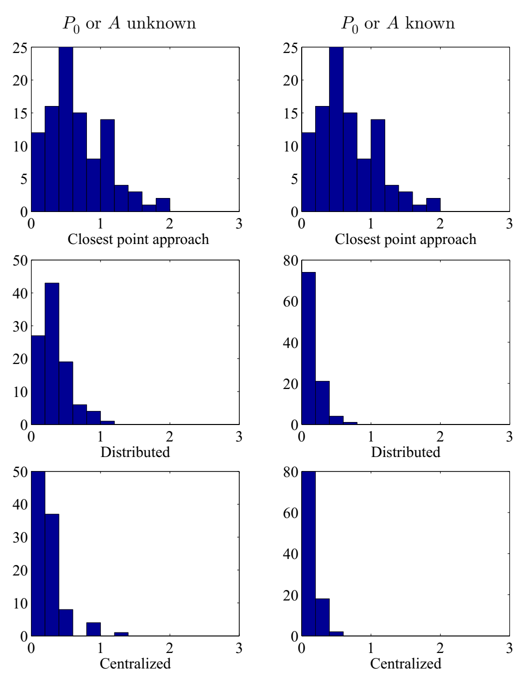

Fig. 1. Histograms of the localization error for 100 realizations of the sensor network

Figure 1 presents the histogram of the L_2 norm of the difference between $\boldsymbol{\theta}^*$ and its estimates provided by the three techniques previously described. The mean and standard deviation of this quantity is also provided in Table 1. The centralized approach performs better than the distributed one, but the distributed approach provides a reasonable estimate at a much lower computation and transmission cost. Both techniques outperforms CPA, the performances of which do not depend on whether A is known.

Figure 2 describes the projection onto the $\boldsymbol{\theta}$-plane of the solution sets provided by the centralized and distributed tech-

Table 1. Mean value m and standard deviation σ of the estimation error for 100 realizations of the sensor network; all quantities are in meters

	known P_0		unknown P_0	
	m	σ	m	σ
CPA	0.66	0.41	0.66	0.41
Distributed	0.16	0.12	0.34	0.21
Centralized	0.14	0.10	0.25	0.21

niques for a given set of measurements. A more precise outer approximation of $\mathbb{P}$ is obtained with centralized localization, but this is achieved at a higher computational cost.

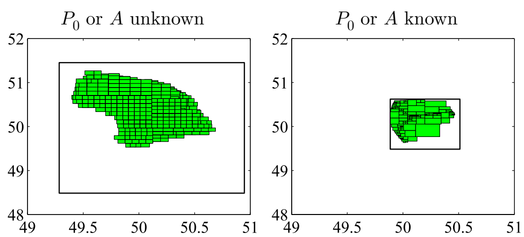

Fig. 2. Zoom on two typical solution sets obtained with centralized localization. The boxes in bold are obtained by the distributed approach

7. CONCLUSIONS AND PERSPECTIVES

Bounded-error techniques using interval analysis have been employed on centralized and distributed source localization in a sensor network. No initial training is required, and the source strength and path loss exponent may be unknown. Moreover, no initial guess of the source location has to be provided. This approach may easily be implemented in a distributed context, as interval computations are only two to four times more complex than floating-point computations.

Deterministic global optimization using interval analysis [10, 13] could also be used in a centralized version of the localization. The results provided by such an approach could serve as a benchmark for other localization techniques. Further research directions include multiple source localization, an extension to source tracking, for which bounded-error state estimation may be put at work [10], robust localization in presence of outliers, and localization with sensors having an imprecise knowledge of their location.

8. REFERENCES

[1] G. Sun, J. Chen, W. Guo, and K. J. Ray Liu, "Signal processing techniques in network-aided positioning," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 12–23, 2005.

[2] A. H. Sayed, A. Tarighat, and N. Khajehnouri, "Network-based wireless location," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 24–40, 2005.

[3] N. Patwari, J. N. Ash, S. Kyperountas, A. O. Hero III, R. L. Moses, and N. S. Correal, "Locating the nodes," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 54–69, 2005.

[4] X. Sheng and Y. H Hu, "Maximum likelihood multiple-source localization using acoustic energy measurements with wireless sensor networks," *IEEE Transactions on Signal Processing*, vol. 53, no. 1, pp. 44 – 53, 2005.

[5] P. Kontkanen, P. Myllymäki, T. Roos, H. Tirri, K. Valtonen, and H. Wettig, "Probabilistic methods for location estimation in wireless networks," in *Emerging Location Aware Broadband Wireless Adhoc Networks*, R. Ganesh, S. Kota, K. Pahlavan, and R. AgustÍ, Eds. 2004, Kluwer Academic Publishers.

[6] F. Gustafsson and F. Gunnarsson, "Mobile positioning using wireless networks," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 41–53, 2005.

[7] S. Gezici, Z. Tian, G. B. Giannakis, H. Kobayashi, A. F. Molish, H. V. Poor, and Z. Sahinoglu, "Localization via ultra-wideband radios," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 70–84, 2005.

[8] M. G. Rabbat and R. D. Nowak, "Decentralized source localization and tracking," in *Proc. ICASSP*, 2004.

[9] A. O. Hero III and D. Blatt, "Sensor network source localization via projection onto convex sets (POCS)," in *Proceedings of ICASSP*, 2005.

[10] L. Jaulin, M. Kieffer, O. Didrit, and E. Walter, *Applied Interval Analysis*, Springer-Verlag, London, 2001.

[11] A. Galstyan, B. Krishnamachari, K. Lerman, and S. Pattem, "Distributed online localization in sensor networks using a moving target," in *Proc. ACM/IEEE International Symposium on Information Processing in Sensor Networks (IPSN)*, Berkeley, CA, 2004.

[12] Y. Okumura, E. Ohmori, T. Kawano, and K. Fukuda, "Field strength ans its variability in VHF and UHF land-mobile radio service," *Rev. Elec. Commun. Lab.*, vol. 16, pp. 9–10, 1968.

[13] R. E. Moore, *Methods and Applications of Interval Analysis*, SIAM, Philadelphia, PA, 1979.

[14] L. Jaulin and E. Walter, "Set inversion via interval analysis for nonlinear bounded-error estimation," *Automatica*, vol. 29, no. 4, pp. 1053–1064, 1993.

[15] E. Davis, "Constraint propagation with interval labels," *Artificial Intelligence*, vol. 32, no. 3, pp. 281–331, 1987.

[16] F. Benhamou and W. Older, "Applying interval arithmetic to real, integer and Boolean constraints," *Journal of Logic Programming*, pp. 1–24, 1997.

[17] M. Yokoo, *Distributed Constraint Satisfaction: Foundations of Cooperation in Multi-Agent Systems*, Springer-Verlag, Berlin, 2001.

[18] L. Jaulin, M. Kieffer, I. Braems, and E. Walter, "Guaranteed nonlinear estimation using constraint propagation on sets," *International Journal of Control*, vol. 74, no. 18, pp. 1772–1782, 2001.

[19] R. Bejar, C. Fernandez, M. Valls, C. Domshlak, C. Gomes, B. Selman, and B. Krishnamachari, "Sensor networks and distributed CSP: Communication, computation and complexity," *Artificial Intelligence Journal*, vol. 161, no. 1-2, pp. 117–148, 2005.

[20] "Special issue on distributed constraint satisfaction," in *Artificial Intelligence*, B. Faltings and M. Yokoo, Eds., 2005, vol. 161, pp. 1–250.