DECENTRALIZED MANAGEMENT OF SENSORS IN A MULTI-ATTRIBUTE ENVIRONMENT UNDER WEAK NETWORK CONGESTION

Michael Maskery and Vikram Krishnamurthy

University of British Columbia
Department of Electrical and Computer Engineering
Vancouver, V6T 1Z4, Canada
Email: mikem@ece.ubc.ca, vikramk@ece.ubc.ca

ABSTRACT

We provide a game theoretic formulation for a sensor activation problem in a multi-attribute environment. Activated sensors randomly select one of M environmental attributes, and transmit data on that attribute to an end user. The goal is to maximize the number of attributes reported while minimizing redundant reports and packet collisions, which both increase with the number of active sensors. Sensor participation is optimized according to an adaptive scheme, in which sensors activate only when their expected utility, given by the number of unique attributes reported minus an energy cost, is positive. We formulate a Nash equilibrium policy that maximizes the expected performance from the perspective of each sensor when transmission is according to a one-shot frequency hopping scheme, and compare this to the global optimum.

1. INTRODUCTION

Due to the advent of inexpensive, low-power electronics and increased computing power, sensor networks are becoming increasingly attractive for large-scale environmental monitoring. The key idea is that large numbers of spatially distributed sensors are ideally suited for monitoring spatially distributed processes. The problem of coordinating large groups of sensors to efficiently gather and report environmental data is an active area of research.

In order to avoid the cost and complexity of coordinating large sensor networks, decision making tasks are shifted from the end user to the sensors themselves. The problems of optimal activation in this case become sensor-centric, which leads naturally to the use of game theory as an analytical tool. We present here a game-theoretic sensor activation scenario, using recent contributions from the economics community to derive a scheme for reporting on a multi-attribute environment over a wireless communication medium.

The sensor activation problem is described as follows. A large array of sensors is deployed on the ground to track a distributed process of interest. For example, the process could be due to animal or human activity, seismic events, etc. Each sensor can detect and report on one component of the process, but does not know whether its counterparts are also reporting on components, and, if they are, whether or not some sensor has selected the same component as itself. If too few sensors provide reports, then insufficient data will be provided to the end user. On the other hand, if too many sensors provide reports, then inefficiency arises for two reasons: 1.) many sensors will report redundant attributes, and 2.) transmission energy will be wasted due to increased packet collisions. We characterize a Nash equilibrium strategy that yields optimal performance from each sensor's perspective, and compare this to the globally optimal strategy.

Although this paper focuses on analytical results, this is motivated by the assumption of a decentralized adaptive scheme leading to the execution of a Nash equilibrium [1]. In the adaptive scheme, sensors periodically receive feedback as to the state of the network (the number of unique attributes reported or the number of sensors on). This feedback is used to to calculate the expected reward and transmission cost. Sensors then learn to activate only if their expected utility, which is computed through strategic reasoning, is positive. Since sensors have limited battery power, decentralized adaptation and decision rules are superior to centralized control, due to the communication energy required to facilitate the latter.

Game theory has been used in many aspects of sensor networks, including transmission power control [2], [3], medium access control [4], and packet routing [5]. Other potential uses for game theory include network connectivity and data fusion. A good overview of game-theoretic models in sensor networks is given in [6] and [7]. This paper is most closely related to the literature on medium access control, but also incorporates an element of data fusion.

The game theoretic approach here resembles the El Farol Bar problem [8] and the related field of minority games. Such games have been studied in the context of sensor networks in [9], but using a different approach. We focus on the generalization developed in [10], which describes a game in which each player must choose either to go to a bar or stay home. The (common) utility of players is based on a global signal plus a function of how crowded the bar is. The model can be extended by the use of *noisy* global signals, which is characteristic of global games [11]. However, this analysis is omitted in this preliminary paper. In this paper, "going to the bar" is identified with sensor participation, and the global signal is identified with energy costs.

2. THE MULTI-ATTRIBUTE SENSOR ACTIVATION GAME

We consider a network of N sensors, deployed to monitor an environment with M attributes. At given time periods, each sensor must decide whether or not to participate in monitoring or enter a "sleep" mode. If the sensor participates, it receives some reward depending on a participation cost θ and the proportion

$$\alpha = n/N \tag{1}$$

of sensors participating in a play of the game (n is the absolute number of participating sensor). Participating sensors randomly select one of the M environmental attributes with equal probability, and

This Research was supported in part by NSERC

report the details of that attribute to an end user over a communication network (of which more will be said below). Non-participating sensors receive zero reward.

Sensors have no control over which attribute is selected. This is characteristic of unattended ground sensor networks that observe a mobile population, since stationary sensors cannot determine which members of the population they observe.

For participating sensors, the expected reward is

$$U(\theta, \alpha) = \theta + f(\alpha). \tag{2}$$

In this paper, $f(\alpha)$ is taken as the expected proportion (relative to M) of unique attributes reported to the base station when proportion α of the sensors participate. θ is assumed to be negative. It is interpreted as the cost of participating, of using energy and hence forgoing future opportunities.

Define $h(\alpha)$ to be the expected proportion of unique attributes reported when proportion α reports are received, and define $g(\alpha)$ to be the expected proportion of successful transmissions given α attempts. Then $f(\alpha)$ obeys the relation

$$f(\alpha) = h(g(\alpha)). \tag{3}$$

Each round of the sensor activation game is summarized in the following algorithm.

Algorithm 2.1 *1. Each sensor observes a common cost θ, possibly through a global feature of the environment or an internal state of the sensor.*

2. Each sensor also estimates α_n based on the state of neighbouring sensors or channel feedback.

3. Based on (θ, α_n), each sensor predicts α_{n+1} and activates at time $n+1$ if doing so maximizes its expected reward.

4. Participating sensors measure one of M environmental attributes, selected at random with equal probability.

5. Reports are transmitted to an end user over one of q shared communication channels. If multiple sensors transmit on a given channel, the transmissions fail, and no retransmission is attempted.

2.1. Attribute Detection and Overlap

We first derive $h(\alpha)$. When α is small, the probability of duplicate attribute detection is small, and h is roughly linear. However, when α becomes large, the probability that sensors duplicate each others' efforts increases, and h grows more slowly. $h(\alpha)$ is given as follows.

Lemma 2.1 *Define the ratio of sensors to attributes by*

$$r = N/M. \tag{4}$$

Then

$$h(\alpha) \approx 1 - e^{-r\alpha}. \tag{5}$$

All proofs are provided in the appendix.

This is an asymptotic result, for large numbers of sensors and attributes. However, it is a good approximation even for smaller numbers. The proof of Lemma 2.1 yields exact values for any given problem size.

2.2. Attribute Reporting and Collisions

Crowded situations lead to packet collisions as well as report duplications. In this section we derive the relation for $g(\alpha)$, under a one-shot frequency hopping transmission scheme, as described in [4]. In this model, n sensors select one of q available channels for transmission. If a single sensor selects a channel, then it transmits successfully. If $m > 1$ sensors select the same channel, none are successful. Since sensors are given limited battery power for transmission, we build energy awareness into our model through the requirement that packet collisions do not result in retransmission. The model leads to the following result.

Lemma 2.2 *Define the ratio of sensors to channels by*

$$\gamma = N/q. \tag{6}$$

Then, for the one-shot $q-$frequency hopping model,

$$g(\alpha) = \alpha e^{-\gamma\alpha}. \tag{7}$$

3. ANALYSIS OF THE GAME

Since sensors are self-interested and do not coordinate explicitly, we seek a Nash equilibrium solution. We have so far established from (3), (5), and (7) that the proportion of unique reports satisfies

$$f(\alpha) = 1 - e^{-r\alpha e^{-\gamma\alpha}}. \tag{8}$$

In [10], we find an analysis of this game for general $f(\alpha)$, as follows:

Assume a participation game with payoff given by (2), where f is quasi-concave, $f(0) = 0$, and $f'(0) > 0$. Then

- For $\theta < -\max_\alpha f(\alpha)$, the unique Nash equilibrium (dominant strategy) is for no sensor to participate.
- For $\theta > -\min(0, f(1))$, the unique Nash equilibrium (dominant strategy) is for all sensors to participate.
- For intermediate θ, there is a single stable mixed strategy equilibrium, where a given proportion of sensors participate, as well as a pure equilibrium where no sensor participates.

A quick check confirms that the multi-attribute sensor activation game satisfies the required conditions on U. First,

$$f'(\alpha) = (1 - \gamma\alpha) r e^{-\alpha(\gamma + r e^{-\gamma\alpha})}. \tag{9}$$

It is now easy to show that $f(0) = 0, f'(0) = r$, and f is quasi-concave with its maximum at

$$(\alpha, f) = \left(1/\gamma, 1 - e^{-\frac{r}{e\gamma}}\right). \tag{10}$$

We consider only the case where $\gamma > 1$. Here $f(\alpha)$ is decreasing for α beyond $1/\gamma$. That is, there is a congestion effect in the game such that the activation of too many sensors decreases system performance; in the region $\alpha < 1/\gamma$, the game is supermodular, otherwise it is submodular. [10] identifies both *weak congestion* and *strong congestion,* depending on whether $f(1) \geq 0$ or $f(1) < 0$, respectively. Since $0 < e^{-x} < 1$ for all $x > 0$, only weak congestion is possible in our case. Nash equilibria for the weak congestion case are analyzed in [10], and we now summarize those results for the case where θ is perfectly observed by each sensor. The equilibrium outlined below are in addition to the dominant strategy (pure) equilibria mentioned above for extreme values of θ.

3.1. Random Reporting under Perfect Observations

When each sensor perfectly observes the cost θ, and this value is between the extremes $-\max_\alpha f(\alpha)$ and $-\min(0, f(1))$, that is for

$$\theta \in \left(e^{-\frac{r}{e\gamma}} - 1, 0\right), \tag{11}$$

two stable Nash equilibria exist. First, it is a Nash equilibrium for no sensor to participate, since the participation reward for small ε is $U(\theta, \varepsilon) \approx \theta < 0$. Second, it is shown in [10] that it is a unique stable mixed strategy Nash equilibrium for proportion $y(\theta)$ of the sensors to participate, where $y(\theta)$ is the unique solution to $(U(\theta, y(\theta)) = 0, y(\theta) \geq 1/\gamma)$. That is,

$$\left(y(\theta)e^{-\gamma y(\theta)} = \frac{-1}{r}\log(\theta+1), y(\theta) \geq \frac{1}{\gamma}\right). \tag{12}$$

Since the null-action Nash equilibrium is clearly worse overall than the mixed equilibrium, we retain only the latter. At this point it is worthwhile to note that the globally optimal strategy (the mixed strategy giving maximal expected global utility) is for proportion $y(\theta) \equiv 1/\gamma$ to participate whenever θ is in the interval given by (11). However, such schemes are unstable in a game theoretic framework since any individual sensor would gain by deviating to a strategy in which it participated with probability one. Such freeriding behaviour is eliminated by restricting our attention to Nash solutions.

The Nash equilibrium would be satisfied if a fixed proportion $y(\theta)$ of the sensors turned on and remained on. However, this is undesirable from a load-sharing perspective. Therefore, we consider randomized policies in which the expected number of "on" sensors is $y(\theta)$. Under the simplest such policy, when each player activates with probability $y(\theta)$, it can be shown numerically that the expected value of $f(y(\theta)) \approx \theta$ for large N. Thus, the policy is nearly Nash.

4. NUMERICAL ANALYSIS

In this section we contrast Nash equilibrium and globally optimal decision rules, and provide a numerical simulation.

4.1. Equilibrium vs. Optimal Decision Rules

For θ given by (11), the globally optimal proportion of $1/\gamma$ is always less than the Nash equilibrium proportion given by (12). The two decision rules are plotted in Figure 1 for $\theta = -0.01$. As θ decreases, the region of inactivity (the flat triangular section in Figure 1) becomes smaller, as it costs more to participate. Also, the equilibrium participation probability increases with r, even though the optimal policy does not. Finally, when γ is small, the unique equilibrium is 100% participation, whereas the optimal policy is not.

4.2. Numerical Example

We show a simulation for $N = 1000$ sensors, $M = 500$ attributes, and $q = 100$ channels. The simulation results are shown in Figure 2, which shows the payoff curve $U(\alpha, -0.0355)$, along with the equilibrium operating point, and the proportion of active sensors operating according to Algorithm 2.1. Sensors predict α as the average of past values, and increase their participation probability sharply as the predicted payoff becomes positive. This random behaviour balances the load; sensors are active roughly the same amount of time.

The participation probability is given as a logistic function:

$$Pr(\text{“on”}|\alpha) = 1 - 1/(1 + e^{-k(\alpha - y(\theta))}). \tag{13}$$

Parameter k must be tuned. Larger values of k results in the mean number of sensors staying close to $y(\theta)$, with very little noise. However, convergence time also increases with k, as does the variance in the proportion of time each sensor is "on". Setting $k = 1000$ in this example resulted in a good trade-off of these considerations.

5. SUMMARY

We have presented a participation-based sensor activation game, in which sensors determine individually optimal strategies for maximizing reporting efficiency in a multi-attribute environment. The game admits a mixed-strategy Nash equilibrium that is close but inferior to the globally optimal policy. Future work will consider correlated attributes, a more sophisticated CDMA transmission model, and a more general formulation for U, so that θ may be used as an estimation parameter such as $\theta = r/\gamma$. A sensitivity analysis will also be performed. In addition, the global game situation, in which each sensor receives only a noisy estimate of (r, γ, θ) will be considered.

6. APPENDIX

6.1. Proof of Lemma 2.1

Consider n sensors making independent observations of M possible attributes. Assume the observations are made in sequence and consider the k^{th} sensor. Given i different attributes have been observed so far, k's probability of observing a new attribute is $\frac{M-i}{M}$.

We formulate a Markov chain indexed over sensors, with state X_k representing the proportion of attributes observed by the first k sensors. Define the state space:

$$S = \left\{0, \frac{1}{M}, \frac{2}{M}, \ldots, \frac{M-1}{M}, 1\right\}, \tag{14}$$

where state $i \in S$ corresponds to the proportion i of attributes sensed. The transition probabilities are:

$$p_{i(i+1)} = 1 - i, p_{ii} = i \tag{15}$$

The chain is a pure birth process, with unique absorbing class $\{1\}$.

Define P to be the transition matrix for the Markov chain with rows and columns corresponding to absorbing states deleted, and let

$$C = (I - P)^{-1}. \tag{16}$$

Then according to [12], the ij^{th} entry of C, c_{ij}, denotes the expected number of steps spent in state j, given that the chain starts in state i.

Given C, the expected number of sensors S needed to sample a given number J of the M attributes is

$$S(J) = \sum_{k=0}^{J-1} c_{0k}. \tag{17}$$

It can easily be verified that C is given by

$$C = (I - P)^{-1} = \begin{bmatrix} 1 & \frac{M}{M-1} & \frac{M}{M-2} & \cdots & M \\ & \frac{M}{M-1} & \frac{M}{M-2} & \cdots & M \\ & & \ddots & & \\ & & & & M \end{bmatrix}. \tag{18}$$

Therefore, we have

$$S(J) = \sum_{k=0}^{J-1} \frac{M}{M-k}. \tag{19}$$

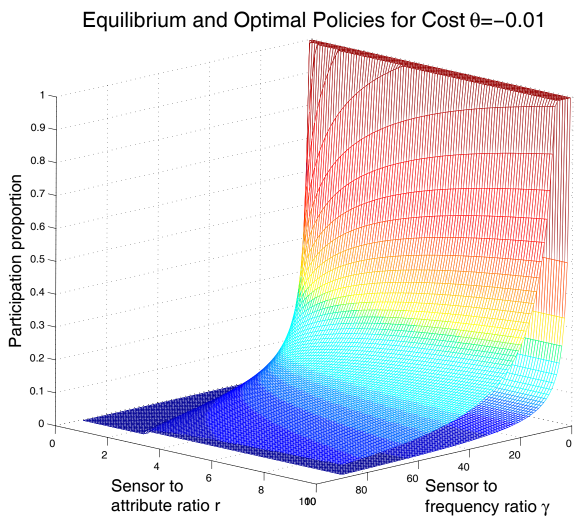

Fig. 1. Nash equilibrium (upper curve) and globally optimal policies (lower curve) for nominal network values.

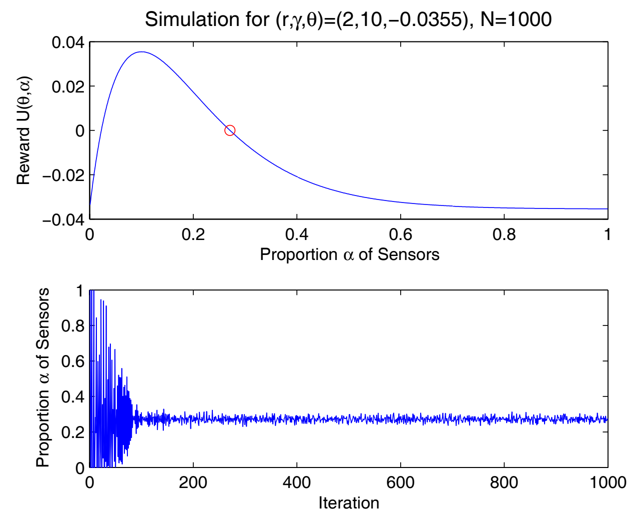

Fig. 2. (Top) Utility curve for nominal example, with Nash equilibrium operating point (red circle). (Bottom) Simulated proportion of active sensors.

Since $S(J)$ gives the number of sensors needed to observe J attributes, the function $S^{-1}(n)$, gives the number of attributes observed by n sensors. This gives the solution to our problem.

We now calculate $S(J)$ and $S^{-1}(n)$ for large M. Let the proportion of attributes sampled relative to M be

$$h(n/M) = J(n/M)/M. \tag{20}$$

From (19), we have that

$$S(J+1) = S(J) + M/(M-J). \tag{21}$$

For large M, this gives

$$\frac{dS}{dJ} \approx \frac{1}{1 - J/M}. \tag{22}$$

Integrating (22), and noting that $S(0) = 1$, we obtain

$$S(J) \approx -M \ln(1 - J/M) + 1. \tag{23}$$

Hence,

$$S^{-1}(n) \approx M\left(1 - e^{\frac{1}{M} - \frac{n}{M}}\right). \tag{24}$$

Noting that $n/M = r\alpha$, we obtain $h(\alpha) \approx 1 - e^{-r\alpha}$. ■

6.2. Proof of Lemma 2.2

From [4], the expected number of successes per sensor in n transmissions is $r(n) = n/N(1-1/q)^{n-1}$. This is rewritten as $r(\alpha) = \alpha(1-\gamma/N)^{N\alpha-1}$, which approaches $r(\alpha) \approx \alpha e^{-\gamma\alpha}$ as $N \to \infty$. ■

7. REFERENCES

[1] D. Fudenberg and D. Levine, *The Theory of Learning in Games*, MIT Press, 1999.

[2] Y. Xing and R. Chandramouli, "Distributed discrete power control in wireless data networks using stochastic learning," *Proc. CISS, Princeton*, 2004.

[3] C. Saraydar, N. Mandayam, and D.Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Transactions on Communications*, vol. 50, no. 2, 2002.

[4] A. MacKenzie and S. Wicker, "Stability of multipacket slotted aloha with selfish users and perfect information," *IEEE Infocom*, 2003.

[5] R. Kannan, S. Sarangi, S.S. Iyengar, and L. Ray, "Sensor-centric quality of routing in sensor networks," *Infocom*, 2003.

[6] S. Wicker and A. Goldsmith, "Design challenges for energy-constrained ad hoc wireless networks," *IEEE Wireless Communications*, 2002.

[7] A. MacKenzie and S. Wicker, "Game theory and the design of selfconfiguring, adaptive wireless networks," *IEEE Communications Magazine*, 2001.

[8] B. Arthur, "Inductive reasoning and bounded rationality," *American Economic Association Papers*, vol. 84, 1994.

[9] A. Galstyan, B. Krishnamachari, and K. Lerman, "Resource allocation and emergent coordination in wireless sensor networks," *AAAI Workshop on Sensor Networks*, 2004.

[10] L. Karp, I Lee, and R. Mason, "A global game with strategic substitutes and complements," *Department of Agricultural and Resource Economics, UCB. CUDARE Working Paper 940.*, 2004.

[11] H. Carlsson and E. van Damme, "Global games and equilibrium selection," *Econometrica*, vol. 61, no. 5, 1993.

[12] S. Ross, *Introduction to Probability Models*, Academic Press, sixth edition, 1997.