ANALYSIS OF THE DEGRADATION IN SOURCE LOCATION ACCURACY IN THE PRESENCE OF SENSOR LOCATION ERROR

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ABSTRACT

It is well known that sensor location uncertainty can seriously deteriorate the source location accuracy. In this paper, we provide the analysis of how much degradation the source location accuracy is expected to be with respect to the amount of sensor location error. We first derive the source location MSE when the estimator assumes no sensor location error but in fact there is. Then, the CRLB is evaluated and compared with the one without sensor location error. The analytical results allow us to decide whether a new algorithm to account for the sensor location error is necessary to improve the source location accuracy.

1. INTRODUCTION

Source localization has been one of the important problems in a variety of fields ranging from radar, sonar, radio astronomy, and seismology to oceanography [1]. Considerable attention has been received for this problem and many different estimation algorithms have been proposed.

Regardless of the localization algorithms used, source localization accuracy can be seriously degraded by the uncertainty in sensor locations [2, 3]. For the past few decades, several authors have proposed better location solutions when the receiving sensors have location errors [2–6]. Rockah and Schultheiss [2, 3] improve the DOA estimation accuracy in the presence of sensor location uncertainty if one sensor location and the direction to a second sensor are known. In [4], Lo and Marple derived a calibration technique that requires the knowledge of directions of two calibrating sources. Ng *et al.* [5] approximate the true array steering vector from the measurement by the first order Taylor-series expansion, and then estimate the source DOA by the MUSIC algorithm. In [6], a closed-form solution was proposed that requires only 2 least-squares (LS) minimization steps to estimate the source location in the presence of sensor position errors.

There are, however, very few studies on the deterioration in the source localization accuracy due to the erroneous sensor locations. This paper provides the analysis of how much degradation the source location accuracy is expected to be with respect to the amount of sensor location errors. This paper derives the source location MSE when an algorithm assumes perfect knowledge of the sensor locations but in fact they have errors. The CRLB is also evaluated and compared with the one without sensor location error. Then the difference in the MSE and the CRLB in the presence of sensor location errors will be compared to see whether a new method that takes the sensor location errors into account is necessary to improve accuracy.

The paper is organized as follows. Section 2 provides the location scenario and introduces the symbols used. The MSE and CRLB analysis in the presence of sensor location errors are derived in Section 3. Section 4 presents simulation results to support the analysis, and Section 5 is the conclusions.

2. LOCATION SCENARIO

Let us consider the problem of locating a moving source at position $\mathbf{u}^o = [x^o, y^o, z^o]^T$ and velocity $\dot{\mathbf{u}}^o = [\dot{x}^o, \dot{y}^o, \dot{z}^o]^T$ using an array of M moving sensors through TDOA and FDOA measurements. We shall denote $\mathbf{s}_i^o = [x_i^o, y_i^o, z_i^o]^T$ and $\dot{\mathbf{s}}_i^o = [\dot{x}_i^o, \dot{y}_i^o, \dot{z}_i^o]^T$, $i = 1, 2, \ldots, M$ as the true sensor positions and velocities for the sensors during measurements and they are not known. The available sensor positions and velocities are noisy and are represented as $\mathbf{s}_i = \mathbf{s}_i^o + \Delta\mathbf{s}_i$ and $\dot{\mathbf{s}}_i = \dot{\mathbf{s}}_i^o + \Delta\dot{\mathbf{s}}_i$, where $\Delta\mathbf{s}_i$ and $\Delta\dot{\mathbf{s}}_i$ are position and velocity errors for sensor i and are assumed to be zero mean Gaussian distributed with covariance matrix $E[\boldsymbol{\beta}\boldsymbol{\beta}^T] = \mathbf{Q}_\beta$, where $\boldsymbol{\beta} = [\Delta\mathbf{s}_1^T, \ldots, \Delta\mathbf{s}_M^T, \Delta\dot{\mathbf{s}}_1^T, \ldots, \Delta\dot{\mathbf{s}}_M]^T$.

The TDOA t_{i1} between sensor pair i and 1 is related to the target ranges to the two sensors as

$$r_{i1}^o = ct_{i1}^o = r_i^o - r_1^o, i = 2, 3, \ldots, M \tag{1}$$

where c is the signal propagation speed and

$$r_i^o = \|\mathbf{u}^o - \mathbf{s}_i^o\| = \sqrt{(\mathbf{u}^o - \mathbf{s}_i^o)^T(\mathbf{u}^o - \mathbf{s}_i^o)}. \tag{2}$$

The time derivative of (1) gives the FDOA(range rate):

$$\dot{r}_{i1}^o = \dot{r}_i^o - \dot{r}_1^o, i = 2, 3, \ldots, M \tag{3}$$

where from (2),

$$\dot{r}_i^o = \frac{(\mathbf{u}^o - \mathbf{s}_i^o)^T(\dot{\mathbf{u}}^o - \dot{\mathbf{s}}_i^o)}{r_i^o}. \tag{4}$$

The TDOA and FDOA measurements are noisy so that the measurement vectors are denoted as $\mathbf{r} = [r_{21}, r_{31}, \ldots, r_{M1}]^T = [r_{21}^o, r_{31}^o, \ldots, r_{M1}^o]^T + \mathbf{n} = \mathbf{r}^o + \mathbf{n}$ and $\dot{\mathbf{r}} = [\dot{r}_{21}, \dot{r}_{31}, \ldots, \dot{r}_{M1}]^T = [\dot{r}_{21}^o, \dot{r}_{31}^o, \ldots, \dot{r}_{M1}^o]^T + \dot{\mathbf{n}} = \dot{\mathbf{r}}^o + \dot{\mathbf{n}}$.

We collect the TDOA and FDOA measurement vectors as $\mathbf{p} = [\mathbf{r}^T, \dot{\mathbf{r}}^T]^T$, and the TDOA and FDOA error vectors as $\boldsymbol{\alpha} = [\mathbf{n}^T, \dot{\mathbf{n}}^T]^T$. In this study, $\boldsymbol{\alpha}$ is assumed to be a zero mean Gaussian vector with covariance matrix $E[\boldsymbol{\alpha}\boldsymbol{\alpha}^T] = \mathbf{Q}_\alpha$. The measurement noise $\boldsymbol{\alpha}$ is assumed to be independent of the sensor location noise $\boldsymbol{\beta}$ for ease of illustration.

3. MSE AND CRLB

In this section, we will first derive the source location MSE assuming that there is no sensor location error, but in fact sensor location error is present. Second, the CRLB is evaluated and compared with the one when the sensor location error is absent.

3.1. Source Location MSE without accounting for Sensor Location Error

When an estimation algorithm finds the source location assuming there is no sensor error but in fact there is, we are interested in determining the increase in the source location MSE. Here we derive the MSE based on the Taylor-series linearization approach. The analytical results are valid for any estimation algorithm approaching the CRLB in the absence of sensor location error.

First we define the following quantities,

$$\begin{aligned} f_{i1}(\boldsymbol{\theta}) &= \|\mathbf{u}-\mathbf{s}_i\| - \|\mathbf{u}-\mathbf{s}_1\| \\ \dot{f}_{i1}(\boldsymbol{\theta}) &= \frac{(\dot{\mathbf{u}}-\dot{\mathbf{s}}_i)^T(\mathbf{u}-\mathbf{s}_i)}{\|\mathbf{u}-\mathbf{s}_i\|} - \frac{(\dot{\mathbf{u}}-\dot{\mathbf{s}}_1)^T(\mathbf{u}-\mathbf{s}_1)}{\|\mathbf{u}-\mathbf{s}_1\|}, \end{aligned} \tag{5}$$

note that $\mathbf{s}_i$ and $\dot{\mathbf{s}}_i$ are the noisy sensor positions and velocities known to the estimator. Let $\boldsymbol{\theta} = [\ \mathbf{u}^T \quad \dot{\mathbf{u}}^T\ ]^T$ be the source location parameter vector. Applying Taylor-series expansion of $\mathbf{f}(\boldsymbol{\theta})$ around certain $\boldsymbol{\theta}_o$ gives

$$\mathbf{f}(\boldsymbol{\theta}) \simeq \mathbf{f}(\boldsymbol{\theta}_o) + \mathbf{F}(\boldsymbol{\theta}_o)(\boldsymbol{\theta}-\boldsymbol{\theta}_o), \tag{6}$$

where $\mathbf{f}(\boldsymbol{\theta}) = [f_{21}(\boldsymbol{\theta}), f_{31}(\boldsymbol{\theta}), \ldots, f_{M1}(\boldsymbol{\theta}), \dot{f}_{21}(\boldsymbol{\theta}), \dot{f}_{31}(\boldsymbol{\theta}), \ldots, \dot{f}_{M1}(\boldsymbol{\theta})]^T$, $\mathbf{F}(\boldsymbol{\theta}_o) = \left.\frac{\partial \mathbf{f}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right|_{\boldsymbol{\theta}_o} = \left.[\ \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \quad \frac{\partial \mathbf{f}}{\partial \dot{\mathbf{u}}}\ ]\right|_{\boldsymbol{\theta}_o}$ and the 2nd and higher order terms are ignored. Subtracting (6) from the measurement vector $\mathbf{p}$ gives the measurement error,

$$\mathbf{e} = \mathbf{p} - \mathbf{f}(\boldsymbol{\theta}) = \mathbf{p} - \mathbf{f}(\boldsymbol{\theta}_o) - \mathbf{F}(\boldsymbol{\theta}_o)(\boldsymbol{\theta}-\boldsymbol{\theta}_o).$$

We find $\boldsymbol{\theta}$ by minimizing

$$\boldsymbol{\xi} = \mathbf{e}^T \mathbf{Q}_\alpha^{-1} \mathbf{e},$$

where $\mathbf{Q}_\alpha^{-1}$ is the weighting matrix. The choice of it will be discussed at the end of this section. Taking derivative with respect to $\boldsymbol{\theta}$ and setting the gradient to zero yields the solution

$$\boldsymbol{\theta} = \boldsymbol{\theta}_o + \left[\mathbf{F}(\boldsymbol{\theta}_o)^T \mathbf{Q}_\alpha^{-1} \mathbf{F}(\boldsymbol{\theta}_o)\right]^{-1} \mathbf{F}(\boldsymbol{\theta}_o)^T \mathbf{Q}_\alpha^{-1} \left(\mathbf{p} - \mathbf{f}(\boldsymbol{\theta}_o)\right).$$

If we choose $\boldsymbol{\theta}_o$ to be the true solution $\boldsymbol{\theta}^o$, then the estimation error of the emitter is

$$\boldsymbol{\theta} - \boldsymbol{\theta}^o = \left[\mathbf{F}(\boldsymbol{\theta}^o)^T \mathbf{Q}_\alpha^{-1} \mathbf{F}(\boldsymbol{\theta}^o)\right]^{-1} \mathbf{F}(\boldsymbol{\theta}^o)^T \mathbf{Q}_\alpha^{-1} \left(\mathbf{p} - \mathbf{f}(\boldsymbol{\theta}^o)\right). \tag{7}$$

We now simplify $\mathbf{p} - \mathbf{f}(\boldsymbol{\theta}^o)$ further in order to obtain the MSE. The elements of $(\mathbf{p} - \mathbf{f}(\boldsymbol{\theta}^o))$ are $r_{i1} - f_{i1}(\boldsymbol{\theta}^o)$ and $\dot{r}_{i1} - \dot{f}_{i1}(\boldsymbol{\theta}^o)$, $i = 2, 3, \ldots, M$. First we will simplify $r_{i1} - f_{i1}(\boldsymbol{\theta}^o)$ by using (2) and (5),

$$\begin{aligned} r_{i1} - f_{i1}(\boldsymbol{\theta}^o) &= n_{i1} + r_{i1}^o - f_{i1}(\boldsymbol{\theta}^o) \\ &= n_{i1} + \|\mathbf{u}^o - \mathbf{s}_i^o\| - \|\mathbf{u}^o - \mathbf{s}_1^o\| \\ &\quad - \left(\|\mathbf{u}^o - \mathbf{s}_i\| - \|\mathbf{u}^o - \mathbf{s}_1\|\right). \end{aligned} \tag{8}$$

Putting $\mathbf{s}_i = \mathbf{s}_i^o + \Delta\mathbf{s}_i$ and applying Taylor-series expansion up to the first order term gives,

$$\|\mathbf{u}^o - \mathbf{s}_i\| \simeq \|\mathbf{u}^o - \mathbf{s}_i^o\| - \mathbf{a}_i^T \Delta\mathbf{s}_i,$$

where $\mathbf{a}_i = \frac{(\mathbf{u}^o - \mathbf{s}_i^o)}{\|\mathbf{u}^o - \mathbf{s}_i^o\|}$ and hence

$$r_{i1} - f_{i1}(\boldsymbol{\theta}^o) = n_{i1} + \mathbf{a}_i^T \Delta\mathbf{s}_i - \mathbf{a}_1^T \Delta\mathbf{s}_1. \tag{9}$$

We can express $\dot{r}_{i1} - \dot{f}_{i1}(\boldsymbol{\theta}^o)$ by using (2), (4) and (5),

$$\begin{aligned} &\dot{r}_{i1} - \dot{f}_{i1}(\boldsymbol{\theta}^o) = \dot{n}_{i1} + \dot{r}_{i1}^o - \dot{f}_{i1}(\boldsymbol{\theta}^o) \\ &= \dot{n}_{i1} + \frac{(\mathbf{u}^o - \mathbf{s}_i^o)^T(\dot{\mathbf{u}}^o - \dot{\mathbf{s}}_i^o)}{\|\mathbf{u}^o - \mathbf{s}_i^o\|} - \frac{(\mathbf{u}^o - \mathbf{s}_1^o)^T(\dot{\mathbf{u}}^o - \dot{\mathbf{s}}_1^o)}{\|\mathbf{u}^o - \mathbf{s}_1^o\|} \\ &- \left(\frac{(\mathbf{u}^o - \mathbf{s}_i)^T(\dot{\mathbf{u}}^o - \dot{\mathbf{s}}_i)}{\|\mathbf{u}^o - \mathbf{s}_i\|} - \frac{(\mathbf{u}^o - \mathbf{s}_1)^T(\dot{\mathbf{u}}^o - \dot{\mathbf{s}}_1)}{\|\mathbf{u}^o - \mathbf{s}_1\|}\right). \end{aligned} \tag{10}$$

Applying Taylor-series expansion and simplifying, yields

$$\frac{(\mathbf{u}^o - \mathbf{s}_i)^T(\dot{\mathbf{u}}^o - \dot{\mathbf{s}}_i)}{\|\mathbf{u}^o - \mathbf{s}_i\|} \simeq \frac{(\mathbf{u}^o - \mathbf{s}_i^o)^T(\dot{\mathbf{u}}^o - \dot{\mathbf{s}}_i^o)}{\|\mathbf{u}^o - \mathbf{s}_i^o\|} - \mathbf{a}_i^T \Delta\dot{\mathbf{s}}_i - \mathbf{b}_i^T \Delta\mathbf{s}_i,$$

where $\mathbf{a}_i$ is defined before (9) and $\mathbf{b}_i = \frac{(\dot{\mathbf{u}}^o - \dot{\mathbf{s}}_i^o)}{\|\mathbf{u}^o - \mathbf{s}_i^o\|} - \frac{(\mathbf{u}^o - \mathbf{s}_i^o)\dot{r}_i^o}{\|\mathbf{u}^o - \mathbf{s}_i^o\|^2}$. Hence

$$\dot{r}_{i1} - \dot{f}_{i1}(\boldsymbol{\theta}^o) = \dot{n}_{i1} + \mathbf{a}_i^T \Delta\dot{\mathbf{s}}_i - \mathbf{a}_1^T \Delta\dot{\mathbf{s}}_1 + \mathbf{b}_i^T \Delta\mathbf{s}_i - \mathbf{b}_1^T \Delta\mathbf{s}_1. \tag{11}$$

As a result, from (9) and (11),

$$\mathbf{p} - f(\boldsymbol{\theta}^o) = \boldsymbol{\alpha} - \mathbf{P}\boldsymbol{\beta}, \tag{12}$$

and

$$\mathbf{P} = \begin{bmatrix} [\mathbf{P}_{11}]_{(M-1)\times(3M)} & [\mathbf{O}]_{(M-1)\times(3M)} \\ [\mathbf{P}_{21}]_{(M-1)\times(3M)} & [\mathbf{P}_{11}]_{(M-1)\times(3M)} \end{bmatrix}.$$

The i^{th} row, $i = 1, 2, \ldots, M-1$ of $\mathbf{P}_{11}$ and $\mathbf{P}_{21}$ are

$$\begin{aligned} \mathbf{P}_{11}(i,:) &= [\ \mathbf{a}_1^T \quad \mathbf{0}_{1\times 3(i-1)} \quad -\mathbf{a}_{i+1}^T \quad \mathbf{0}_{1\times 3(M-i-1)}\ ] \\ \mathbf{P}_{21}(i,:) &= [\ \mathbf{b}_1^T \quad \mathbf{0}_{1\times 3(i-1)} \quad -\mathbf{b}_{i+1}^T \quad \mathbf{0}_{1\times 3(M-i-1)}\ ], \end{aligned}$$

where $\mathbf{a}_i$ and $\mathbf{b}_i$ are defined before (9) and (11) respectively.

Putting (12) into (7), and under the assumption that both $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are zero mean, the expectation of (7) gives,

$$E[\boldsymbol{\theta} - \boldsymbol{\theta}^o] = \left[\mathbf{F}(\boldsymbol{\theta}^o)^T \mathbf{Q}_\alpha^{-1} \mathbf{F}(\boldsymbol{\theta}^o)\right]^{-1} \mathbf{F}(\boldsymbol{\theta}^o)^T \mathbf{Q}_\alpha^{-1} E\left[\boldsymbol{\alpha} - \mathbf{P}\boldsymbol{\beta}\right] = \mathbf{0}$$

which shows that the estimator $\boldsymbol{\theta}$ is unbiased up to linear approximation, although we did not account for the sensor noise in estimating the emitter location and velocity.

Multiplying (7) with its transpose, taking expectation and using (12) forms

$$\begin{aligned} MSE(\boldsymbol{\theta}) = &\left(\mathbf{F}(\boldsymbol{\theta}^o)^T \mathbf{Q}_\alpha^{-1} \mathbf{F}(\boldsymbol{\theta}^o)\right)^{-1} + \\ &\left(\mathbf{F}(\boldsymbol{\theta}^o)^T \mathbf{Q}_\alpha^{-1} \mathbf{F}(\boldsymbol{\theta}^o)\right)^{-1} \mathbf{F}(\boldsymbol{\theta}^o)^T \mathbf{Q}_\alpha^{-1} \mathbf{P} \\ &\mathbf{Q}_\beta \mathbf{P}^T \mathbf{Q}_\alpha^{-1} \mathbf{F}(\boldsymbol{\theta}^o) \left(\mathbf{F}(\boldsymbol{\theta}^o)^T \mathbf{Q}_\alpha^{-1} \mathbf{F}(\boldsymbol{\theta}^o)\right)^{-1}. \end{aligned} \tag{13}$$

Let us now choose $\mathbf{Q}_\alpha$ as the covariance matrix of $\boldsymbol{\alpha}$ and consider Gaussian noise. Then that the 1^{st} term in (13) is the CRLB in the absence of sensor location noise [7]. The second term therefore represents the additional error resulted from sensor location inaccuracy. The trace of the second term is the increase in MSE.

3.2. CRLB

The CRLB is the lowest possible variance that an unbiased linear estimator can achieve. It is given by the inverse of the Fisher information matrix $\mathbf{J}$ defined as [8],

$$\mathbf{J} = E\left[\left(\frac{\partial \ln g(\mathbf{v};\boldsymbol{\phi})}{\partial \boldsymbol{\phi}}\right)^T \left(\frac{\partial \ln g(\mathbf{v};\boldsymbol{\phi})}{\partial \boldsymbol{\phi}}\right)\right]. \quad (14)$$

where $\mathbf{v} = [\mathbf{p}^T, \mathbf{q}^T]^T$ is the data vector and $\boldsymbol{\phi} = [\boldsymbol{\theta}^T, \mathbf{q}^T]^T$ is the unknown vector with $\boldsymbol{\theta} = [\mathbf{u}^T, \dot{\mathbf{u}}^T]^T$ and $\mathbf{q} = [\mathbf{s}^T, \dot{\mathbf{s}}^T]^T$. Since the receiver locations are unknown, they act as auxiliary parameters when deriving the CRLB of the source location. $g(\mathbf{v};\boldsymbol{\phi})$ is the probability density function of $\mathbf{v}$ that is parameterized on the vector $\boldsymbol{\phi}$. We assume that TDOA and FDOA noise and sensor location noise are independent and Gaussian distributed with known covariance matrices $\mathbf{Q}_\alpha$ and $\mathbf{Q}_\beta$ respectively. Therefore we can write $g(\mathbf{v};\boldsymbol{\phi}) = g(\mathbf{p};\boldsymbol{\phi})g(\mathbf{q};\boldsymbol{\phi})$. Normally $\mathbf{Q}_\alpha$ and $\mathbf{Q}_\beta$ are obtained by using of a source of known location and the amounts of perturbations in the receiver locations, and they are not known exactly in practice. We assume they are known here for ease of derivation. After taking natural log and performing differentiation, the CRLB is,

$$CRLB(\boldsymbol{\phi}) = \mathbf{J}^{-1} = -\begin{bmatrix} \mathbf{X} & \mathbf{Y} \\ \mathbf{Y}^T & \mathbf{Z} \end{bmatrix}^{-1} \quad (15)$$

The partial derivative of the upper left corner is

$$\mathbf{X} = -E\left[\frac{\partial^2 \ln g(\mathbf{v};\boldsymbol{\phi})}{\partial \boldsymbol{\theta}\partial \boldsymbol{\theta}^T}\right] = \left(\frac{\partial \mathbf{p}}{\partial \boldsymbol{\theta}}\right)^T \mathbf{Q}_\alpha^{-1}\left(\frac{\partial \mathbf{p}}{\partial \boldsymbol{\theta}}\right), \quad (16)$$

and

$$\frac{\partial \mathbf{p}}{\partial \boldsymbol{\theta}} = \left[\frac{\partial \mathbf{p}}{\partial \mathbf{u}}, \frac{\partial \mathbf{p}}{\partial \dot{\mathbf{u}}}\right],$$

where $\frac{\partial \mathbf{p}}{\partial \mathbf{u}} = [\mathbf{a}_2 - \mathbf{a}_1, \ldots, \mathbf{a}_M - \mathbf{a}_1, \mathbf{b}_2 - \mathbf{b}_1, \ldots, \mathbf{b}_M - \mathbf{b}_1]^T$, $\frac{\partial \mathbf{p}}{\partial \dot{\mathbf{u}}} = \left[\mathbf{O}_{3\times(M-1)}, \mathbf{a}_2 - \mathbf{a}_1, \ldots, \mathbf{a}_M - \mathbf{a}_1\right]^T$, $\mathbf{a}_i = \frac{(\mathbf{u}^o - \mathbf{s}_i^o)}{\|\mathbf{u}^o - \mathbf{s}_i^o\|}$ and $\mathbf{b}_i = \frac{(\dot{\mathbf{u}}^o - \dot{\mathbf{s}}_i^o)}{\|\mathbf{u}^o - \mathbf{s}_i^o\|} - \frac{(\mathbf{u}^o - \mathbf{s}_i^o)\dot{r}_i^o}{\|\mathbf{u}^o - \mathbf{s}_i^o\|^2}$.

The partial derivative of the lower right corner is,

$$\mathbf{Z} = -E\left[\frac{\partial^2 \ln g(\mathbf{v};\boldsymbol{\phi})}{\partial \mathbf{q}\partial \mathbf{q}^T}\right] = \left(\frac{\partial \mathbf{p}}{\partial \mathbf{q}}\right)^T \mathbf{Q}_\alpha^{-1}\left(\frac{\partial \mathbf{p}}{\partial \mathbf{q}}\right) + \mathbf{Q}_\beta{}^{-1}. \quad (17)$$

and

$$\frac{\partial \mathbf{p}}{\partial \mathbf{q}} = \left[\frac{\partial \mathbf{p}}{\partial \mathbf{s}}, \frac{\partial \mathbf{p}}{\partial \dot{\mathbf{s}}}\right],$$

where $\frac{\partial \mathbf{p}}{\partial \mathbf{s}} = \begin{bmatrix} [\mathbf{P}_{11}]_{(M-1)\times(3M)} \\ [\mathbf{P}_{21}]_{(M-1)\times(3M)} \end{bmatrix}$, $\frac{\partial \mathbf{p}}{\partial \dot{\mathbf{s}}} = \begin{bmatrix} [\mathbf{O}]_{(M-1)\times(3M)} \\ [\mathbf{P}_{11}]_{(M-1)\times(3M)} \end{bmatrix}$, and $\mathbf{P}_{11}$ and $\mathbf{P}_{21}$ are defined below (12).

The partial derivative of the upper right corner is,

$$\mathbf{Y} = -E\left[\frac{\partial^2 \ln g(\mathbf{v};\boldsymbol{\phi})}{\partial \boldsymbol{\theta}\partial \mathbf{q}^T}\right] = \left(\frac{\partial \mathbf{p}}{\partial \boldsymbol{\theta}}\right)^T \mathbf{Q}_\alpha^{-1}\left(\frac{\partial \mathbf{p}}{\partial \mathbf{q}}\right), \quad (18)$$

Putting (16),(17) and (18) into (15) and invoking the partitioned matrix inversion formula [9] gives

$$CRLB(\boldsymbol{\theta}) = \mathbf{X}^{-1} + \mathbf{X}^{-1}\mathbf{Y}(\mathbf{Z} - \mathbf{Y}^T\mathbf{X}^{-1}\mathbf{Y})^{-1}\mathbf{Y}^T\mathbf{X}^{-1}. \quad (19)$$

Note that $\mathbf{X}^{-1}$ is the CRLB of $\boldsymbol{\theta}$ when there is no sensor position and velocity noise [7]. Hence the second term in (19) represents the increase in CRLB in the presence of sensor location error. The trace of (19) is the minimum possible source location MSE that any linear unbiased estimator can achieve.

Table 1. Positions and Velocities of Sensors. The unit of the position is meter and that of the velocity is m/s.

sensor no. i	x_i	y_i	z_i	$\dot{x}_i$	$\dot{y}_i$	$\dot{z}_i$
1	-591	-650	-493	30	-20	20
2	90	444	562	-30	10	20
3	590	860	-843	10	-20	10
4	-750	-515	191	10	20	30
5	35	222	-150	-20	10	10
6	-260	180	340	20	-10	10

4. SIMULATIONS

In this section, we will provide the simulation results to support the theoretical development in the previous section. The sensor positions and velocities used for simulation are shown in Table I. The TDOA noise is Gaussian with power $\sigma_t^2 = 10^{-4}/c^2$ and the FDOA noise is also Gaussian with power $\sigma_f^2 = 10^{-5}/c^2$. $\mathbf{Q}_\alpha$ is $\begin{bmatrix} \mathbf{R}_t & \mathbf{O} \\ \mathbf{O}^T & \mathbf{R}_f \end{bmatrix}$, where $\mathbf{R}_t$ is a $(M-1)\times(M-1)$ matrix with $c^2\sigma_t^2$ in the diagonal and $0.5c^2\sigma_t^2$ in all other elements [7], and $\mathbf{R}_f = \mathbf{R}_t\sigma_f^2/\sigma_t^2$. $\mathbf{Q}_\beta$ is $\begin{bmatrix} \mathbf{R}_s & \mathbf{O} \\ \mathbf{O}^T & \dot{\mathbf{R}}_s \end{bmatrix}$, where $\mathbf{R}_s = \sigma_s^2\mathbf{I}_{3M\times3M}$, $\dot{\mathbf{R}}_s = \dot{\sigma_s}^2\mathbf{I}_{3M\times3M}$, and $\dot{\sigma_s}^2 = 0.5\sigma_s{}^2$.

The emitter is located at $\mathbf{u} = [600, 650, 550]^T m$ and has a velocity of $\dot{\mathbf{u}} = [-20, 15, 40]^T m/s$. We shall examine the location accuracy as σ_s^2 varies. Fig. 1 plots the trace of the upper left 3×3 submatrix of (13) which is the source position MSE, and the trace of the lower 3×3 submatrix of (13) which is the source velocity MSE. The traces of $CRLB(\mathbf{u})$, $CRLB(\dot{\mathbf{u}})$ with the sensor location errors as obtained from (19) and those without sensor location noise [7] are also shown for comparison. The CRLB deviates farther and farther away from the case without sensor location error as $\sigma_s{}^2$ increases. Even at a very small sensor error power $\sigma_s{}^2 = 10^{-4}m^2$, the increase in CRLB for position $\mathbf{u}$ is $5.16dB$ and that for velocity $\dot{\mathbf{u}}$ is $10.33dB$. The MSE performance is clearly worse than the CRLB, when the sensor location error is not taken into account in a location algorithm.

Fig. 2 is the results for the same near-field source but with different noise powers at different sensor locations, where $\mathbf{R}_s = \sigma_s^2 diag[1,1,1,2,2,2,10,10,10,4,4,4,20,20,20,3,3,3]$ and $\dot{\mathbf{R}}_s = 0.5\mathbf{R}_s$. The results are consistent with those in Fig. 1. There is, however, even bigger difference between the MSE without taking location error into account and the CRLB. If the sensor location noise power σ_s^2 is bigger than $10^{-4}m^2$, the difference between the two is about $4dB$ for position and $3dB$ for velocity.

The performance of a far-field source at $\mathbf{u} = [2000, 2500, 3000]^T$ m and $\dot{\mathbf{u}} = [-20, 15, 40]^T m/s$ is shown in Fig. 3, where the sensors have different location error as those in Fig. 2. The increase in CRLB in the presence of sensor errors is much more dramatic in a far-field source than a near-field source. The difference between the MSE and the CRLB with sensor location errors also becomes bigger as the source is farther away from the sensors. The increase in error is about $5dB$ for position and $4dB$ for velocity when σ_s^2 is bigger than $10^{-4}m^2$.

Although only the theoretical MSEs are shown in the figure, we have verified the MSE formula (13) with the location algorithm [7] as well as the Taylor series iterative method.

The simulation results provides two observations. First, the CRLB is very sensitive to the sensor location error. Even a very small sensor location error can lead to a very big increase in CRLB with respect to

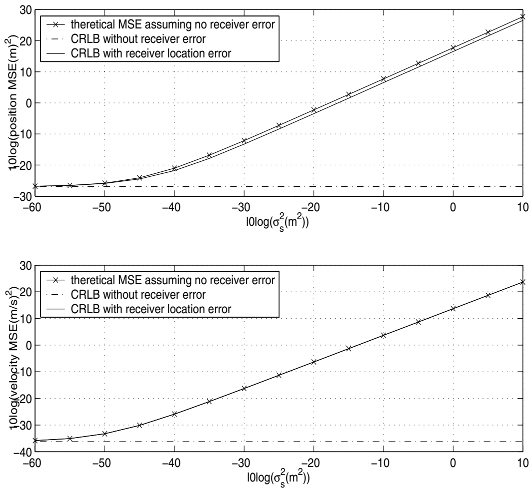

Fig. 1. Comparison of the CRLBs and the source location MSE when assuming no sensor location noise, near-field moving source and equal sensor location noise power.

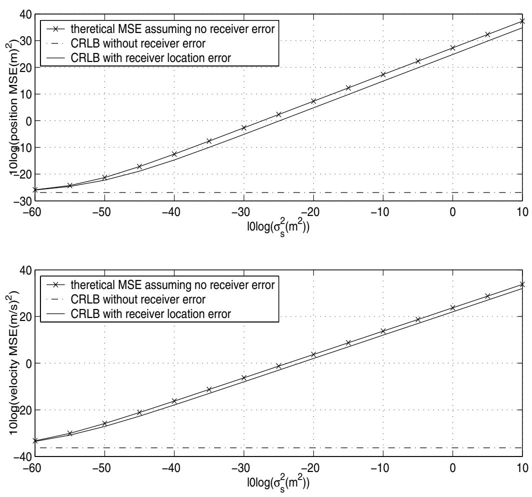

Fig. 2. Comparison of the CRLBs and the source location MSE when assuming no sensor location noise, near-field moving source and unequal sensor location noise power.

the case of no sensor location error. The increase will become larger if the source is farther away from the sensors. Second, the MSE that ignores the sensor location errors has significant difference with the CRLB, especially when the noise power in the sensor locations are not equal. Consequently, an estimation algorithm should take the sensor location error into account in order to improve accuracy.

5. CONCLUSIONS

We have derived analytically the increase in location MSE when an algorithm assumes the sensor locations are accurate but in fact have errors, and the increase in CRLB due to random and Gaussian sensor location error. The analytical results provide us some guidance on whether a new algorithm that accounts for the sensor location error is necessary to improve performance under different location scenarios.

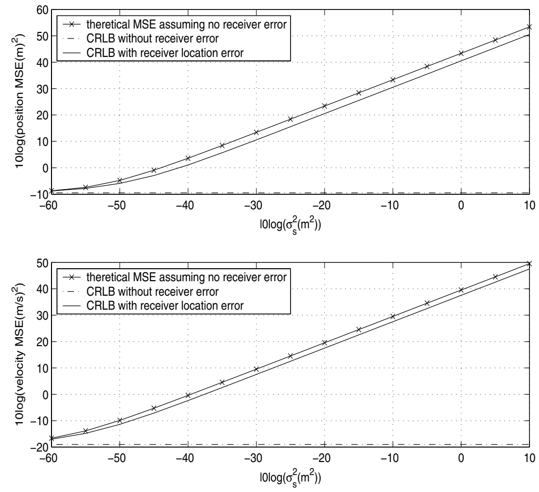

Fig. 3. Comparison of the CRLBs and the source location MSE when assuming no sensor location noise, far-field moving source and unequal sensor location noise power.

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