DIVIDE-AND-CONQUER BASED CLOSED-FORM POSITION ESTIMATION FOR AOA AND TDOA MEASUREMENTS

Andreu Urruela , Alba Pagès-Zamora and Jaume Riba

Signal Processing and Communications Group, Technical University of Catalonia
Campus Nord, Ed. D5, Jordi Girona 1-3, D5, 08034, Barcelona (SPAIN)
Emails: {andreu,alba,jriba}@gps.tsc.upc.edu

ABSTRACT

Mobile location using Time Of Arrival (TOA), Time Difference Of Arrival (TDOA) or Angle Of Arrival (AOA) measurements has received considerable attention over the last years. Several closed-form algorithms have been presented for the TOA and TDOA case based on approximations of the Maximum-Likelihood (ML) estimator. In the case of AOA measurements, only ad-hoc estimators have been presented in order to avoid the classical linearization solution that needs an initial guess. This paper presents an approximation of the ML position estimator based on AOA measurements applying the Divide-And-Conquer approach dividing the ML estimation in smaller problems each one with a closed-form solution. Numerical simulations show that the proposed algorithm outperforms the previous contributions and presents a generic way to combine AOA and TDOA measurements.

1. INTRODUCTION

A problem of growing importance in mobile radio systems is finding the position of mobile terminals. This need is motivated not only by the mandatory requirements imposed by the US FCC for emergency calls but also by the potential market related to location-based applications. Recently, several publications have provided methods to estimate the mobile position using measurements of different nature, i.e., Time of Arrival (TOA), Time-Difference-Of-Arrival (TDOA), Signal Strength (SS) and Angle of Arrival (AOA). For an overview of the existing techniques, we refer the reader to [1] and [2] and references therein. This paper focuses on AOA-based positioning methods, combined with TDOA measurements as an option, to estimate the mobile position. Since wireless 3G systems and beyond already include an antenna array at the Node B, we assume the Base Station (BS) can measure the AOA of the incoming signal. A relevant feature of AOA-based positioning methods is that, in a 2-dimensional problem, they require only two AOA measurements to estimate the mobile position, whereas the other methods need at least three. However, the position accuracy of AOA-based positioning methods decreases with the distance between the mobile and the BS. Therefore, a good performance of AOA-based methods is expected when the mobile is located near the BS. The ML estimation of the position follows a non-linear relationship between the AOA measurements and the mobile position. The classical solution to this problem was presented in [3] where a linearization of the non-linear relation is proposed based on the first order Taylor approximation. This technique requires an initial position estimate and the solution is obtained in an iterative fashion. The procedure is valid for homogeneous and hybrid methods based on TOAs, TDOAs, SSs and AOAs measurements. However, the convergence of this technique is not guaranteed and the final accuracy of the position estimate is determined by the initial position. As an alternative to the linearized ML estimator, non-optimal closed-form methods have been proposed. In [4], a closed-form position estimate method with AOA measurements is provided based on the Least-Squares principle. This method proved to be useful providing an initial position estimation for the linearized iterative ML estimator. This paper presents a new algorithm to estimate the mobile position based only on AOA measurements or based on AOAs combined with TDOA measurements. The proposed algorithm is based on the divide and conquer principle and is an extension of the closed-form estimator presented in [5] for TDOA measurements.

This work was financed by the Spanish/Catalan Science and Technology Commissions and FEDER funds from the European Commission: TIC2003- 05482, TEC2004-04526 and 2001SGR-00268.

2. SIGNAL MODEL AND PREVIOUS APPROACHES

Let us assume a scenario where N BSs can measure N AOA measurements of a single mobile terminal. These N measurements denoted by θ_n for $n = [1, N]$ are:

$$\theta_n = f_n(\mathbf{z}) + w_n \tag{1}$$

where $\mathbf{z} = \begin{bmatrix} x & y \end{bmatrix}^T$ is the unknown position of the mobile and w_n is a zero-mean uncorrelated Gaussian noise term with known variance, i.e. $E[w_n w_{n'}] = \sigma_n^2 \delta_{n-n'}$. The non-linear relationship between the AOA measurements and the mobile position $f_n(\mathbf{z})$ can be expressed as:

$$f_n(\mathbf{z}) = \arctan\left(\frac{y - y_n}{x - x_n}\right) \tag{2}$$

where $\mathbf{z}_n = \begin{bmatrix} x_n & y_n \end{bmatrix}^T$ is the known position of the n-th BS. Stacking the N AOA measurements of (1) in a vector, we have:

$$\boldsymbol{\theta} = \mathbf{f}(\mathbf{z}) + \mathbf{w} \tag{3}$$

where $\boldsymbol{\theta} = \begin{bmatrix} \theta_1 & \dots & \theta_N \end{bmatrix}^T$, $\mathbf{w} = \begin{bmatrix} w_1 & \dots & w_N \end{bmatrix}^T$ and $\mathbf{f}(\mathbf{z}) = \begin{bmatrix} f_1(\mathbf{z}) & \dots & f_N(\mathbf{z}) \end{bmatrix}^T$. From (3) and taking into account that $\mathbf{w}$ is a zero-mean Gaussian noise vector, the ML estimator of the position can be expressed as:

$$\widehat{\mathbf{z}}_{ML} = \arg\min_{\mathbf{z}} \phi_{ML}^{AOA}(\mathbf{z}) \tag{4}$$

where

$$\phi_{ML}^{AOA}(\mathbf{z}) = (\boldsymbol{\theta} - \mathbf{f}(\mathbf{z}))^T \mathbf{R}^{-1} (\boldsymbol{\theta} - \mathbf{f}(\mathbf{z})) \tag{5}$$

and $\mathbf{R} = E\left[\mathbf{w}\mathbf{w}^T\right] = \mathrm{diag}\left(\begin{array}{ccc} \sigma_1^2 & \ldots & \sigma_N^2 \end{array}\right)$. The classical solution to this minimization problem consists in linearizing the non-linear function $\mathbf{f}(\mathbf{z})$ by expanding it in a Taylor serie around a reference point denoted by $\mathbf{z}_0$ as follows:

$$\mathbf{f}(\mathbf{z}) \approx \mathbf{f}(\mathbf{z}_0) + \mathbf{G}(\mathbf{z}_0)(\mathbf{z} - \mathbf{z}_0) \tag{6}$$

where $\mathbf{G}(\mathbf{z})$ is the Jacobian matrix obtained as:

$$\mathbf{G}(\mathbf{z}) = \nabla_{\mathbf{z}} \mathbf{f}(\mathbf{z}) = \begin{bmatrix} \mathbf{g}_1(\mathbf{z})^T \\ \vdots \\ \mathbf{g}_N(\mathbf{z})^T \end{bmatrix} \tag{7}$$

and

$$\mathbf{g}_n(\mathbf{z}) = \frac{1}{||\mathbf{z} - \mathbf{z}_n||} \begin{bmatrix} -\sin f_n(\mathbf{z}) \\ \cos f_n(\mathbf{z}) \end{bmatrix} \tag{8}$$

Considering (6), the minimization shown in (4) has a closed-form solution, also known as Torrieri method [3], as:

$$\widehat{\mathbf{z}}_{ML} \approx \mathbf{z}_0 + \left(\mathbf{G}(\mathbf{z}_0)^T \mathbf{R}^{-1} \mathbf{G}(\mathbf{z}_0)\right)^{-1} \mathbf{G}(\mathbf{z}_0)^T \mathbf{R}^{-1} (\boldsymbol{\theta} - \mathbf{f}(\mathbf{z}_0))$$

One of the major drawbacks of this solution is the requirement of an initial guess $\mathbf{z}_0$. As it will be shown in the simulations section, the performance of this algorithm is very sensitive to the accuracy of this initial guess. The main advantage is that this algorithm can be implemented iteratively using new AOA measurements at each iteration. Indeed, this leads to an implementation of the classical Extended Kalman Filter.

In [4], a closed-form position estimator that solves the problem of the initial guess was proposed. This approach is the Least Squares (LS) solution of the signal model presented in (3) in the absence of noise. Taking (1) without the noise terms, we have:

$$\tan\theta_n = \frac{\sin\theta_n}{\cos\theta_n} = \frac{y - y_n}{x - x_n} \tag{9}$$

Rearranging the previous equations and stacking the N equations corresponding to the N AOA measurements, we obtain the following linear system:

$$\mathbf{H}(\boldsymbol{\theta})\mathbf{z} = \mathbf{b}(\boldsymbol{\theta}) \tag{10}$$

where

$$\mathbf{H}(\boldsymbol{\theta}) = \begin{bmatrix} -\sin\theta_1 & \cos\theta_1 \\ \vdots & \vdots \\ -\sin\theta_N & \cos\theta_N \end{bmatrix} \tag{11}$$

$$\mathbf{b}(\boldsymbol{\theta}) = \begin{bmatrix} -x_1\sin\theta_1 + y_1\cos\theta_1 \\ \vdots \\ -x_N\sin\theta_N + y_N\cos\theta_N \end{bmatrix} \tag{12}$$

Since the measurement noise term $\mathbf{w}$ of (3) affects both the model matrix $\mathbf{H}(\boldsymbol{\theta})$ and the vector $\mathbf{b}(\boldsymbol{\theta})$, it is not clear how to find an estimate of $\mathbf{z}$. The authors in [4] proposed the following position estimate based on the LS technique:

$$\widehat{\mathbf{z}} = \left(\mathbf{H}(\boldsymbol{\theta})^T \mathbf{H}(\boldsymbol{\theta})\right)^{-1} \mathbf{H}(\boldsymbol{\theta})^T \mathbf{b}(\boldsymbol{\theta}) = \mathbf{H}(\boldsymbol{\theta})^{\#} \mathbf{b}(\boldsymbol{\theta}) \tag{13}$$

Although this estimator does not come from the ML principle, the performance is comparable with that of the Torrieri algorithm. One important drawback of this approach is that it is difficult to include information about the variance of the initial AOA measurements since the errors affect the matrix $\mathbf{H}(\boldsymbol{\theta})$ and the vector $\mathbf{b}(\boldsymbol{\theta})$ in a non-linear way.

3. DIVIDE-AND-CONQUER AOA-BASED ALGORITHM

In this section we present a closed-form algorithm to the minimization problem shown in (4). The idea here is to apply the divide and conquer approach presented in [5] to the location problem using AOA measurements. Assuming that N is multiple of two (see [5] for a straightforward extension to an odd number of measurements), let us divide the incoming vector of AOA measurements $\boldsymbol{\theta}$ in $M = N/2$ subsets of two measurements as follows:

$$\tilde{\boldsymbol{\theta}}_m = \left[\begin{array}{cc} \theta_{2m-1} & \theta_{2m} \end{array}\right]^T \quad m = [1, \ldots, M] \tag{14}$$

Then, taking into account that $\mathbf{R}$ is diagonal, the cost function $\phi_{ML}^{AOA}(\mathbf{z})$ shown in (5) can be expressed as

$$\phi_{ML}^{AOA}(\mathbf{z}) = \sum_{m=1}^{M} \left(\tilde{\boldsymbol{\theta}}_m - \tilde{\mathbf{f}}_m(\mathbf{z})\right)^T \tilde{\mathbf{R}}_m^{-1} \left(\tilde{\boldsymbol{\theta}}_m - \tilde{\mathbf{f}}_m(\mathbf{z})\right) \tag{15}$$

where $\tilde{\mathbf{f}}_m(\mathbf{z}) = \left[\begin{array}{cc} f_{2m-1}(\mathbf{z}) & f_{2m}(\mathbf{z}) \end{array}\right]^T$ is the vector containing the non-linear functions shown in (2) refereed only to the AOA measurements included in $\tilde{\boldsymbol{\theta}}_m$ and $\tilde{\mathbf{R}}_m$ contains their variances as $\tilde{\mathbf{R}}_m = \mathrm{diag}\left(\begin{array}{cc} \sigma_{2m-1}^2 & \sigma_{2m}^2 \end{array}\right)$. Now from (15) and applying the derivative with respect to the unknown position we have

$$\nabla_{\mathbf{z}} \phi_{ML}^{AOA}(\mathbf{z}) = 2\sum_{m=1}^{M} \tilde{\mathbf{G}}_m(\mathbf{z})^T \tilde{\mathbf{R}}_m^{-1} \left(\tilde{\boldsymbol{\theta}}_m - \tilde{\mathbf{f}}_m(\mathbf{z})\right) \tag{16}$$

and

$$\tilde{\mathbf{G}}_m(\mathbf{z}) = \nabla_{\mathbf{z}} \tilde{\mathbf{f}}_m(\mathbf{z}) = \begin{bmatrix} \mathbf{g}_{2m-1}(\mathbf{z})^T \\ \mathbf{g}_{2m}(\mathbf{z})^T \end{bmatrix} \tag{17}$$

where $\mathbf{g}_n(\mathbf{z})$ is defined in (8). It can be observed in (16) that we can make zero the term $\left(\tilde{\boldsymbol{\theta}}_m - \tilde{\mathbf{f}}_m(\mathbf{z})\right)$ for each m, but not for all of them at the same time. Anyway, since $\tilde{\boldsymbol{\theta}}_m$ contains only two AOA measurements, a partial position estimator $\widehat{\mathbf{z}}_m$ can be found only using these measurements satisfying the following set of two non-linear equations:

$$\tilde{\mathbf{f}}_m(\widehat{\mathbf{z}}_m) = \tilde{\boldsymbol{\theta}}_m \quad m = [1, M] \tag{18}$$

Indeed, this closed-form position estimate is the ML estimation of the mobile position using only the pair of measurements included in $\tilde{\boldsymbol{\theta}}_m$, or what is the same, $\widehat{\mathbf{z}}_m$ is the intersection of the two straight lines defined by the two measured angles included in $\tilde{\boldsymbol{\theta}}_m$. After some straightforward mathematical manipulations, we can find the solution of the non-linear equations presented in (18) as:

$$\widehat{\mathbf{z}}_m = \begin{bmatrix} \frac{y_{2m} - y_{2m-1} + x_{2m-1}\tan(\theta_{2m-1}) - x_{2m}\tan(\theta_{2m})}{\tan(\theta_{2m-1}) - \tan(\theta_{2m})} \\ \\ \frac{x_{2m} - x_{2m-1} + y_{2m-1}\tan^{-1}(\theta_{2m-1}) - y_{2m}\tan^{-1}(\theta_{2m})}{\tan^{-1}(\theta_{2m-1}) - \tan^{-1}(\theta_{2m})} \end{bmatrix} \tag{19}$$

Now from (18) and assuming that the non-linear function $\tilde{\mathbf{f}}_m(\mathbf{z})$ can be linearized in the neighborhood of $\widehat{\mathbf{z}}_m$ as

$$\tilde{\mathbf{f}}_m(\mathbf{z}) \approx \tilde{\mathbf{f}}_m(\widehat{\mathbf{z}}_m) + \tilde{\mathbf{G}}_m(\widehat{\mathbf{z}}_m)(\mathbf{z} - \widehat{\mathbf{z}}_m) \tag{20}$$

and that $\tilde{\mathbf{G}}_m(\mathbf{z}) \approx \tilde{\mathbf{G}}_m(\widehat{\mathbf{z}}_m)$ (in the neighborhood of $\widehat{\mathbf{z}}_m$), we can approximate equation (16) as

$$\nabla_{\mathbf{z}} \phi_{ML}^{AOA}(\mathbf{z}) \approx -2 \sum_{m=1}^{M} \tilde{\mathbf{C}}_m^{-1} (\mathbf{z} - \widehat{\mathbf{z}}_m) \tag{21}$$

where

$$\tilde{\mathbf{C}}_m = \left[\tilde{\mathbf{G}}_m(\widehat{\mathbf{z}}_m)^T \tilde{\mathbf{R}}_m^{-1} \tilde{\mathbf{G}}_m(\widehat{\mathbf{z}}_m) \right]^{-1} \tag{22}$$

Finally from (21), it can be observed that $\nabla_{\mathbf{z}} \phi_{ML}^{AOA}(\widehat{\mathbf{z}}_{ML}) = 0$ has a closed-form solution as:

$$\widehat{\mathbf{z}}_{ML} \approx \sum_{m=1}^{M} \mathbf{W}_m \widehat{\mathbf{z}}_m \tag{23}$$

where

$$\mathbf{W}_m = \left[\sum_{m'=1}^{M} \tilde{\mathbf{C}}_{m'}^{-1} \right]^{-1} \tilde{\mathbf{C}}_m^{-1} \tag{24}$$

In fact, equation (23) states that the ML position estimator using AOA measurements can be approximated by a linear combination of the partial position estimates $\widehat{\mathbf{z}}_m$ $m = [1, M]$. These partial estimates are obtained as the intersection of the straight-lines defined by the AOAs grouped in sets of two measurements.

4. FUSION WITH TDOA MEASUREMENTS

It is well known that fusing AOA and TDOA measurements is a very good strategy to get more accurate position estimates. The natural solution for this problem is again the application of the Torrieri method with AOA and TDOA measurements at the same time using a common initial guess $\mathbf{z}_0$. Simulation results show a great improvement between the performance of the Torrieri method using only AOAs and using AOA and TDOA when it is optimally initialized. However, as it will be shown in the simulations, the problem is still the same: the performance of the algorithm is very sensitive to the goodness of the initial guess.

The idea now is to extend the algorithm presented in section 3 to the generic case with N AOA measurements and $N-1$ TDOA measurements in order to achieve the same performance as the Torrieri method optimally initialized but without the need of an initial guess. If we can collect N AOA and $N-1$ TDOA measurements, our signal model in (3) becomes:

$$\begin{bmatrix} \boldsymbol{\theta} \\ \mathbf{t} \end{bmatrix} = \begin{bmatrix} \mathbf{f}(\mathbf{z}) \\ \mathbf{f}'(\mathbf{z}) \end{bmatrix} + \begin{bmatrix} \mathbf{w} \\ \mathbf{w}' \end{bmatrix} \tag{25}$$

where $\mathbf{t} = [t_1, \ldots, t_{N-1}]$ is the vector containing the N-1 TDOA measurements, $\mathbf{f}'(\mathbf{z}) = \left[f_1'(\mathbf{z}), \ldots, f_{N-1}'(\mathbf{z}) \right]$ are the non-linear relationship between the TDOA measurements and the unknown position $\mathbf{z}$ and $\mathbf{w}' = \left[w_1', \ldots, w_{N-1}' \right]$ is the Gaussian noise vector added to the TDOA measurements. The non linear relations $f_i'(\mathbf{z})$ can be expressed now as:

$$f_i'(\mathbf{z}) = ||\mathbf{z} - \mathbf{z}_{i+1}|| - ||\mathbf{z} - \mathbf{z}_1|| \tag{26}$$

As in the AOA measurements case, we will also assume that the TDOA measurements are independently corrupted as:

$$\mathbf{R}' = E\left[\mathbf{w}' \mathbf{w}'^T \right] = \mathrm{diag}\left({\sigma_1'}^2, \cdots, {\sigma_{N-1}'}^2 \right). \tag{27}$$

As far as we are assuming that both kind of measurements are not correlated $E\left[\mathbf{w}' \mathbf{w}^T \right] = \mathbf{0}$, we can formulate the ML position estimate as:

$$\widehat{\mathbf{z}}_{ML} = \arg\min_{\mathbf{z}} \phi_{ML} \quad ; \quad \phi_{ML} = \phi_{ML}^{AOA}(\mathbf{z}) + \phi_{ML}^{TDOA}(\mathbf{z}) \tag{28}$$

where $\phi_{ML}^{AOA}(\mathbf{z})$ is defined in (4) and $\phi_{ML}^{TDOA}(\mathbf{z})$ can be expressed as

$$\phi_{ML}^{TDOA}(\mathbf{z}) = (\mathbf{t} - \mathbf{f}'(\mathbf{z}))^T \mathbf{R}'^{-1} (\mathbf{t} - \mathbf{f}'(\mathbf{z})) \tag{29}$$

From (15) to (21), it has been shown that

$$\nabla_{\mathbf{z}} \phi_{ML}^{AOA}(\mathbf{z}) = -2 \sum_{m=1}^{M} \tilde{\mathbf{C}}_m^{-1} (\mathbf{z} - \widehat{\mathbf{z}}_m) \tag{30}$$

and with the same procedure, we obtain the equivalent expression for the TDOA measurements as:

$$\nabla_{\mathbf{z}} \phi_{ML}^{TDOA}(\mathbf{z}) = -2 \sum_{m=1}^{M'} \left[\tilde{\mathbf{C}}_m' \right]^{-1} (\mathbf{z} - \widehat{\mathbf{z}}_m') \tag{31}$$

where M' is the number of subsets of TDOA measurements (as M was for AOA measurements), $\widehat{\mathbf{z}}_m'$ is obtained as the intersection of the two hyperbolas defined by a pair of TDOAs and $\tilde{\mathbf{C}}_m'(\widehat{\mathbf{z}}_m)$ are obtained in the same way as $\tilde{\mathbf{C}}_m(\widehat{\mathbf{z}}_m)$ in (22) using the specific non-linear function for TDOA measurement shown in (26). Note that, as in the AOA case in (19), $\widehat{\mathbf{z}}_m'$ is also obtained using a closed-form expression [5]. Finally, from (28) , (30) and (31) we have

$$\nabla_{\mathbf{z}} \phi_{ML} = -2 \sum_{m=1}^{M} \tilde{\mathbf{C}}_m^{-1}(\widehat{\mathbf{z}}_m)(\mathbf{z} - \widehat{\mathbf{z}}_m) - 2 \sum_{m=1}^{M'} \left[\tilde{\mathbf{C}}_m' \right]^{-1} (\mathbf{z} - \widehat{\mathbf{z}}_m') \tag{32}$$

where we can extract a closed-form position for the position estimate as:

$$\widehat{\mathbf{z}}_{ML} = \sum_{m=1}^{M} \mathbf{W}_m^{AOA} \widehat{\mathbf{z}}_m + \sum_{m=1}^{M'} \mathbf{W}_m^{TDOA} \widehat{\mathbf{z}}_m' \tag{33}$$

where

$$\begin{aligned} \mathbf{W}_m^{AOA} &= \left[\sum_{m'=1}^{M} \tilde{\mathbf{C}}_{m'}^{-1} + \sum_{m'=1}^{M'} \left[\tilde{\mathbf{C}}_{m'}' \right]^{-1} \right]^{-1} \tilde{\mathbf{C}}_m^{-1} \\ \mathbf{W}_m^{TDOA} &= \left[\sum_{m'=1}^{M} \tilde{\mathbf{C}}_{m'}^{-1} + \sum_{m'=1}^{M'} \left[\tilde{\mathbf{C}}_{m'}' \right]^{-1} \right]^{-1} \left[\tilde{\mathbf{C}}_m' \right]^{-1} \end{aligned}$$

The conclusion is that we can extend the proposed algorithm in a natural way to use AOA and TDOA measurements. The general idea is, first compute the intersections between pairs of straight lines defined by pairs of AOA measurements. Second, compute the intersection between the hyperbolas defined by pairs of TDOA measurements. Finally all the partial position estimates, obtained with simple intersections, are combined in (33) to obtain the final position estimate.

5. NUMERICAL SIMULATIONS

The scenario selected for numerical simulations consists in 4 BSs uniformly distributed on a circle of radius 750 m centered

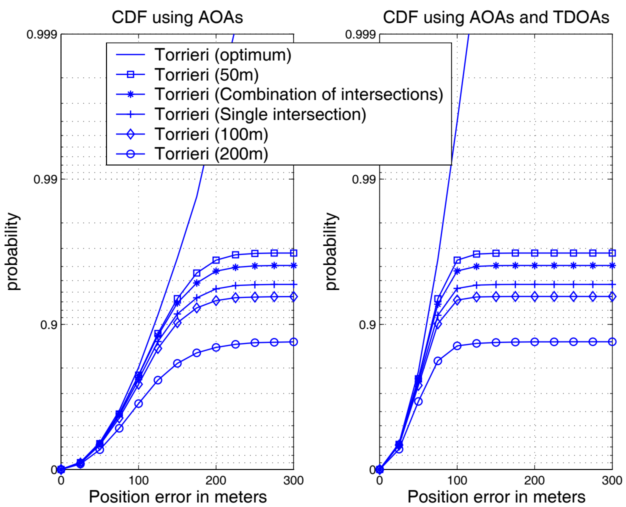

Fig. 1. Torrieri algorithm performance

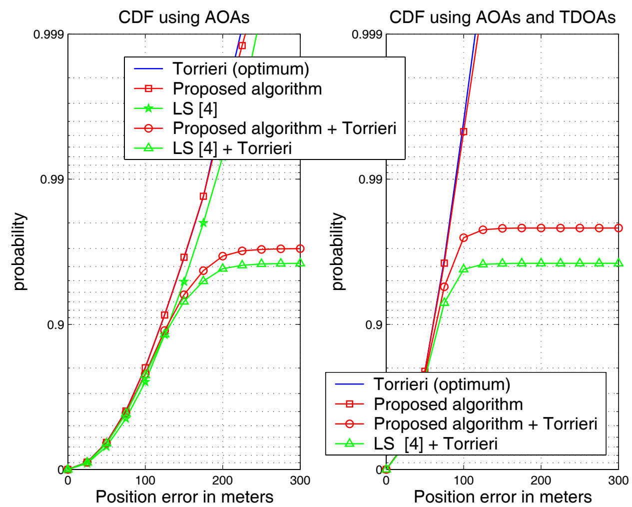

Fig. 2. Proposed algorithm performance

at the origin of coordinates. The mobile is randomly placed following a uniform distribution inside a circle of radius 1500 m. The AOA measurements are all corrupted with a Gaussian noise term of $\sigma_n = 3$ degrees for all the BS and the TDOA measurements are corrupted with Gaussian noise of $\sigma'_n = 60$ meters. All the simulations use 10000 trials. Figure (1) shows the Cumulative Distribution Function (CDF) of the position error of the Torrieri method using AOA measurements (left side) and AOA+TDOAs (right side). In this figure, we can see the performance of the Torrieri algorithm when it is initialized at the correct point (optimum). This non-realistic performance is used as a bound for the rest of methods. It can be seen in both sides of the figure that the performance is degraded as the error of the initial guess increases from 50 to 200 meters. Also in both sides, we can see the performance if we use as initial guess the intersection of two AOAs (Single intersection) or the mean of the two intersections between the four AOAs grouped in pairs (Combination of intersections). We can compare both sides to see the improvements produced by the TDOA measurements.

Figure (2) shows the CDF of the proposed algorithm using the same AOA measurements (left side) or the same AOA and TDOA measurements (right side) as in figure (1). It can be seen that the performance of the proposed technique is close to the performance of the Torrieri algorithm ideally initialized in both scenarios (AOA or AOA+TDOA). Note that as far as the Torrieri algorithm attains the Cramer-Rao Bound when it converges, our approach also attains the CRB. In order to compare with previous AOA-based approaches, the performance of the closed-form algorithm presented in [4] has also been plotted in the left side. Note that this previous approach did not present a way to fuse TDOAs. Finally, in both sides, we show the performance of the Torrieri approach initialized with the closed-form solutions [4] and the one proposed here. It can be observed that [4] is, in some cases, outperformed by the the Torrieri approach initialized with [4]. This is not the case of the proposed algorithm that outperforms the Torrieri approach initialized with any initial guess. In conclusion, it can be seen that the proposed algorithm outperforms the closed-form algorithm presented in [4] and the Torrieri algorithm initialized with any initial guess, even with the closed-form proposed in this paper.

6. CONCLUSION

In this paper we have proposed a novel closed-form algorithm to compute the mobile position using AOA measurements, combined with TDOA measurements as an option. This algorithm is based on the Divide-and-Conquer approach dividing the original set of AOA and TDOA measurements in subsets of two measurements. Once the partial position estimate is computed for each subset as simple intersections, they are optimally linearly combined. Numerical simulations show that the proposed algorithm, outperforms the previous closed-form presented in the literature and also outperforms the classical implementation of the Torrieri algorithm using as initialization any reasonable guess.

7. REFERENCES

[1] J.H Reed, K.J Krizman, B.D Woerner, and T.S. Rappaport, "An overview of the challenges and progress in meeting the E-911 requirement for location service," *IEEE Magaz. on Comm.*, pp. 30–37, April 1998.

[2] Jr. Caffery and G.L. Stuber, "Subscriber location in CDMA cellular networks," *IEEE Trans. on Vehicular Technology*, vol. 47,Issue:2, pp. 406–416, May 1998.

[3] D.J. Torrieri, "Statistical theory of passive location sytems," *IEEE Trans. on Aerospace and Electronic Systems*, March 1984.

[4] A. Pagès-Zamora, J. Vidal, and D.H. Brooks, "Closed-form solution for positioning based on angle of arrival measurements," *IEEE PIMRC*, vol. 4, pp. 1522–1526, September 2002.

[5] A. Urruela and J. Riba, "Novel closed-form ML position estimator for hyperbolic location," *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, vol. 2, pp. 149–52, May 2004.