TRACKING WIDE-BAND RAPIDLY MOVING TARGETS

Paul D. Teal

Industrial Research Limited
P.O. Box 31-310, Lower Hutt, New Zealand
Email: p.teal@irl.cri.nz

ABSTRACT

A method is derived for passively locating wide-band targets (typically acoustic targets) which may be moving at speeds sufficient to produce significant Doppler shift. The method involves a generalisation of standard beam forming techniques. It is shown that conventional beam forming techniques have less discrimination in the direction of motion of the sources, whereas the proposed technique exhibits no such degradation. The derivative and Hessian of the likelihood function can be used for locating the maximum likelihood solution or for deriving a Gaussian approximation to the likelihood function for particle filtering applications. The expressions are applicable for subsonic and supersonic sources.

1. INTRODUCTION

This paper addresses the problem of optimal localisation and tracking of objects which are emitting or reflecting wide-band signals, and moving with a speed which is a significant proportion of the speed (c) of the wave being emitted.

The most basic localisation/tracking scenario involves a single, narrow-band target moving slowly in a straight line in a homogeneous medium. Most real systems involve one or more complicating factors, such as reverberation, multiple targets, or wide-band signals. Wide-band systems, for example, face the issue of how to optimally (coherently or incoherently) combine the information contained across the available bandwidth [1].

This paper considers another complicating factor which arises when the targets are moving rapidly. Most localisation techniques are based on instantaneous snapshots of the environment of the sensor array, with the assumption that the travel distance within the duration of the snapshot is not significant. The problem can be avoided to some extent by making the snapshots very short, and this has been by far the most common approach. Zhou et. al. [2] make a point of explicitly modelling the time variation caused by target movement, but only in the narrow-band situation, and with the implicit assumption that Doppler shift has not caused the signal to move significantly out of this narrow band. The formulation in this paper is for signals with a high bandwidth-time-duration product, although it allows for pass bands of any frequency or width.

If the velocity of the targets is large, the Doppler shift can be significant. The Doppler shift is particularly difficult to deal with when the target is close to the sensor array. At this point, the Doppler shift is not even approximately constant, as it rapidly changes from a positive frequency shift to a negative frequency shift. A significant Doppler shift is more likely to be encountered with an acoustic wave than an electromagnetic wave, hence the formulation presented in this paper is for a non-relativistic situation. As is shown in Section 4, target speeds as low as $0.03c$ may be significant for accurate estimation of source location.

The approach presented in this paper requires that the location and velocity of the targets be modelled explicitly. In one sense this is a disadvantage, as the dimensionality of the problem (i.e., the number of unknowns) is doubled. On the other hand, most tracking schemes use Bayesian techniques of some kind (such as Kalman filters or particle filters) in which the velocity of the targets is already an intrinsic part of the model [3], even though the velocity information contained in the model is not used as part of the update phase. So in this sense, the complexity of the state model is unchanged by explicit inclusion of the target velocity in the *a posteriori* probability calculations.

The paper is organised as follows. In the next section we present the signal model which explicitly includes the effects of Doppler shift. In Section 3, the likelihood function is derived for unknown spatial parameters associated with a received signal based on the model. In Section 4 some simulations are presented which demonstrate the effectiveness of the model in reducing uncertainty in the target location caused by the target motion. Section 5 concludes the paper.

2. RECEIVED SIGNAL MODEL

Consider a source, initially at location $\boldsymbol{d}_s$, travelling with velocity $\boldsymbol{v}_s$, and emitting a signal at time t_0. The signal is detected at time $t = t_0 + \tau$ by a sensor which is initially at $\boldsymbol{d}_r$, and travelling with velocity $\boldsymbol{v}_r$. For both the source and sensor, 'initially' means time $t = 0$, which in general is not equal to t_0. The distance travelled by the wave before detection is

$c\tau$, where c is the velocity of the wave. Thus

$$\|(\boldsymbol{d}_s + t_0\boldsymbol{v}_s) - (\boldsymbol{d}_r + t\boldsymbol{v}_r)\| = c\tau \quad (1)$$

This can be rearranged as

$$\|\boldsymbol{d} + \boldsymbol{v}_s(t-\tau) - \boldsymbol{v}_r t\| = c\tau \quad (2)$$

where $\boldsymbol{d} = \boldsymbol{d}_s - \boldsymbol{d}_r$. Defining $d \triangleq \|\boldsymbol{d}\|$, $v_s \triangleq \|\boldsymbol{v}_s\|$, $\boldsymbol{v} \triangleq \boldsymbol{v}_s - \boldsymbol{v}_r$, $v \triangleq \|\boldsymbol{v}\|$, (2) can be arranged as

$$(c^2 - v_s^2)\tau^2 + 2((\boldsymbol{d}+\boldsymbol{v}t)\cdot\boldsymbol{v}_s)\tau - \|\boldsymbol{d}+\boldsymbol{v}t\|^2 = 0 \quad (3)$$

Solving for τ, we find that

$$\tau(t) = \frac{-(\boldsymbol{d}+\boldsymbol{v}t)\cdot\boldsymbol{v}_s \pm \sqrt{\begin{array}{c}((\boldsymbol{d}+\boldsymbol{v}t)\cdot\boldsymbol{v}_s)^2+ \\ \|\boldsymbol{d}+\boldsymbol{v}t\|^2(c^2-v_s^2)\end{array}}}{(c^2-v_s^2)} \quad (4)$$

$$= \frac{\pm\sqrt{a_0^2t^2 + 2b_0t + c_0^2} + d_0t + e_0}{(c^2-v_s^2)} \quad (5)$$

where

$$\begin{aligned} a_0^2 &= (\boldsymbol{v}\cdot\boldsymbol{v}_s)^2 + v^2(c^2-v_s^2) \\ b_0 &= (\boldsymbol{d}\cdot\boldsymbol{v}_s)(\boldsymbol{v}\cdot\boldsymbol{v}_s) + (\boldsymbol{d}\cdot\boldsymbol{v})(c^2-v_s^2) \\ c_0^2 &= (\boldsymbol{d}\cdot\boldsymbol{v}_s)^2 + d^2(c^2-v_s^2) \\ d_0 &= -\boldsymbol{v}\cdot\boldsymbol{v}_s \\ e_0 &= -\boldsymbol{d}\cdot\boldsymbol{v}_s \end{aligned}$$

For subsonic source velocities, only the addition of the surd produces a positive delay. For supersonic source velocities ($v_s > c$), there is the possibility of both addition and subtraction yielding a positive time, and also the possibility that the wave never reaches the sensor at all.

Assuming that the target is an isotropic radiator, we can represent the signal emitted by the source as $s(t)$. Assuming that the signal power decreases with the square of the distance, and neglecting turbulence effects, the signal detected at the sensor is given by $r(t) = s(t-\tau(t))/c\tau(t)$. The emitted signal $s(t)$, in the time window $t = 0$ to time $t = T$, can be represented by its Fourier series decomposition as $s(t) = \sum_{\ell=-\infty}^{\infty} S_\ell e^{j\omega_0\ell t}$, where $\omega_0 = 2\pi/T$. This implies that the signal is periodic with period T, and in practice some window function will normally be applied to the data to minimise the error caused by this assumption. It is important at this point to note that the parameter $\boldsymbol{d}_s$ is defined as the target location at $t = 0$, i.e., the time at which the signal in this window begins to be received at the sensor array, which will not be the same location as when the signal was first emitted.

In most cases the received signal will be sampled at rate f_s samples per unit time, with sample $r_k \triangleq r(k/f_s)$. These samples can be expressed in terms of the emitted signal as

$$r_k = \sum_{\ell=-\infty}^{\infty} S_\ell \frac{1}{c\tau(k/f_s)} e^{j\omega_0\ell(k/f_s - \tau(k/f_s))}. \quad (6)$$

The discrete Fourier transform $R_q, q \in [-K/2, K/2-1]$ ($K = Tf_s$, and for notational simplicity we assume that K is even) of the received signal r_k is given by

$$\begin{aligned} R_q &= \sum_{k=0}^{K-1} r_k e^{-j\frac{2\pi kq}{K}} \\ &= \sum_{\ell=-\infty}^{\infty} S_\ell \sum_{k=0}^{K-1} \frac{1}{c\tau(k/f_s)} e^{j\omega_0(k/f_s(\ell-q)-\ell\tau(k/f_s))}. \end{aligned} \quad (7)$$

If we assume that the received signal is band-limited, then only values of ℓ in the range $[-L/2, L/2-1]$ need be considered. Defining vectors of signals $\mathbf{r} \triangleq (r_0, \ldots, r_{(K-1)})^T$, $\mathfrak{S} = (S_{-L/2}, \ldots, S_{L/2-1})^T$, and $\mathfrak{R} = (R_{-K/2}, \ldots, R_{K/2-1})^T$, then (6) can be written in matrix form as $\mathbf{r} = \mathfrak{B}\mathfrak{S}$ and (7) as $\mathfrak{R} = \mathfrak{A}\mathfrak{S}$, with the columns of $\mathfrak{A}$ being the discrete Fourier transform of the columns of $\mathfrak{B}$.

We use M to denote the number of sensors, and (extending the model to include multiple targets) N to denote the number of targets. The emitted and received signal vectors may be stacked to obtain

$$\begin{bmatrix} \mathfrak{R}_1 \\ \vdots \\ \mathfrak{R}_M \end{bmatrix} = \begin{bmatrix} \mathfrak{A}_{11} & \ldots & \mathfrak{A}_{1N} \\ \vdots & & \vdots \\ \mathfrak{A}_{M1} & \ldots & \mathfrak{A}_{MN} \end{bmatrix} \begin{bmatrix} \mathfrak{S}_1 \\ \vdots \\ \mathfrak{S}_N \end{bmatrix} \quad (8)$$

which can be expressed as $\mathbf{r} = \mathbf{A}\mathbf{s} + \boldsymbol{\eta}$. We have here introduced a normal noise term $\boldsymbol{\eta}$, with $E\{\boldsymbol{\eta}\boldsymbol{\eta}^H\} = \sigma^2\mathbf{I}$, where $E\{\}$ is the expectation operator and $\mathbf{I}$ is the identity matrix of size MK. We are thus assuming the noise is both temporally and spatially white and of equal power at each frequency and location. (In the presence of ambient localised noise sources this assumption will of course be inaccurate).

Rather than the structure shown in (8), the elements of $\mathbf{r}$ can alternatively be arranged as K lots of M, and the elements of $\mathbf{s}$ can be arranged as L lots of N, with consequent rearrangement of the structure of $\mathbf{A}$. This may have computational advantages. If small entries can be approximated as zero, and $\mathbf{A}$ is sparse, then the rearrangement of $\mathbf{r}$ (but not of $\mathbf{s}$) results in the non-zero elements of $\mathfrak{A}$ being concentrated near the diagonal.

3. LIKELIHOOD FUNCTION

Given the assumed noise model, the likelihood of observing a particular signal $\mathbf{r}$ is thus

$$p(\mathbf{r}|\mathbf{A}(\boldsymbol{\theta}), \mathbf{s}, \sigma^2) = \frac{e^{-(\mathbf{r}-\mathbf{A}\mathbf{s})^H(\mathbf{r}-\mathbf{A}\mathbf{s})/(2\sigma^2)}}{(2\pi\sigma^2)^{MK}}. \quad (9)$$

The maximum likelihood solution to this equation with respect to the spatial parameters (location and velocity) $\boldsymbol{\theta} = (\boldsymbol{d}_{s1}, \boldsymbol{v}_{s1}, \ldots, \boldsymbol{d}_{sN}, \boldsymbol{v}_{sN})$ is [2]

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} J \quad (10)$$

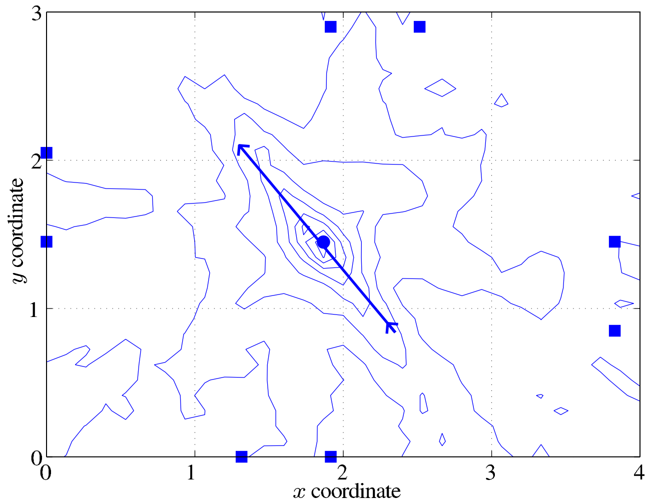

Fig. 1. Plot of conventional delay and sum beam former likelihood function for a source moving at $0.03c$. The line shows the trajectory of the source, from the location at which the first sound is emitted which arrives at the sensors at $t = 0$, to the location of the source at $t = T$. The arrowhead part way along this line is the location of the source at $t = 0$, and is the location we wish to determine. The squares represent the sensor locations, and the circle represents the location of the largest value of the likelihood function.

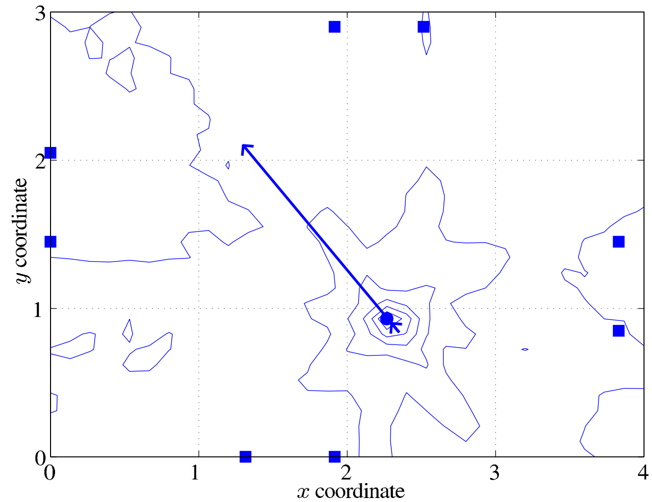

Fig. 2. Plot of likelihood function based on the proposed signal model for a source moving at $0.03c$. This is a section through a a peak of a likelihood function of double the dimension of Fig. 1.

where $J = \mathbf{r}^H(\mathbf{I} - \mathbf{A}\mathbf{A}^+)\mathbf{r} = \mathbf{r}^H(\mathbf{I} - \mathbf{P})\mathbf{r}$, $\mathbf{A}^+$ is the generalised inverse of $\mathbf{A}$, and $\mathbf{P} = \mathbf{A}\mathbf{A}^+$.

For some applications, finding the maximum likelihood solution is all that is required. For particle filtering, we may wish to assume some prior distribution for $\mathbf{s}$, and possibly σ^2, so as to find an *a posteriori* probability density for $\mathbf{r}$. One approach, analogous to that taken by [3], assumes a Jeffrey's prior for σ^2, and (to avoid problems with rank deficiency in $\mathbf{A}^H\mathbf{A}$), a prior on the signal which is proportional to $|\mathbf{A}^H\mathbf{A}|$, resulting in the density

$$p(\mathbf{r}|\mathbf{A}(\boldsymbol{\theta})) \propto (\mathbf{r}^H(\mathbf{I} - \mathbf{P})\mathbf{r})^{LN-KM}. \tag{11}$$

An alternative approach, which achieves good results, is described in [4]. The situation described there is for only one target, but can be extended to $N > 1$. What they call a "pseudo-likelihood" is used which amounts to

$$p(\mathbf{r}|\mathbf{A}(\boldsymbol{\theta})) \propto (\mathbf{r}^H\mathbf{P}\mathbf{r})^W. \tag{12}$$

where $W \in \mathbb{R}^+$ is chosen so as to achieve a suitable compromise between localisation accuracy and particle impoverishment.

To apply a Newton descent type algorithm for finding the maximum likelihood solution, we wish to find the first and second derivative of (10) with respect to the unknown spatial parameters. We also need the derivative of the likelihood function if we wish to use a Gaussian approximation of the optimal importance function of a particle filter. The calculations of these derivatives is tedious, and they are omitted here for lack of space.

4. SIMULATIONS

This Section briefly presents a few simulations which demonstrate the problem with the use of conventional beam forming approaches to source localisation, when the speed of the source is reasonably high.

The first simulation is similar to that of [4], except that in this case, the effects of reverberation are neglected, and the source speed is greater. There are eight microphones, each sampling at 8000 Hz, located at $(1.315, 0)$, $(1.916, 0)$, $(3.83, 0.85)$, $(3.83, 1.45)$, $(2.515, 2.9)$, $(1.915, 2.9)$,$(0, 2.05)$, and $(0, 1.45)$. (These dimensions are in metres.) A single target moves from $(2.3, 0.9)$ to $(1.3, 2.1)$ at a speed of 10 ms^{-1} ($0.03c$), at the same height as the sensors, emitting a signal which is approximately white up to 400 Hz. This takes 160 ms, or 1250 samples, which are windowed using a Prolate spheroidal window of concentration 2. The receiver signal to noise ratio is 0 dB. Fig. 1 shows contours of the likelihood as a function of the hypothesised source location for a conventional beam former. It can be seen that the region of large likelihood is elongated in the direction of travel of the source, and consequently there is some ambiguity in the source location. We would expect the Cramer Rao bound on the variance of the location parameters in this direction to be larger than in the direction perpendicular to the direction of travel. The maximum of the likelihood function lies at a point roughly halfway along the source trajectory. By contrast, Fig. 2 shows

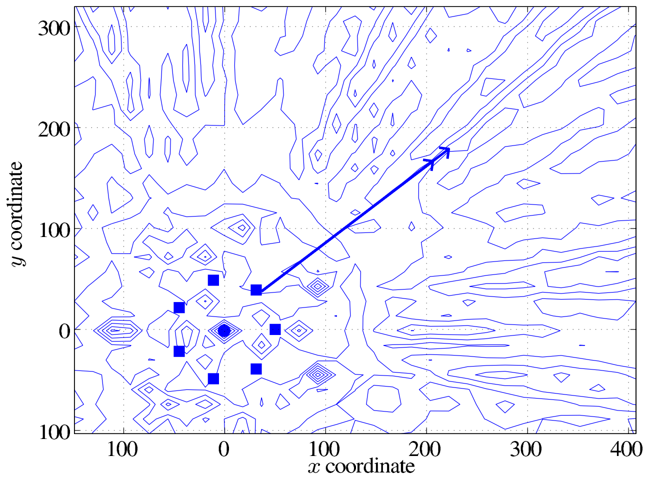

Fig. 3. Plot of delay and sum beam former likelihood function for a source moving at $1.5c$.

contours of the likelihood using the proposed signal model. In this case the likelihood is a function of the source velocity as well as the source location, so the function shown is a two dimensional slice through a peak in the four dimensional likelihood function. The peak was found using a Newton type algorithm. It is clear that the likelihood function shows an unambiguous peak at the true source location, with no loss of discrimination in the direction of travel of the source.

The second example is a single supersonic source, moving at $1.5c$. The seven sensors are arranged in a horizontal circular array of radius 50 m. The source passes horizontally overhead at a height of 100 m, passing through the point $(0, 10, 100)$, at an angle of 0.65 radians to the x axis. The signal is the same as for the first example, with duration 40 ms, or 320 samples. In this case the likelihood of the delay and sum beam former shows several large peaks located near the array elements (Fig. 3—the result is not significantly different if d^2 loss is taken into consideration). The likelihood function for the proposed model (Fig. 4) is much less ambiguous with a large peak near the true location of the source. The simulation used for generating the sound received at the sensors includes both forwards and backwards travelling waves (both addition and subtraction in (5)).

5. CONCLUSIONS

We have derived expressions for the likelihood of a set of target position and velocity parameters. The expressions explicitly include Doppler shift of wide band signals. It has been shown by simulation that the proposed model resolves any ambiguity in the source locations caused by source motion.

Explicit modelling the velocity of the sources in this way effectively doubles the dimensionality of the problem. For tracking applications, where the velocity of the targets is already part of the model, this is an advantage rather than a problem.

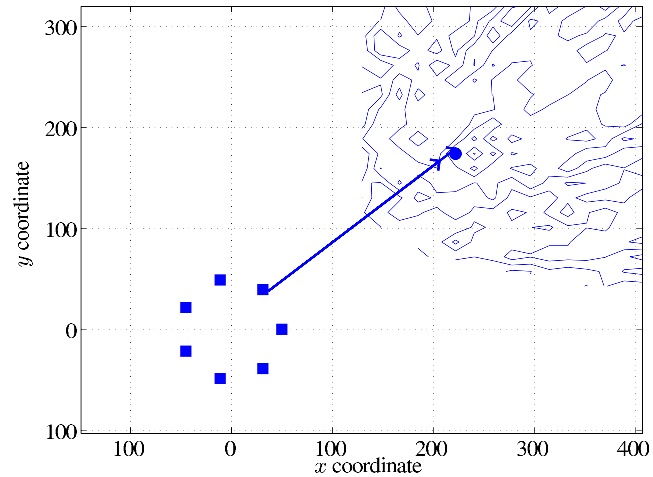

Fig. 4. Plot of Likelihood function based on the proposed model for a source moving at $1.5c$. The source is moving at supersonic speed, so for the conjectured velocity represented by this plane, only certain conjectured locations are feasible, hence the likelihood function is not defined for the entire plane.

The proposed algorithm has a large processing requirement. It may be possible to find a more efficient algorithm which also explicitly models the Doppler shift, but in a simplified manner.

6. REFERENCES

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