AN EXPLICIT HIGH-RESOLUTION DOA ESTIMATION FORMULA FOR TWO WAVE SOURCES

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ABSTRACT

This paper presents a novel and simple high-resolution DOA (Direction-Of-Arrival) estimation algorithm which corresponds to the case of two incident waves received by ULA (Uniform Linear Array). The proposed DOA estimation formula does not employ eigendecomposition, iteration, optimization or space averaging, therefore it requires very low computational cost but achieves high resolution. Surprisingly, the present simple DOA estimator has almost the same accuracy with (and sometimes better than) major conventional DOA estimation algorithms. The estimation accuracy and the computational cost are evaluated through computer simulation in comparison with those by some major conventional algorithms.

1. INTRODUCTION

Mobile communication technology has remarkably developed in the last decade, it is important to accurately detect radio wave propagation environment around base station and mobile terminals. DOA (Direction-Of-Arrival) estimation is one of mandatory techniques for such detection of coherent or non-coherent waves. For such purpose, the high resolution DOA estimation algorithm using array antenna is attracted attention to achieve those demands [1].

Many DOA estimation algorithms have already been studied like MUSIC (eigendecomposition & angular search) [2], Root-MUSIC (eigendecomposition) [3], TLS-ESPRIT (some eigendecompositions) [4], MODE (maximum likelihood) [5] and Structured Matrix (no eigendecomposition but still large computation) [6]. These algorithms generally achieve high accuracy but also needs large computational cost.

In this paper, we concentrate on the case of two incident waves received by ULA (Uniform Linear Array) antenna, which is often used in DOA estimation system in practical situation [7],[8]. Indeed many of commercial smart antenna systems employ small number of array elements, therefore they aim at estimating DOAs of small number of incident waves. Instead, high accuracy and very high-speed computation is desired for such practical situation.

We develop a simple but accurate DOA estimation formula by regarding the DOA estimation problem as the problem of solving simultaneous equation. The proposed formula has many properties as follows: given by an explicit non-iterative formula, very low computational cost, achieve high accuracy, applicable to coherent and correlated waves without space averaging. The estimation accuracy and the computational cost are evaluated through computer simulation in comparison with those by conventional methods.

The rest of the paper is as follows. Section II prepares mathematical modeling and its formulation. Section III gives a new DOA estimation formula and its derivation. After showing some simulation results in Section IV, we will make some concluding remarks in Section V.

2. PRELIMINARIES

In this section, we formulate signals received at array antenna and their correlation matrix.

Assume that L incident waves $s_i(i = 1, 2, \cdots, L)$ arrive at K-element ULA of half-wave length interval each. The complex received signal x_k at the k-th antenna is written as

$$
\begin{aligned}
x_k(n) &= \sum_{i=1}^{L} s_i(n) e^{-j2\pi \frac{(k-1)d}{\lambda} \sin\theta_i} + u_k(n) \\
&= \sum_{i=1}^{L} s_i(n) e^{-j\pi(k-1)\sin\theta_i} + u_k(n),
\end{aligned}
$$

for $k = 1, 2, \ldots, K$, where $d = \lambda/2$ denotes the element interval. The angle θ_i is the DOA of i-th incident wave which changes within $(-90°, 90°)$. Besides, u_k denotes the receiver noise of k-th element where its average is zero.

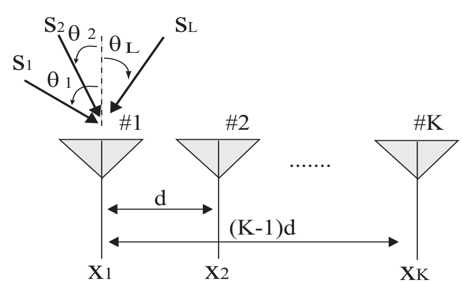


Fig. 1. System configuration

The array input vector $\boldsymbol{x}(n)$ and its correlation matrix $\boldsymbol{R}_{xx}$ (approximated by sampled correlation matrix) are respectively given as follows.

$$
\boldsymbol{x}(n) = [x_1(n), x_2(n), \ldots, x_K(n)]^T,
$$

$$
\boldsymbol{R}_{xx} = E\left[\boldsymbol{x}(n)\boldsymbol{x}^H(n)\right] \simeq \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{x}(n)\boldsymbol{x}^H(n),
$$

where N and $E[\cdot]$ denote the number of snapshots and ensemble average, respectively. Here the following proposition holds.

Proposition 1
The k-th row ℓ-th column component $R_{k\ell}$ of the correlation matrix $\boldsymbol{R}_{xx}$ can be represented as a function of DOAs as follows.

$$\begin{aligned} R_{k\ell} &= E[x_k(n)x_\ell^*(n)] \\ &\simeq \sum_{i=1}^{L}\sum_{j=1}^{L}\left[S_{ij}\Theta_i^{-(k-1)}\Theta_j^{\ell-1}\right] + \sigma^2\delta_{k\ell}, \end{aligned}$$

where S_{ij} and Θ_i are given by

$$\begin{aligned} S_{ij} &= E\left[s_i(n)s_j^*(n)\right] \simeq \frac{1}{N}\sum_{n=1}^{N} s_i(n)s_j^*(n), \\ \Theta_i &= e^{j\pi\sin\theta_i}. \end{aligned}$$

Besides, σ^2 and $\delta_{k\ell}$ denote noise power and Kronecker's delta, respectively.
(Space does not permit to prove Proposition 1.) ■

3. PROPOSED APPROACH

In this section, we present the proposed DOA estimation formula and its derivation.

3.1. Case of Three Antennas

First we study the case that 2 waves are received by 3-element array antenna, where the minimum number of antenna elements to estimate DOAs of 2 incident waves. In this case, the diagonal and upper-triangular elements of correlation matrix $\boldsymbol{R}_{xx}$ can be written as follows.

$$\begin{aligned} R_{11} &= S_{11} + S_{12} + S_{21} + S_{22} + \sigma^2, && (1) \\ R_{22} &= S_{11} + S_{12}\Theta_1^{-1}\Theta_2 + S_{21}\Theta_1\Theta_2^{-1} + S_{22} + \sigma^2, && (2) \\ R_{33} &= S_{11} + S_{12}\Theta_1^{-2}\Theta_2^2 + S_{21}\Theta_1^2\Theta_2^{-2} + S_{22} + \sigma^2, && (3) \\ R_{12} &= S_{11}\Theta_1 + S_{12}\Theta_2 + S_{21}\Theta_1 + S_{22}\Theta_2, && (4) \\ R_{13} &= S_{11}\Theta_1^2 + S_{12}\Theta_2^2 + S_{21}\Theta_1^2 + S_{22}\Theta_2^2, && (5) \\ R_{23} &= S_{11}\Theta_1 + S_{12}\Theta_1^{-1}\Theta_2^2 + S_{21}\Theta_1^2\Theta_2^{-1} + S_{22}\Theta_2, && (6) \end{aligned}$$

where the diagonal components R_{11}, R_{22} and R_{33} take real values, and the upper-triangular components R_{12}, R_{13} and R_{23} take complex values. Using four of these components, we have an explicit DOA estimation formula as indicated in Proposition 2.

Proposition 2
In the case that 2 waves are received by 3-element array antenna, DOAs θ_1 and θ_2 are estimated by:

$$\theta_i = \sin^{-1}\left\{\frac{1}{\pi}\tan^{-1}\left(\frac{\mathrm{Im}[\Theta_i]}{\mathrm{Re}[\Theta_i]}\right)\right\}, \quad (7)$$

for $i = 1, 2$, where $\tan^{-1}(\cdot)$ denotes four-quadrant arctangent. Moreover, Θ_i in (7) is given by

$$\Theta_i = \frac{2R_a}{R_b + (-1)^i\sqrt{R_b^2 - 4|R_a|^2}}, \quad (8)$$

where

$$\begin{aligned} R_a &= R_{23} - R_{12}, \\ R_b &= R_{33} - R_{11}. \end{aligned}$$

(Proof)
From (4) and (6), R_a can be written as follows:

$$\begin{aligned} R_a &= S_{12}(\Theta_1^{-1}\Theta_2^2 - \Theta_2) + S_{21}(\Theta_1^2\Theta_2^{-1} - \Theta_1) \\ &= \Theta_2\tilde{S}_{12} + \Theta_1\tilde{S}_{21}, \end{aligned}$$

where $\tilde{S}_{ij}$ is given by:

$$\tilde{S}_{ij} = S_{ij}\left(\Theta_i^{-1}\Theta_j - 1\right).$$

On the other hand, R_b is written as follows by (1) and (3):

$$\begin{aligned} R_b &= S_{12}\left(\Theta_1^{-2}\Theta_2^2 - 1\right) + S_{21}\left(\Theta_1^2\Theta_2^{-2} - 1\right) \\ &= \left(\Theta_2\tilde{S}_{12} + \Theta_1\tilde{S}_{21}\right)\left(\Theta_2^{-1} + \Theta_1^{-1}\right) \\ &= R_a\left(\Theta_2^{-1} + \Theta_1^{-1}\right). && (9) \end{aligned}$$

Also the complex conjugate R_a^* of R_a satisfies

$$R_a^* = \Theta_1^{-1}\Theta_2^{-1}R_a. \quad (10)$$

From (9) and (10), we formulate a 2nd-order polynomial equation of Θ_1 and Θ_2:

$$R_a\Theta_i^{-2} - R_b\Theta_i^{-1} + R_a^* = 0, \quad i = 1, 2,$$

where the solution is given by (8). Therefore we have the DOA estimation formula (7). ■

3.2. Case of Four or More Antennas

When with four or more array elements, similar approach can be applied but a little bit complicated.

In the case of four array elements, we could have a DOA estimation formula by the way of Proposition 3 and some more additional procedure.

Proposition 3
In the case that 2 incident waves are received by 4-element array antenna, DOAs θ_1 and θ_2 are estimated by:

$$\theta_i = \sin^{-1}\left\{\frac{1}{2\pi}\tan^{-1}\left(\frac{\mathrm{Im}[\Theta_i]}{\mathrm{Re}[\Theta_i]}\right)\right\}, \quad i = 1, 2, \quad (11)$$

where Θ_i is given by

$$\Theta_i = \frac{2R_c}{R_d + (-1)^i\sqrt{R_d^2 - 4|R_c|^2}},$$

where

$$\begin{aligned} R_c &= R_{24} - R_{13}, \\ R_d &= R_{44} - R_{33} + R_{22} - R_{11}. \end{aligned}$$

(Proof)
Similar to the proof of Proposition 2 but partially more complicated. (Space does not permit to prove Proposition 3.) ■

Note that the difference between (7) and (11) is that $1/\pi$ in (7) is replaced by $1/2\pi$ in (11). This expands the range of $\sin^{-1}(\cdot)$ operation and often derive redundant (more than two) DOAs. Therefore we put some additional procedures as follows in order to derive only two DOAs.

1. Estimate DOAs $\tilde{\theta}_i$ by (11). Here we have four (or sometimes more) DOAs even though the correct numbers of DOAs are only two.

2. For a reference, simply estimate DOAs $\hat{\theta}_j$ by (7) using only 3 array elements.

3. Choose two DOAs from $\tilde{\theta}_i$ so that the error between $\tilde{\theta}_i$ and $\hat{\theta}_i$ is minimized.

In a similar manner, we can estimate DOAs for the cases with 5 or more antenna elements.

4. SIMULATION

In this section, the estimation accuracy and the computational cost of the proposed method are evaluated through computer simulation in comparison with those by Root-MUSIC, TLS-ESPRIT and MODE methods. These algorithms are selected because they don't employ angular search that is the most time-consuming process.

Specifications of simulation are shown in Tables 1 and 2. The estimation accuracy is evaluated by RMSE (Root Mean Square Error), which is calculated as the average of 1,000 Monte-Carlo simulation results because the influence of noise is not uniform in each single estimation. The behavior of RMSEs are shown in Figs. 2 and 3. We have the following observations from those figures.

Table 1. Common specifications

array configuration	4 elements ULA
array element interval	half wavelength
incident waves	2 coherent waves
carrier frequency	2.0 GHz
modulation	QPSK
number of snapshots	200

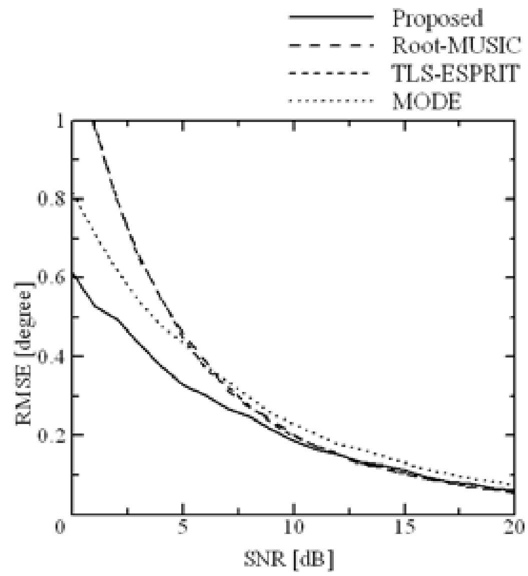
Table 2. Detailed specifications

	Fig.2(a)	Fig.2(b)	Fig.3(a)	Fig.3(b)
DOA θ_1	0°	0°	0°	0°
DOA θ_2	30°	10°	change	change
SNR	change	change	5[dB]	10[dB]

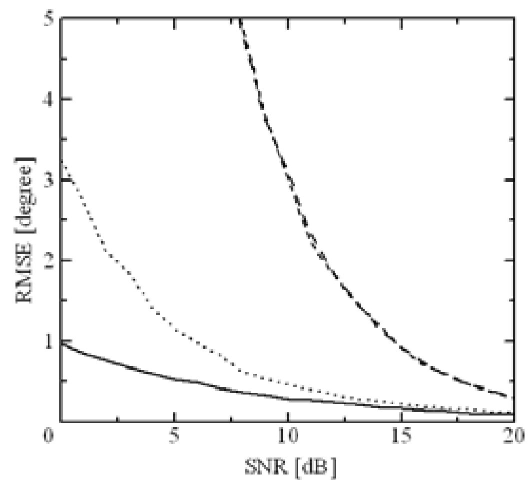
- From Figs. 2 and 3, we see that the accuracy of Root-MUSIC and TLS-ESPRIT is usually the same, and often worse than that of MODE and proposed method. That may be the effect of space averaging used only in Root-MUSIC and TLS-ESPRIT to be able to estimate DOAs of coherent waves. Note that MODE and the proposed methods do not require space averaging for coherent (or correlated) wave estimation.

- For some values of DOAs, the proposed method can achieve higher accuracy than MODE method. For example, Figs.3(a) and 3(b) says that the proposed method achieves the best accuracy when θ_2 is around $\pm 30°$. Also Fig.2(a) indicates the superiority of the proposed method.

- For the DOAs close to $\pm 90°$, the proposed method cannot well estimate DOAs as seen from Fig.3(a) and 3(b). The reason might be that we didn't use the relations (2) and (5). That should be discussed as a future study.

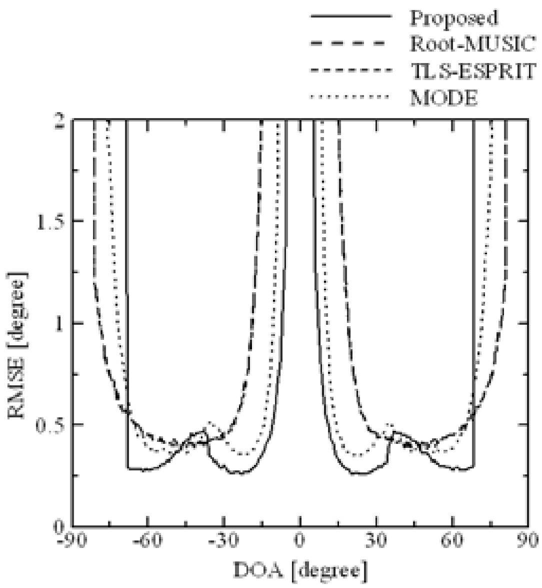


(a) Case of $\theta_2 = 30°$

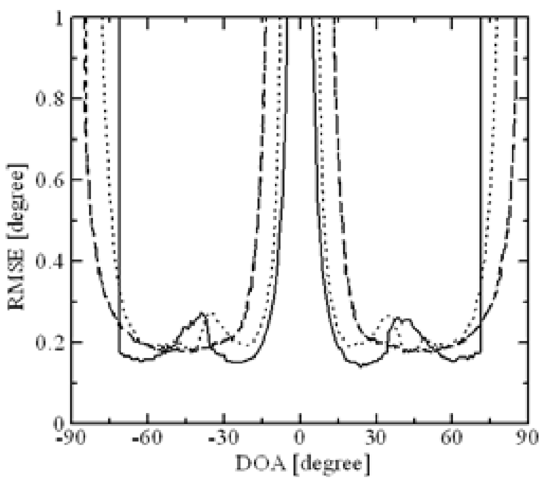


(b) Case of $\theta_2 = 10°$

Fig. 2. Behavior of RMSE in the case of $\theta_1 = 0°$ for various SNRs



(a) Case of SNR is 5[dB]



(b) Case of SNR is 10[dB]

Fig. 3. Behavior of RMSE in the case of $\theta_1 = 0°$ and various angles of θ_2

4.1. Computational cost

The computational cost of the proposed method is evaluated through computer simulation in comparison with Root-MUSIC, TLS-ESPRIT and MODE methods. Under the same condition, the computation time required to estimate DOAs of 10,000 times is indicated in Table 3. Note that the unit of computation time is [sec], and $(\cdot)$ represents the ratio when the time for the proposed method is set to one. All the simulations have been done by MATLAB, the specifications of computer are: OS Windows XP SP2, CPU Pentium IV 3.0GHz, and Memory 1GB.

From Table 3, we found that the computational cost of the proposed method is smaller than that by conventional methods. Note that the proposed method will become more computationally efficient if we can develop a fast procedure for computing $\sin^{-1}(\cdot)$ and $\tan^{-1}(\cdot)$.

Table 3. Comparison of computational cost

	Root-MUSIC	TLS-ESPRIT	MODE	Proposed
$\star$1	5.23 (4.5)	3.84 (3.3)	10.46 (9.0)	1.16 (1.0)
$\star$2	5.72 (3.9)	4.33 (3.0)	11.44 (7.8)	1.45 (1.0)

$\star$1: cost for DOA estimation only
$\star$2: cost for computing $\boldsymbol{R}_{xx}$ and DOA estimation

5. CONCLUDING REMARKS

In this paper, we proposed a simple but high resolution DOA estimation algorithm based on the fact that the correlation between the received signals at different antenna elements can be written as a function of DOAs. We gave an explicit DOA estimation formula like (7) or (11), while the conventional DOA estimation algorithms never give such explicit formula because they employ either eigendecomposition, iteration or optimization. As an expansion of this work, the case of three (and more) incident waves will be presented in the near future.

6. REFERENCES

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