EXPLOITING SIGNAL SUBSPACE INVARIANCE TO RESOLVE NON-STATIONARY, NON-ORTHOGONAL SENSOR ARRAY SIGNALS IN CORRELATED NOISE FIELDS

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ABSTRACT

We investigate a new approach for the problem of source separation/classification of non-orthogonal, non-stationary noisy signals impinging on an array of sensors. We propose a solution to the problem when the contaminating noise is temporally and spatially correlated. The observations are projected onto a nested set of multiresolution spaces prior to classical eigendecomposition. An inherent invariance property of the signal subspace is observed in a subset of the multiresolution spaces that depends on the level of approximation expressed by the orthogonal basis. This feature, among others revealed by the algorithm, is eventually used to separate the correlated signal sources in the context of 'best basis' selection. The technique shows robustness to source non-stationarity as well as anisotropic properties of the channel characteristics under no constraints on the array design. We illustrate the high performance of the technique on simulated and experimental multichannel neurophysiological data measurements.

1. INTRODUCTION

Multichannel signal processing aims at fusing data collected at several sensors in order to carry out an estimation task of signal sources [1]. Many array signal processing algorithms rely on eigenstructure subspace methods performed in the time domain [2], in the frequency domain [3], or in the composite time-frequency domain [4]. Regardless of which domain is used, eigenstructure based algorithms offer an optimal solution to many array processing applications provided that the assumptions about the underlying signal and noise processes are appropriate (e.g.: independent source signals, uncorrelated signals and noise, spatially and temporally white noise processes, etc…).

For some applications, many of these assumptions cannot be intrinsically made, such that when the sources have correlated waveform shapes and the noise is correlated among sensors, or when the propagating medium is anisotropic. Many approaches have been suggested in the literature to mitigate the effects of unknown spatially correlated noise fields to enable better source separation of the array mixtures and showed various degrees of success [5]-[7]. Nevertheless, the particular case where signal sources are non-orthogonal and may inherently possess considerable correlation with the noise has not received considerable attention. This situation may occur, for example, when the noise is the result of the presence of an exceptionally large number of *weak* sources that generate signal waveforms identical to those of the desired ones.

The objective of this paper is to develop a new technique for separating and potentially classifying a number of correlated sources impinging on an array of sensors in the presence of strong correlated noise. In that respect, we make the following assumptions about the problem at hand:

1- The observations are a mixture of *wide-band* signals and the array mixing matrix is unknown but time invariant.
2- Sources are non-orthogonal, with signals that are *transient-like*, and may be fully or partially coherent across the array.
3- The number of sources within the observation window is unknown and is less than the number of channels.
4- The noise is a mixture of two components:
  (a) Zero mean *iid* Gaussian white noise.
  (b) Temporally and spatially correlated noise component with unknown covariance resulting from numerous interfering weak sources.

2. THEORY

A plausible model of P correlated signals over the N samples time interval assumes that the p^{th} source signal can be expressed in terms of an independent signal $\boldsymbol{s}_1 = [s_1[0]\dots s_1[N-1]]$ as

$$s_p[n] = \sum_{k=1}^{K_p} h_p(k) s_1(k-n) \qquad n = 0,\dots,N-1 \tag{1}$$

which is traditionally expressed in terms of the convolution matrix operator H_p as $\boldsymbol{s}_p = H_p \boldsymbol{s}_1$, $p = 2,\dots,P$. It is obvious that $h_1(k) = \delta_K(k)$, where $\delta_K(k)$ is the *Kronecker* delta function. It is assumed that the filters $\boldsymbol{h}_p$, $\forall\, p = 2,\dots,P$, that represent the anisotropic channel characteristics are deterministic but unknown, whereas the source $\boldsymbol{s}_1$ is Gaussian distributed with zero mean and variance $\sigma^2_{s_1}$.

If the P source signals impinge on an array of M sensors, the n^{th} snapshot of the array can be expressed by the $M \times 1$ vector

$$\boldsymbol{x}(n) = \boldsymbol{A}\left[\boldsymbol{h}_{1n}^T \boldsymbol{s}_1 \quad \boldsymbol{h}_{2n}^T \boldsymbol{s}_1 \quad \dots \quad \boldsymbol{h}_{Pn}^T \boldsymbol{s}_1\right]^T, \; n = 0,\dots,N-1 \tag{2}$$

where $\boldsymbol{A} \in \Re^{M\times P}$ denotes a full rank time invariant *mixing* matrix, and $\boldsymbol{h}_{pn}$ denotes the n^{th} row of the convolution matrix operator H_p. In matrix form equation (2) can be expressed as

$$\begin{aligned}\boldsymbol{X} &= [\boldsymbol{x}(0) \quad \boldsymbol{x}(2) \quad \dots \quad \boldsymbol{x}(N-1)] \\ &= \boldsymbol{A}[H_1\boldsymbol{s}_1 \quad H_2\boldsymbol{s}_1 \quad \dots \quad H_P\boldsymbol{s}_1]^T = \boldsymbol{AS}\end{aligned} \tag{3}$$

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As mentioned previously, it is assumed that neither $\boldsymbol{A}$ nor H_p are known, $\forall\ p = 2,...,P$.

From linear algebra theory, it is known that the matrix $\boldsymbol{X}$ would be rank deficient, since $\boldsymbol{S}$ is rank deficient. The column space of $\boldsymbol{X}$, is spanned by all the linearly independent columns of $\boldsymbol{A}$, while the row space of $\boldsymbol{X}$ is spanned by the rows of $\boldsymbol{S}$. Identifying the signal subspace by spectral factorization of $\boldsymbol{X}$ using Singular Value Decomposition (SVD) yields

$$\boldsymbol{X} = \boldsymbol{U}_X \boldsymbol{D}_X \boldsymbol{V}_X^T \tag{4}$$

where the first P singular values along the diagonal of $\boldsymbol{D}_X$ are nonzero and correspond to the P leftmost columns of $\boldsymbol{U}_X$ that span the subspace $\{\boldsymbol{A}\}$, spanned by the columns of $\boldsymbol{A}$. The remaining $M-P$ eigenvalues are zero with probability one. This analysis is guaranteed to separate the sources provided they are mutually orthogonal and fails otherwise because of the rank deficiency of $\boldsymbol{X}$. In the presence of additive noise, the observation matrix $\boldsymbol{Y} \in \Re^{M \times N}$ expressed as

$$\boldsymbol{Y} = \boldsymbol{X} + \boldsymbol{Z} = \boldsymbol{A}\boldsymbol{S} + \boldsymbol{Z} \tag{5}$$

where $\boldsymbol{Z} \in \Re^{M \times N}$ denotes a zero-mean additive noise with arbitrary spatial and temporal covariances $\boldsymbol{R}_Z \in \Re^{M \times M}$ and $\boldsymbol{C}_Z \in \Re^{N \times N}$, respectively. Similar to (4), using SVD

$$\boldsymbol{Y} = \boldsymbol{U}_Y \boldsymbol{D}_Y \boldsymbol{V}_Y^T = \sum_{i=1}^{M} \delta_i \boldsymbol{u}_i \boldsymbol{v}_i^T \tag{6}$$

where δ_m denotes the m^{th} singular value corresponding to the m^{th} diagonal entry in $\boldsymbol{D}_Y$. The eigenvectors $\boldsymbol{V}_Y = [\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_N] \in \Re^{N \times N}$ span the row space of $\boldsymbol{Y}$, which is now full rank due to noise presence. On the other hand, the rightmost $M-P$ columns of $\boldsymbol{U}_Y$ span the noise subspace $\{\boldsymbol{Z}\}$.

When the source signals are non-orthogonal, each singular value δ_i and the corresponding eigenvector $\boldsymbol{u}_i$ do not correspond to a single source. This can be seen by considering the $(i,j)^{\text{th}}$ entry of the signal covariance given by

$$\boldsymbol{R}_S(i,j) = \rho_{ij}\sigma_{\boldsymbol{s}_1}^2 = \boldsymbol{h}_{in}^T \boldsymbol{h}_{jn} \sigma_{\boldsymbol{s}_1}^2 \tag{7}$$

where $\rho_{ij} = \boldsymbol{h}_{in}^T \boldsymbol{h}_{jn}$ expresses the unknown correlation coefficient between the i^{th} and j^{th} sources. The i^{th} eigenvalue of the spatial covariance $\boldsymbol{R}_Y = \boldsymbol{A}\boldsymbol{R}_S\boldsymbol{A}^T + \boldsymbol{R}_Z$ will correspond to

$$\lambda_i = \sigma_{\boldsymbol{s}_1}^2 \sum_{p=1, p\neq i}^{P} \rho_{ip} + \sigma_{z_i}^2 \qquad i = 1,...P \tag{8}$$

where $\sigma_{z_i}^2$ is the i^{th} eigenvalue of $\boldsymbol{R}_Z$. Thus, λ_i is a weighted sum of the variances of the sources that have nonzero projection along the direction of eigenvector $\boldsymbol{u}_i$. Therefore, the correlation of the sources causes a significant ambiguity problem in signal subspace determination. Additionally, when the SNR is small, or when the noise is cross-correlated with the signal, *leakage* occurs between the directions $\boldsymbol{u}_P$ and $\boldsymbol{u}_{P+1}$ in the form of a *fuzzy* gap between the signal singular values and those of the noise.

3. ORTHOGONAL TRANSFORMATION

The objective is to attempt to reduce the contribution of the unknown correlation coefficient terms ρ_{ij} on the eigenvalues of the noise-free observation matrix $\boldsymbol{X}$, as well as enhance the separation gap between the signal and noise singular values in the noisy observation matrix $\boldsymbol{Y}$. Our approach relies on exploiting an alternative solution to signal subspace determination. Recall from equation (4), the signal subspace is P dimensional that can be determined from the span of the columns of $\boldsymbol{A}$. Alternatively, it can be determined from the P rows of $\boldsymbol{S}$ if signal correlation is minimized by appropriate signal subspace *rotation*. If the rotation does not alter the span of the signal subspace $\{\boldsymbol{A}\}$, then it can be used as a mean to separate the correlated sources. Subspace rotation can be achieved by a wide range of orthogonal transformations. The idea is to find a particular orthogonal transformation that minimizes the signal correlation, subject to preservation of the signal subspace $\{\boldsymbol{A}\}$. This can be conveniently achieved if sparse representation of the signals is obtained.

In the case of unknown signals, sparse representation can be obtained using a multiresolution orthogonal transformation by means of the Discrete Wavelet Transform (DWT) [8][9] and its overcomplete representation, the Discrete Wavelet Packet Transform (DWPT) [10], particularly when the source signal $\boldsymbol{s}_1$ is unknown. Additionally, the denoising property of the DWT makes it very powerful in suppressing noise by simple thresholding [10][12]. Let's denote by $\mathbf{W}^{(j)}$ a $N \times N$ linear DWT operator (convolution matrix) for subband j, where $j = 0,1,...,J$. The matrix $\boldsymbol{Y}_j$, obtained through orthogonal transformation of equation (5) with $\mathbf{W}^{(j)}$ can be decomposed using SVD to yield

$$\boldsymbol{Y}_j = \boldsymbol{A}\boldsymbol{S}_j + \boldsymbol{Z}_j = \boldsymbol{U}_Y^j \boldsymbol{D}_Y^j \boldsymbol{V}_Y^{j^T} = \sum_{i=1}^{M} \lambda_i^j \boldsymbol{u}_i^j \boldsymbol{v}_i^{j^T} \tag{9}$$

where $\boldsymbol{S}_j$ and $\boldsymbol{Z}_j$ expresses the projection of the signal and noise matrices onto the space Ω_j of all piecewise smooth functions in $\mathrm{L}_2(\Re)$. These are spanned by the integer-translated and dilated copies $\phi_{j,k} \overset{def}{=} 2^{j/2}\phi(2^j.-k)$ of a scaling function ϕ that has compact support [9]. Note that the unknown mixing matrix $\boldsymbol{A}$ was assumed transformation independent and thus remains *invariant* under the transformation operator. This assumption is valid if the signal mixing is stationary within the analysis interval.

The outer products form in (9) allows us to observe that the span of $\boldsymbol{V}_Y^j$ directly impacts the span of the column space of $\boldsymbol{U}_Y^j$. This is because the p^{th} row of the matrix $\boldsymbol{S}_j$ can be expressed as

$$\boldsymbol{s}_p^j = W^{(j)}\boldsymbol{s}_p = W^{(j)} H_p \boldsymbol{s}_1 = H_p^j \boldsymbol{s}_1 \tag{10}$$

Equation (10) expresses the projection of the sources onto the space Ω_j in terms of the unknown parameters H_p. Furthermore, it establishes the relation between the row space of $\boldsymbol{X}_j$ and that of $\boldsymbol{X}$ using the transformed convolution operator H_p^j. Specifically, If $\mathbf{W}^{(j)}$ spans the *null* space of H_p, then the projection $\boldsymbol{s}_p^j$ will be zero. Conversely, if $\mathbf{W}^{(j)}$ spans the *range* space of H_p, then it is guaranteed that $\boldsymbol{s}_p^j$ will belong to the row space of $\boldsymbol{X}_j$, and consequently will have a corresponding eigenvector in the matrix $\boldsymbol{V}_Y^j$.

To be more specific, let us denote by $\Delta\{J\}$ the dictionary of basis obtained for *L* decomposition levels ($J = 2^{L+1} - 1$ subbands). Now consider the simple illustration in Figure 1, in which the dictionary has only three basis, $\boldsymbol{\beta}_i$, $\boldsymbol{\beta}_l$ and $\boldsymbol{\beta}_k$. The projections of the row space of $\boldsymbol{X}$, denoted $\{\boldsymbol{X}\}$, and the row space of $\boldsymbol{Z}$, denoted $\{\boldsymbol{Z}\}$, are indicated by the components $\{\boldsymbol{X}\}_l$ and $\{\boldsymbol{X}\}_k$, while $\{\boldsymbol{X}\}_i$ is zero. Similarly, $\{\boldsymbol{Z}\}$ results in $\{\boldsymbol{Z}\}_{//}$ that represents the correlated noise component, and $\{\boldsymbol{Z}\}_\perp$ that represents the white noise component. In turn, $\{\boldsymbol{Z}\}_{//}$ results in $\{\boldsymbol{Z}\}_l$ and $\{\boldsymbol{Z}\}_k$, while $\{\boldsymbol{Z}\}_\perp$ results in $\{\boldsymbol{Z}\}_i$. It is thus assumed that $\boldsymbol{\beta}_i$ does not represent any of the signal sources, i.e., $H_p^i = 0_{N\times N}$, $\forall\, p = 1,...,P$. Careful examination of these projections yields the following:

(a) Any signal source that belongs to $\{\boldsymbol{X}\}_l$ is dominant over noise projections $\{\boldsymbol{Z}\}_l$.

(b) Any noise component that belong to $\{\boldsymbol{Z}\}_k$ is dominant over the signal projection $\{\boldsymbol{X}\}_k$.

(c) Any noise projection that belong to $\{\boldsymbol{Z}\}_\perp$ is fully accounted for by the basis $\boldsymbol{\beta}_i$.

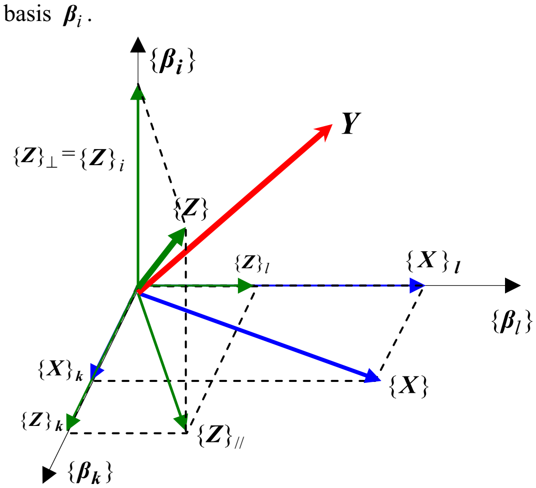

Figure 1: Projection of the signal and noise subspaces $\{\boldsymbol{X}\}$, and $\{\boldsymbol{Z}\}$, respectively, onto a fixed orthogonal basis space. The space is assumed to be *completely* spanned by three orthogonal basis, $\{\boldsymbol{\beta}_l\}$, $\{\boldsymbol{\beta}_k\}$ and $\{\boldsymbol{\beta}_i\}$ for clarity.

Generalizing to an arbitrary size dictionary, the subset of basis following (a) is interpreted as the collection of wavelet basis that best represent the signal sources for which $\mathbf{W}^{(j)}$ spans the range space of H_p, i.e. $H_p^j \neq 0_{N\times N}$. For each of these source, the set will be denoted $\Im_p$, with cardinality J_p. The signal subspace spanned by the principal eigenvectors in $\boldsymbol{U}_Y^j$, denoted $\{\boldsymbol{A}\}_j$, will be restricted to those basis that belong to $\Im_p$. Accordingly, the signal subspace dimension in subband *j*, denoted P_j, will be upper bounded by P.

To identify the signal subspace in the context of source separation, we can interpret the above findings in two different ways:

1. Within subband *j*, the source *separation* process amounts to finding the signal eigenvalues corresponding to the group of sources that possess nonzero correlation with the *j*th wavelet basis. These will be ranked in decreasing order of magnitude as

$$\lambda_1^j > \lambda_2^j > ... > \lambda_{P_j}^j \Leftrightarrow \left\|H_{p_1}^j\right\|^2 > \left\|H_{p_2}^j\right\|^2 > ... > \left\|H_{p_j}^j\right\|^2$$
$$\textit{such that } \; p_1 = \underset{p\in\{1,...,P_j\}}{\arg\max} \left\|H_p^j\right\|^2 \quad j \in \Im_p \tag{12}$$

2. Given a specific source indexed by $p^* \in \{1,...,P\}$, the source *classification* process amounts to specifying an operator B_{p^*}, that finds the set of subband indices among all $j \in \Delta\{J\}$ for which there exist a nonzero eigenvector $\boldsymbol{v}_{p^*}^j$ that corresponds to an *invariant* eigenvector $\boldsymbol{u}_{p^*}^j$. This set of basis, now denoted $\Im_{p^*} \subset \Delta\{J\}$ will constitute the 'best basis' representing the source p^*.

The last interpretation falls under the class of best basis selection schemes, originally introduced in [10]. In that context, best signal representation is obtained by defining a cost function for *pruning* the binary tree. In [10], it was suggested to prune the tree by minimizing an entropy cost function between the parent and children nodes. The cost of each node in the binary tree is compared to the cost of its children. A parent node is marked as a terminal node if it yields a lower cost than its children's cost. Other cost functions were suggested, such as Mean Square Error (MSE) minimization [11]. In our context, the cost function can be expressed in terms of the invariance property of the signal subspace $\{\boldsymbol{A}\}_j$ of children nodes compared to their parent node. Specifically, a child node is considered a candidate for further splitting if the ℓ_2 norm of the distance between the signal subspace in the parent node and that of the child is minimized, i.e.

$$Cost(j,p) = \min_{j\in\Im_p} \left\|\boldsymbol{u}_p^{Parent} - \boldsymbol{u}_p^{Child}\right\|^2 \tag{13}$$

The cost definition ensures that children nodes that do not have a "similar" signal subspace to that of the parent, they will not be marked as candidates for further splitting.

4. RESULTS

We tested the proposed approach on multichannel neurophysiological recordings obtained with microelectrode arrays implanted in the brain. The signals of interest consist of neural activity in the form of rapid, short-duration transient waveforms elicited by a small population of neurons in the vicinity of the electrode array. The correlated noise component is the spontaneous and stimulus-driven activity from hundreds of neurons in the background that are far from the array, and white noise from the signal conditioning electronics. Characteristics obtained from the algorithm are illustrated in Figure 2 for a single source waveform sampled along the array. In Figure 3, a complex waveform (middle panel) from two fully coherent sources (left panel) is demonstrated across the 4-channel array. The features obtained (right panel) are indicated by the two distinct sets of the principal eigenvector in the best basis nodes, demonstrating the separation ability of the algorithm. Invariance to signal *fading* is illustrated in Figure 4, in which ten consecutive events from one source were simulated to undergo ~45% decrease in energy over time. In the context of neurophysiology, this is a typical characteristic of the extracellular space [14] that makes the separation of multi-neuron signals a formidable task. As can be seen, this is efficiently captured by the percentage change in the principal eigenvalue -relative to that of the first event- in the best basis that characterize the signal.

5. CONCLUSION

The analysis carried out in this work provides the mathematical basis for the notion of invariance of signal subspace under orthogonal transformation in sensor array processing. We described an intuitive and efficient algorithm best suited for blind source separation and classification in multichannel signal environments when source signals are linearly dependent and exhibit temporal nonstationarity in the presence of strong spatially and temporally correlated noise fields. In the absence of knowledge of the mixing matrix, the correlation among source signals complicates the source separation problem, especially in colored noise. It is obvious that the wavelet basis choice is of crucial importance to the separation process. When the signal or the channel parameters are partially available, they can be used to design admissible wavelets to maximize a classification metric. Ideally, the wavelet basis should be selected to provide maximum separability between different sources. Extension beyond second order statistics is also feasible and is currently under investigation.

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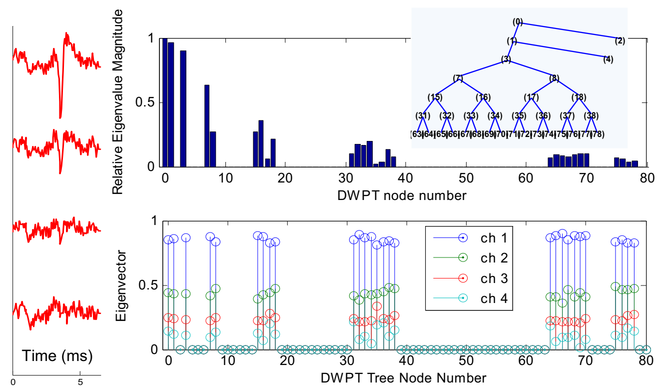

Figure 2: Feature set for a single source (wavelet basis = *symlet*4)

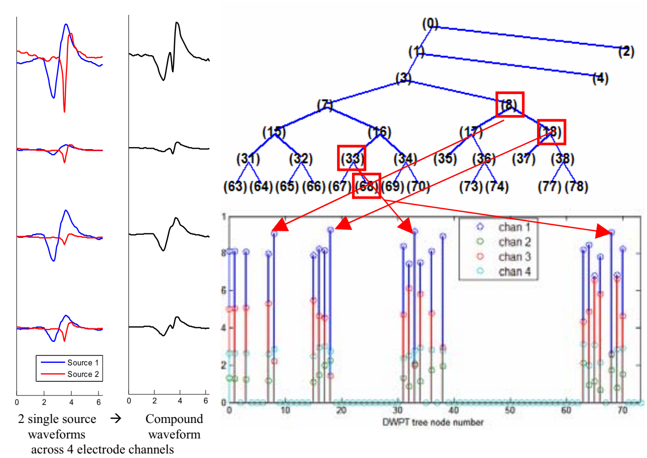

Figure 3: Separation of two fully coherent signal sources

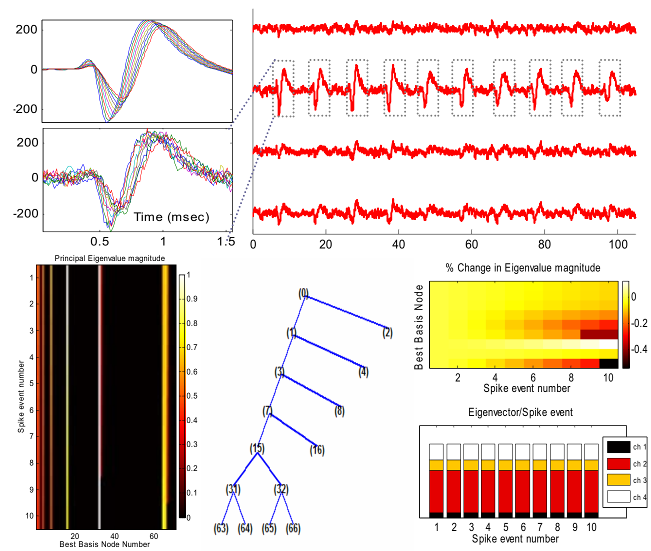

Figure 4: Invariance of signal subspace to source nonstationarity