CONJUGATE MUSIC FOR NON-CIRCULAR SOURCES

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Abstract-This paper proposes a method for the direction of arrival (DOA) estimation for non-circular sources, such as binary phase shift keying (BPSK) and M-ary amplitude shift keying (MASK). The proposed scheme can be applied to both scenarios; the sources are noncoherent or coherent. Comparisons that are made with the well-known classical subspace algorithms, such as MUSIC and forward/backward smoothing by Pillai et al., show that the proposed method has several advantages. First, no forward/backward spatial smoothing for the covariance matrix is needed, whereas the Pillai method requires it, which increases the computational load and is time-consuming. Second, the proposed method can detect *N-1* coherent sources when *N* antennas are used, whereas the well-known Pillai method can only detect *2N/3* coherent sources. Third, the proposed method is more suitable for real-time implementation since it only requires a single or few snapshots in order to give an accurate DOA estimation. However, both MUSIC and Pillai require a high number of snapshots, which increases the computational cost and memory storage. Simulation results show that the proposed method has better performance, compared to the forward/backward spatial smoothing by the Pillai and MUSIC Algorithm.

I. INTRODUCTION

The direction of arrival estimation (DOA) of multiple signals impinging on an array of sensors has attracted many researchers [1-3]. The idea of DOA can be applied in many commercial and military applications, such as sonar, radar, and mobile communication systems.

Many systems nowadays deal with non-circular sources. For example, in telecommunication or satellite systems, the signals are often used as an amplitude modulation (AM) or BPSK modulation. The non-circular incoming signals have been proposed in various literatures [4-9].This paper proposes a method that applies to non-circular sources.

The most popular classical subspace algorithms, MUSIC and ESPRIT, are well-known algorithms for estimating the DOA [1-2]. However, these algorithms use the time averaging of the covariance matrix based on a high number of snapshots to obtain a good performance. Also, these methods are known for giving a high-resolution for DOA estimation when they are applied only in situations where the sources are uncorrelated; however, in multipath environments, MUSIC and ESPRIT performances are either degraded or failed to detect all incident sources.

Several methods are suggested in order to resolve the situation when the incident sources are coherent [10-12]. Evans et al. [10] and Shan et al. [11] proposed the forward spatial smoothing technique, which is based on the partition of the total number of antenna arrays into the number of subarrays, and then finding the covariance matrix for each subarray before averaging the output covariance matrices. However, the forward estimation techniques require a large number of antenna arrays in order to estimate the *N/2* coherent sources, and the scheme requires *N* antenna elements. In addition, this method requires that the covariance matrix be found for each subarray using a high number of snapshots and then averaging the covariance matrices for all subarrays, which results in a high computational load and is very time-consuming, especially when the number of antennas and coherent sources increase.

Pillai et al. proposed a scheme called "forward/backward spatial smoothing" to improve the forward spatial smoothing [12]. These combinations of forward/backward spatial smoothing increase the number of detected sources from *N/2* to *2N/3*, where *N* is the number of antenna elements.

In this paper, the proposed method (without using forward/ backward spatial smoothing) will detect an additional number of coherent sources compared to the spatial smoothing techniques [10-12]. The proposed method is called Conjugate MUSIC (C-MUSIC). The proposed method (C-MUSIC) enables an increase in the number of detected sources to *N-1* coherent sources when *N* number of antenna is used instead of detecting *N/2 or 2N/3* coherent sources using other methods [12]. Subsequently, there is a reduction in the computational complexity and cost, and a lesser number of antenna arrays are required to estimate the same number of sources compared to the method in reference [12]. In addition, C-MUSIC uses a single or few snapshots to estimate the DOA for incident sources, whereas the method in [12] required a high number of snapshots, i.e., 100 to 200 snapshots taken in order to have a good estimate. It should be noted that C-MUSIC has a good performance estimation and works efficiently even for a low signal-to-noise ratio environment with just a few snapshots.

Section 2 describes the system model. Section 3 will present simulation results and show the performance superiority of our proposed method over MUSIC and forward/backward spatial smoothing techniques. Section 4 will make conclusions.

II. PROPOSED ALGORITHM

Consider a uniform linear array (ULA) composed of *N* elements and assume that *K* noncoherent and narrow band one-dimensional signals are received at the ULA with different DOAs, $\theta_1, \theta_2, ..., \theta_K$. The $N\times 1$ received signal vector can be written as:

$$Z(t) = \sum_{k=1}^{K} \underline{a}(\theta_k) s_k(t) + \underline{n}(t) \tag{1}$$

where $s_k(t)$ represents the signal from the *k*-th sources with DOA equal to θ_k, $\underline{a}(\theta_k)$, which denotes the $N\times 1$ array response vector, and $\underline{n}(t)$ is the $N\times 1$ AWGN vector with each component of mean zero and variance equal to σ^2. The array response vector can be written as:

$$\underline{a}(\theta_k) = \left(1, u_k, u_k^2, ..., u_k^{N-1}\right)^T, \quad k = 1,..., N \tag{2}$$

where the superscript *T* stands for the transpose,

$$u_k = \exp\left(-j\frac{2\pi d\cos\theta_k}{\lambda}\right), \tag{3}$$

λ is the wavelength, and *d* is the spacing between two successive antenna array elements. The $N\times K$ array response matrix and $K\times 1$ signal vector can be written as:

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$$A(\theta) = \left(\underline{a}(\theta_1), \underline{a}(\theta_2), ..., \underline{a}(\theta_K)\right) \quad (4)$$

$$S(t) = \left(s_1(t), s_2(t), ..., s_K(t)\right)^T, \quad (5)$$

respectively. The received signal vector can be rewritten as:

$$Z(t) = A(\theta)S(t) + \underline{n}(t). \quad (6)$$

In the following paragraph, C-MUSIC is presented so that it estimates the azimuth DOAs for multiple incident sources θ_k .

A. Azimuth DOA Estimation with proposed method (C-MUSIC)

In the proposed method, the measurement is taken from all of the array elements and put into a column vector. Figure 1 shows the inputs to N subarrays. For example, the inputs to the subarray processing 1 are $(z_1, z_2, z_3, ..., z_N)$, the subarray processing 2 are $(z_2^*, z_1, z_2, ..., z_{N-1})$, and the subarray processing N are $(z_N^*, z_{N-1}^*, z_{N-2}^*, ..., z_1)$. The way that this proposal constructs the input data is completely different compared to other well-known algorithms that have been proposed to find the DOAs [1-2, 10-12]. For example, C-MUSIC is unlike the Pillai method [12]. First, our proposed algorithm (C-MUSIC) does not use any doublet concept. Each subarray processed in C-MUSIC uses the maximum number of array elements equal to N, whereas each subarray processed in Pillai can be $(N-1)$ at most. Second, C-MUSIC composed the input data from N number of subarrays without using either the concept of maximum overlapping or forward/backward averaging, whereas the number of maximum overlapping subarrays in Pillai is $(N-1)$ in forward direction and the same number of subarrays in backward direction.

The element is treated in the origin of the array as the reference with respect to the other elements, as shown in Figure 1. In this case, the $N\times1$ input vector to subarray-1 can be written as:

$$Z_1 = \begin{bmatrix} z_1 & z_2 & \cdots & z_N \end{bmatrix}^T \quad (7)$$

In this case, the $N\times1$ input vector in (7) can be written in vector form as follows:

$$Z_1(t) = A(\theta)S(t) + \underline{n}_1(t) \quad (8)$$

$$\underline{n}_1(t) = [n_1(t), n_2(t), ..., n_N(t)]^T \quad (9)$$

where $A(\theta)$ is the array response vector with dimension $N\times K$ (not $N-1\times K$ used for the Pillai in each directions), $S(t)$ is the narrow band signal vector with dimension $K\times 1$, and $\underline{n}_1(t)$ is the $N\times 1$ AWGN vector whose component has a zero mean and a variance equal to σ^2.

The input vector to the subarray-2 processing is written as:

$$Z_2 = [z_2^*, z_1, ..., z_{N-1}]^T. \quad (10)$$

For the one dimensional signals such as BPSK and MASK where $s_k = s_k^*$, this input vector can be rewritten in terms of $A(\theta)$, $S(t)$, and Φ^* as:

$$Z_2(t) = A(\theta)\Phi^* S(t) + \underline{n}_2(t) \quad (11)$$

$$\Phi^* = diag(u_1^*, u_2^*, ..., u_K^*) \quad (12)$$

$$\underline{n}_2(t) = [n_2^*(t), n_1(t), ..., n_{N-1}(t)]^T \quad (13)$$

Now the input vector to the subarray-3 processing is written as:

$$Z_3 = [z_3^*, z_2^*, ..., z_{N-2}]^T. \quad (14)$$

Equation (14) can be rewritten in terms of $A(\theta)$, $S(t)$, and Φ^* as:

$$Z_3(t) = A(\theta)\left(\Phi^*\right)^2 S(t) + \underline{n}_3(t) \quad (15)$$

$$\underline{n}_3(t) = [n_3^*(t), n_2^*(t), ..., n_{N-2}(t)]^T \quad (16)$$

Finally, the input vector to the subarray-N processing can be written as:

$$Z_N = [z_N^*, z_{N-1}^*, ..., z_1]^T \quad (17)$$

Equation (17) can be rewritten in terms of $A(\theta)$, $S(t)$, and Φ^* as:

$$Z_N(t) = A(\theta)\left(\Phi^*\right)^{N-1} S(t) + \underline{n}_N(t) \quad (18)$$

$$\underline{n}_N(t) = [n_N^*(t), n_{N-1}^*(t), ..., n_1(t)]^T \quad (19)$$

These observation vectors $(Z_1, Z_2, Z_3, ..., Z_N)$ will be employed in the C-MUSIC. Hence, the total output data $Y(t)$ can be written as:

$$Y(t) = \begin{bmatrix} Z_1 & Z_2 & Z_3 & \cdots & Z_N \end{bmatrix} = \begin{bmatrix} z_1 & z_2 & z_3 & \cdots & z_N \\ z_2^* & z_1 & z_2 & \cdots & z_{N-1} \\ z_3^* & z_2^* & z_1 & \cdots & z_{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z_N^* & z_{N-1}^* & z_{N-2}^* & \cdots & z_1 \end{bmatrix}^T \quad (20)$$

where the total output date $Y(t)$ is $(N \times N)$ matrix. In equation (20) Y(t) is a symmetric Toeplitz matrix with dimension N x N and the first row $\begin{bmatrix} z_1 & z_2 & z_3 & \cdots & z_N \end{bmatrix}$ in Y(t) is the observation from the antenna array. In C-MUSIC, the symmetric Toeplitz matrix property was used to form the collection of the observation data. Forming the data in symmetric Toeplitz form the incident sources and become decorated; this enables the proposed method to detect $N-1$ sources even though the sources are coherent. In C-MUSIC, the DOAs of incident signals were estimated for the cases when the sources are coherent and noncoherent through the following steps.

Step 1: Obtain the measurements $\begin{bmatrix} z_1 & z_2 & z_3 & \cdots & z_N \end{bmatrix}$ from the antenna array.

Step 2: Use the measurement in Step 1 to construct the output data in symmetric Toeplitz matrix form as in (20).

Step 3: Obtain an estimate R using a single or few snapshots from the collection data Y(t) as follows:

$$R = \frac{Y(1)*Y(1)^H + Y(2)*Y(2)^H + \cdots + Y(L)*Y(L)^H}{L} = \frac{1}{L}\sum_{t=1}^{L} Y(t)Y(t)^H \quad (21)$$

where R is the average spatial covariance matrix with dimension N x N and L is the number of a few snapshots.

Step 4: Apply eigen-decomposition to R, i.e.:

$$R = E_s \Lambda_s E_s^H + E_n \Lambda_n E_n^H \quad (22)$$

where Es=[e1,e2,…,ek] is the collection of eigenvectors for the signal space and $\Lambda_s = \begin{bmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_k \end{bmatrix}$ is the collection of the largest eigenvalues, which correspond to the k number of sources. Also, En=[e_{k+1},e_{k+2}, … ,e_N] is the collection of eigenvectors for the noise space and $\Lambda_s = \begin{bmatrix} \lambda_{k+1} & \lambda_{k+2} & \cdots & \lambda_N \end{bmatrix}$ is the collection of $N-K$ smallest eigenvalues of the noise space.

Step 5: Employ the power spectrum estimation to search for the peaks, which correspond to the DOAs of incident sources.

$$P(\theta) = \frac{1}{a(\theta)^H E_n E_n^H a(\theta)} \quad (23)$$

where En is the eigenvectors of noise space, which was obtained in the proposed method from (22).

III. SIMULATION RESULTS

In this section, the performance of C-MUSIC is compared with the MUSIC algorithm for noncoherent sources and with forward/backward spatial smoothing by Pallai for coherent sources [12]. The performance is verified through simulations for coherent and noncoherent sources. The following sources, *K=3, K=6, and K=8* are considered to be different SNR values and different DOAs, and a ULA consisting of four sensors *N=4* for a noncoherent case, and *N=9* for coherent sources with spacing equal to a half wavelength.

For Fig. 2 and 3, three coherent signals are assumed to have been received with *azimuth* DOAs at *[40 55 70]* degrees and a low SNR of 0 dB for all sources. It is clear in Fig. 2 that even with a few snapshots, our proposed algorithm produces accurate DOA estimations for all sources, and all three peaks are observed from the exact direction at *40º, 55º, and 70º,* whereas in Fig. 3, it is clear that MUSIC can detect the sources with less accuracy and very low resolution, even though one hundred snapshots were taken.

For Fig. 4 and 5, six coherent signals are assumed to have been received with *azimuth* DOAs at *[20 40 60 80 100 120]* degrees, nine antenna elements *N=9*, and SNR of *10* dB for all sources. It is clear in Fig. 4 that even with a single snapshot our proposed algorithm produces accurate DOA estimations for all sources, and all eight peaks are observed from the exact direction at *20º, 40º, 60º, 80º, 100º, and 120º,* whereas in Fig. 5, it is clear that the forward/backward spatial smoothing algorithm fails to detect the incident sources.

For Fig. 6 and 7, eight coherent signals are assumed to have been received with *azimuth* DOAs at *[40 55 70 85 100 115 130 145]* degrees, nine antenna elements *N=9*, and SNR of *5* dB for all sources. It is clear in Fig. 6 that our proposed algorithm produces accurate DOA estimations for *N-1* sources, and all eight peaks are observed from the exact direction at *40º, 55º, 70º, 85º, 100º, 115º, 130º, and 145º,* whereas in Fig. 7 it is clear that the forward/backward spatial smoothing algorithm fails to detect the incident sources.

IV. CONCLSIONS

This paper has proposed a method which is called C-MUSIC for the direction of arrival angle estimation employing the Symmetric Toeplitz data. The proposed method can correctly estimate the DOAs even from the correlated/coherent sources with a single or a few snapshots, whereas MUSIC and forward/backward spatial smoothing fail under a few snapshots. Moreover, C-MUSIC can detect *N-1* sources where *N* is the number of antenna, whereas forward/backward spatial smoothing can detect *2N/3* at most. These advantages make our proposed method more appropriate for real time implementation.

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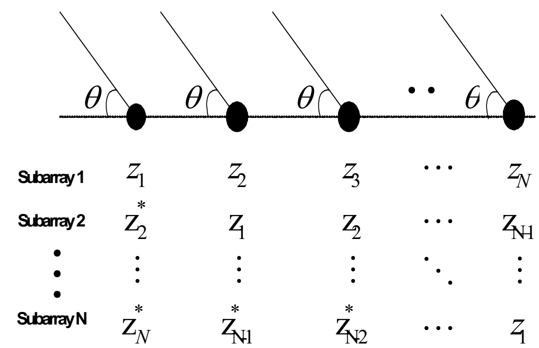

Fig.1. The inputs to subarray (1, 2, …, N) processing for the proposed proposed algorithm, where $z_1, \ldots, z_N$ are the received signals at element 1,…, N, of an array, respectively.

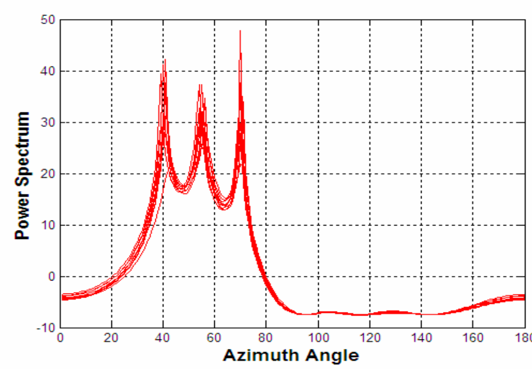

Fig. 2. Power Spectrum of DOA Estimation for 10 independent trials with three noncoherent sources *K=3* at *[40 55 70]* degrees when *SNR=0 dB*, number of samples *10,* and *N=6* elements by using **the *Proposed* method.**

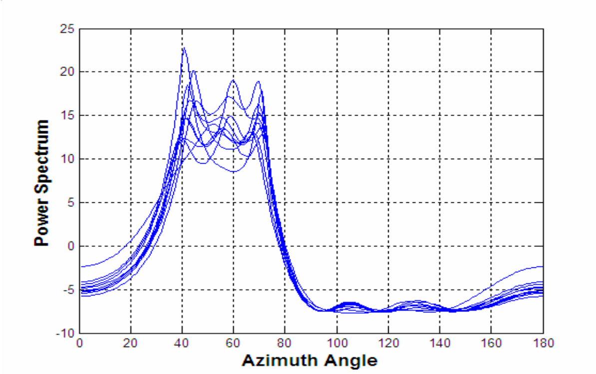

Fig. 3. Power Spectrum of DOA Estimation for 10 independent trials with three noncoherent sources $K=3$ at *[40 55 70]* degrees when *SNR=0 dB*, number of samples *100,* and $N=6$ elements by using the *MUSIC method.*

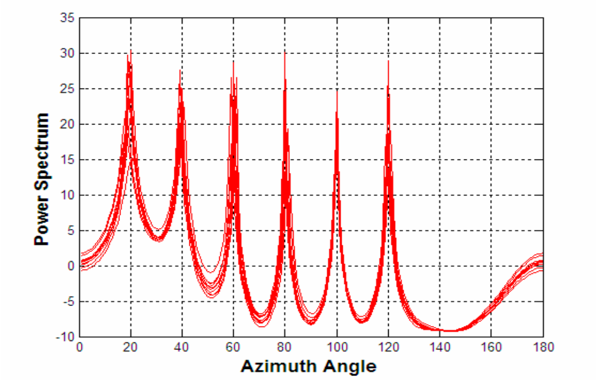

Figure 4: Power Spectrum of DOA Estimation for 10 independent trials with six coherent sources $K=6$ at *[20 40 60 80 100 120]* degrees when *SNR=10 dB*, *single snapshot,* and $N=9$ elements by using the ***Proposed*** **method.**

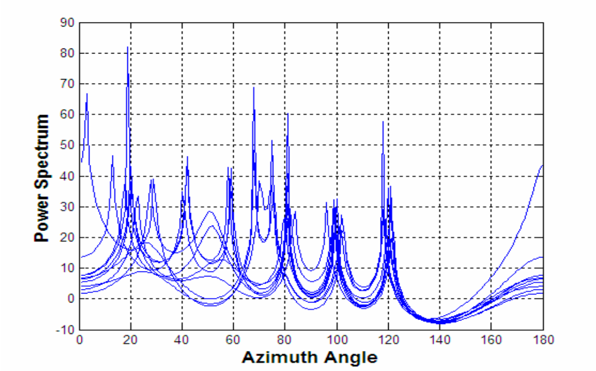

Fig. 5. Power Spectrum of DOA Estimation for 10 independent trials with six coherent sources $K=6$ at *[20 40 60 80 100 120]* degrees when *SNR=10 dB*, *single snapshot,* and $N=9$ elements by using the *forward/backward spatial smoothing method*.

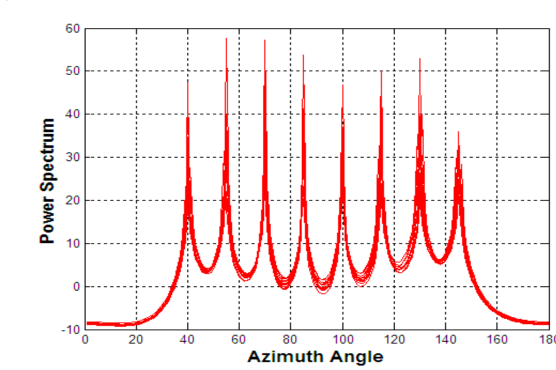

Fig. 6. Power Spectrum of DOA Estimation for 10 independent trials with eight coherent sources $K=8$ at *[40 55 70 85 100 115 130 145]* degrees when *SNR=5 dB*, *10 snapshot,* and $N=9$ elements by using the ***proposed method.***

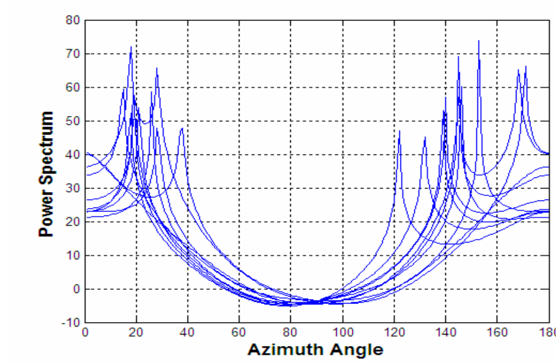

Fig. 7. Power Spectrum of DOA Estimation for 10 independent trials with eight coherent sources $K=8$ at *[40 55 70 85 100 115 130 145]* degrees when *SNR=5 dB*, *10 snapshot,* and $N=9$ elements by using the *forward/backward spatial smoothing method.*