TRACKING OF TIME-VARYING NUMBER OF MOVING TARGETS IN WIRELESS SENSOR FIELDS BY PARTICLE FILTERING

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ABSTRACT

In this paper, we consider tracking time-varying number of targets which move along a two-dimensional area monitored by a network of wireless sensors. We propose a novel fusion algorithm based on particle filtering that accounts for both detection of the number of active targets in the field and estimation of their positions and velocities. The method uses measurements collected by acoustic sensors, where the measurements represent superposition of received powers of signals transmitted by the targets. Computer simulations are provided to illustrate the feasibility of the proposed method in scenarios with zero, one, and two targets.

1. INTRODUCTION

Multiple target tracking is a challenging problem that has been widely addressed in the signal processing literature [1, 2]. The most interesting and general case of it includes the task of tracking when the number of targets is unknown. This implies that besides estimation, the tracker has to implement detection, or in general, model selection, where distinct models include different number of targets, which entails that their state spaces are of different dimensions. One possibility of addressing multiple target tracking consists of estimating the number of targets separately from the target estimates [3]. Alternative approaches include the use of Stone's model [4], particle filters [5, 6] and random sets [7].

In most of the previous work, the available measurements are associated with one target at a time which implies that the problem itself involves the resolution of the problem of data association. In this paper we address the tracking of time-varying number of targets in a field of wireless sensors, where the sensor data represent measurements of superimposed powers of acoustic signals emanating from one or more moving targets. The sensor measurements include also background noise. Thus, the problem of data association is not relevant and instead, other difficulties arise in the detection.

The measurements do not have to represent acoustic power. In fact they can be of any type, for example measurements of radio frequency or vibration signals. In the scenarios that we have simulated, the measured acoustic powers by the sensors are reported to a fusion center. The fusion center processes them and at any time has estimates for the number of active targets in the sensor field and of their positions and velocities.

In this paper we propose a particle filtering approach to the multiple target tracking problem as defined above. All the estimates of the unknowns (number of targets, positions, and velocities) are expressed as posterior distributions. We discuss the details of the method that we have developed and provide results that demonstrate its feasibility.

The remaining of this paper is organized as follows. In Section 2, we introduce the dynamic system that describes mathematically our tracking problem of time-varying number of targets. In Sections 3 and 4, we explain the algorithm that we propose for tracking and discuss its advantages and drawbacks, respectively. Computer simulation results illustrating the performance of the method are presented in Section 5. Finally, brief concluding remarks are made in Section 6.

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2. SYSTEM MODEL

Consider a network of N acoustic sensors deployed in a two-dimensional field. At time instant t an unknown number of targets, K_t, move in the field according to a standard model formulated as [2]

$$\mathbf{x}_t \quad = \quad \mathbf{G}_x\mathbf{x}_{t-1} + \mathbf{G}_u\mathbf{u}_t \tag{1}$$

where $\mathbf{x}_t^\top = [\mathbf{x}_{1,t}^\top, \ldots, \mathbf{x}_{K_t,t}^\top]^\top \in \mathbb{R}^{4K_t}$ indicates the position and the velocity of the targets in the field, i.e., $\mathbf{x}_{k,t} = [x_{1,k,t} \;\; x_{2,k,t} \;\; \dot{x}_{1,k,t} \;\; \dot{x}_{2,k,t}]^\top$, $k = 1, \ldots, K_t$.

The transition matrices, $\mathbf{G}_x$ of size $4K_t \times 4K_t$, and $\mathbf{G}_u$ of size $4K_t \times 2K_t$, are block diagonal matrices with respective blocks defined by,

$$\mathbf{G}'_x = \begin{pmatrix} 1 & 0 & T_s & 0 \\ 0 & 1 & 0 & T_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } \mathbf{G}'_u = \begin{pmatrix} \frac{T_s^2}{2} & 0 \\ 0 & \frac{T_s^2}{2} \\ T_s & 0 \\ 0 & T_s \end{pmatrix}$$

where T_s is the sampling period. The noise in the state equation, $\mathbf{u}_t \in \mathbb{R}^{2K_t}$, accounts for small acceleration turbulences and is modeled as a Gaussian process with zero mean and covariance matrix $\mathbf{C}_u = \mathrm{diag}\left(\sigma^2_{u_{1,1}}, \sigma^2_{u_{1,2}}, \ldots, \sigma^2_{u_{K_t,1}}, \sigma^2_{u_{K_t,2}}\right)$.[1] The n-th sensor is located at a known position $\mathbf{r}_n \in \mathbb{R}^2$, $n = 1, \ldots, N$ and receives the signal power transmitted from the targets that are present in the field according to [8],

$$\begin{aligned} y_{n,t} \quad &= \quad g_n(\mathbf{x}_t) + w_{n,t} \qquad\qquad n = 1, \ldots, N \\ &= \quad \sum_{k=1}^{K_t} \frac{\Psi_k d_0^\alpha}{|\mathbf{r}_n - \mathbf{l}_{k,t}|^\alpha} + w_{n,t}, \end{aligned} \tag{2}$$

[1] We note that our method, since it is based on particle filtering, can operate on any type of noise.

where $g_n(\cdot)$ is a function that models the received signal power by the $n-$th sensor, Ψ_k is the emitted power of the $k-$th target measured at a reference distance d_0, $\mathbf{l}_{k,t} = [x_{1,k,t},\ x_{2,k,t}]^\top$ is the location of the $k-$the target at time t, α is an attenuation parameter that depends on the transmission medium and is considered known and the same for all sensors, and

$$|\mathbf{r}_n - \mathbf{l}_{k,t}| = \sqrt{(r_{1,n} - x_{1,k,t})^2 + (r_{2,n} - x_{2,k,t})^2}.$$

The observation noise, $v_{n,t}$, which is independent from $\mathbf{u}_t$, is modeled according to $\mathcal{N}(\mu_v, \sigma_v^2)$, where $\mu_v = \sigma^2$ and $\sigma_v^2 = 2\sigma^4/L$, with σ^2 and L being the power of the background noise of one sample and the number of samples used to obtain the measured power, respectively.

The objective is to detect the number of targets K_t in the sensor field and track each of them, i.e., estimate $\mathbf{x}_t$, using the measurements of the N sensors, $y_{n,t}$, $n = 1, 2, \cdots N$.

3. PARTICLE FILTERING ALGORITHM

In this section we propose a detection/estimation scheme for time-varying number of targets that is based on particle filtering [5]. Note that our objective is actually finding the posterior distribution, $p(K_{0:t}, \mathbf{x}_{0:t}|y_{1:t})$,[2]. It immediately becomes clear that finding the analytical form of the posterior is a formidable task, so therefore we settle for less. To that end, we approximate the posteriors at every time instant t by discrete random measures which are constructed by using the mechanism of particle filtering.

The posterior distribution $p\left(K_{0:t}, \mathbf{x}_{0:t}|y_{1:t}\right)$ is approximated by the random measure $\chi_{0:t} = \{K_{0:t}^{(m)}, \mathbf{x}_{0:t}^{(m)}, w_{0:t}^{(m)}\}_{m=1}^M$, where $K_{0:t}^{(m)}$ and $\mathbf{x}_{0:t}^{(m)}$ are streams of particles with corresponding weights $w_{0:t}^{(m)}$, and M is the total number of particles. For example, the $m-$th particle at time instant t may have a value of $K_t^{(m)} = 2$, which means that it corresponds to two targets and that the states of the two targets $\mathbf{x}_t^{(m)}$ are expressed by an 8-dimensional vector. Thus, the size of $\mathbf{x}_t^{(m)}$ in general is $4K_t^{(m)} \times 1$, and it may change with time. When the $m-$th particle represents no targets, $K_t^{(m)} = 0$ and there is only a weight associated with it. Obviously, the state in this case does not exist, and it is suppressed in the representation of the random measure. Some other specifics about the particles that represent no targets are presented in Section 4.

Each of the particle streams represent one possible evolution of the number of targets in the field and their positions and velocities. If at time instant t the $m-$th particle has $K_t^{(m)} > 0$ targets, we assume that at time instant $t+1$ one of the following events will happen:

1. the number of targets does not change and the targets are the same,
2. a new target enters the sensor field and therefore the number of targets is increased by one, and
3. one of the $K_t^{(m)}$ of targets from time instant t either leaves the field or simply becomes inactive, thereby decreasing the number of targets by one.

In all of these scenarios, we do not allow for a change of more than one target in the field between two time instants. With this assumption we keep the number of possibilities relatively small. It is clear that if a particle at time t corresponds to $K_t^{(m)} > 0$ targets, it will propagate at time instant $t+1$ to $K_t^{(m)} + 2$ new particles. With this strategy we allow for testing of models that represent creation of new targets and disappearance of old targets. The computed weights of the targets provide information about the posterior probabilities of these models.

When the particle represents no targets in the field at time instant t, there are only two possible actions at $t+1$. One is to create a target, and the other is to maintain no targets in the field. Recall that the particle with no targets in the field does not have a state vector.

The weight computation in our approach is the standard one. If we draw the particles from their priors, the weights are obtained from the likelihoods according to

$$w_t^{(m)} \propto \prod_{n=1}^{N} p\left(y_{n,t}\,|\,\mathbf{x}_t^{(m)}, K_t^{(m)} > 0\right) \tag{3}$$

where $\mathbf{x}_t^{(m)}$ is a particle of a state vector of $K_t^{(m)}$ targets. When the particle represent no targets, its weight is given by

$$w_t^{(m)} \propto \prod_{n=1}^{N} p\left(y_{n,t}\,|K_t^{(m)} = 0\right). \tag{4}$$

An important issue in the computation of the weights of all the particles is their comparability. In other words, can we compare the weights of the particle that correspond to different models? The answer to this question is affirmative. This can easily be deduced from (3) and (4), where the factors in the product represent one-dimensional probability density functions.

The proposed particle filtering procedure is summarized below.

At each time instant $t > 0$, perform the following operations:
1. Particle generation: For all m, where $K_t^{(m)} > 0$, generate $(K_t^{(m)} + 2)$ particles, where
(a) One particle that maintains the number of $K_t^{(m)}$ targets,
(b) $K_t^{(m)}$ particles that correspond to one of the $K_t^{(m)}$ different combinations of $K_t^{(m)} - 1$ targets, and
(c) One particle that adds one more target, i.e., that corresponds to $K_t^{(m)} + 1$ targets.
If $K_t^{(m)} = 0$, generate only one particle for a model with one target.
2. Weight computation: Compute the weights of the particles generated in step 1 according to (3) and (4) and normalize them.
3. Estimation: Obtain the estimates of the positions of the targets according to each of the models.
4. Resampling: Resample M particles from the set of generated particles.

4. DISCUSSION

Here we briefly discuss a few important issues of our approach. We start with the particles that represent no targets in the field. Suppose that at time instant t there are no targets and that all the particles represent no targets. These are "abstract particles" all collapsed into one particle that has no state. For the next time instant, we generate M particles with one target and compute their weights. We also

[2]The notation $\mathbf{x}_{0:t}$ means $\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_t$, and similarly for the other variables.

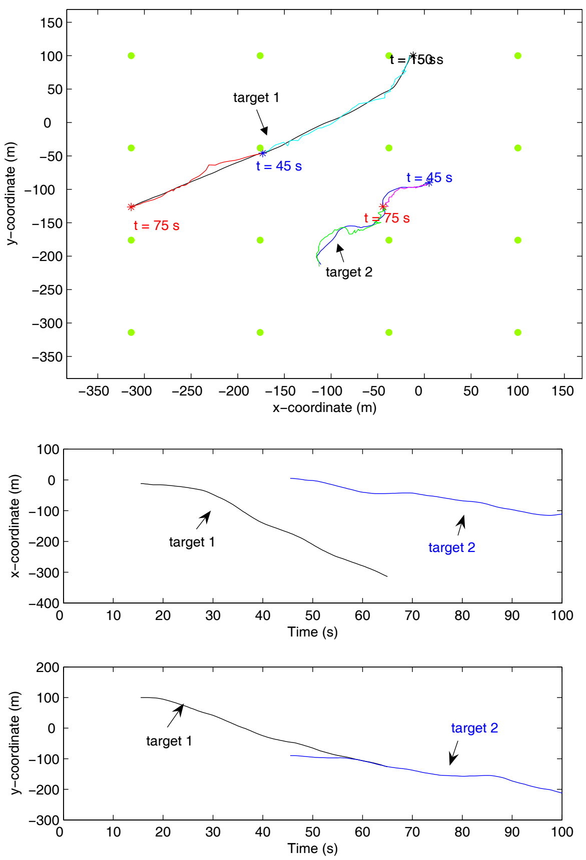

Fig. 1. Top: The sensor field and the true and estimated trajectories of two targets. The time instants of its appearance/disappearance are indicated. Middle: x-coordinates of the targets as functions of time. Bottom: y-coordinates of the targets as functions of time.

compute the weight of the particle with no targets. Then we formally clone that particle M times (because these particles are all identical) and proceed with resampling from $2M$ to M particles. If at time instant $t+1$, M_1 particles were resampled that correspond to one target, and $M-M_1$ particles of no targets, then the M_1 particles would create $3M_1$ new particles, and the $M-M_1$ particles of no targets, $2(M-M_1)$ new particles. All the new particles would be resampled in the usual way. The method proceeds along these lines for time instants $t+2, t+3$, and so on.

When we have multiple targets, it is important to label them so that we can use the particles to estimate the states of the targets. Namely, the elements of the state vector of each particle must be ordered in a way that would allow for element-wise comparison of the state vectors. For example, if there are $K_t^{(m)} > 1$ targets in the field, and the first element of the state vector represents the x coordinate of the "first" target, then the first elements of all the particles must represent the coordinate of that target. We achieve this by labeling (indexing) the targets.

Finally, we point out the importance of the initialization of the targets. Clearly, if we have prior information about the problem (for example, roads where the targets move), it should be exploited in generating the initial possible positions of the targets. In absence of any knowledge, we can either set the initial positions of the targets by drawing from a uniform distribution, or we can use a method that is specifically devised for this purpose and based on

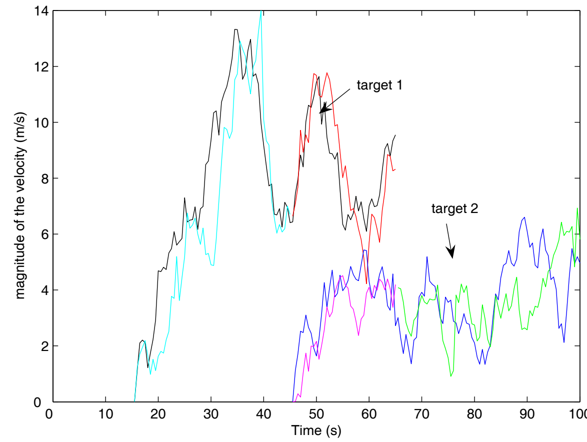

Fig. 2. Estimates of the magnitude of the velocity with respect to time.

the computation of likelihoods of the position of the new target. The former would require the use of a larger number of particles and therefore more computation. In comparison to scenarios of good priors, the latter, too, will have increased computation which will be due to the calculations of the likelihoods.

5. COMPUTER SIMULATIONS

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In this section, we present simulation results that show the performance of the tracking algorithm discussed in Section 3. We ran an experiment where we generated data according to model (1)-(2), which corresponded to the evolution of a system during $T = 100$s with sampling period $T_s = 0.5$s. We considered that the distribution of the state noise was $\mathbf{u}_t \sim \mathcal{N}(\mathbf{0}, .5\mathbf{I}_{2K_t})$; the parameters of the observation model were, $\Psi_k = \Psi = 10^7$, $d_0 = 1$m, and $\alpha = 2$; and the observation noise distribution was generated with $\sigma^2 = 0.02$ and $L = 100$. At the beginning of the simulation no targets were present in the field; at time instant $t = 15$s a first target appeared; at time instant $t = 37.5$s a second target got into the sensor area; and finally, at $t = 75$s the first target disappeared. The sensor network was composed of $N = 16$ sensors placed on a deterministic grid within the field (see Figure 1 for system configuration were the sensors are marked with dots.)

We applied the algorithm proposed in Section 3 using $M = 500$ particles[3] and initialized it by assuming the starting areas of the trajectories approximately known. The obtained estimates for the positions of the targets in one simulation run are shown in Figure 1 (top). It can be seen that the algorithm successfully follows the trajectories of the targets. Figure 2 depicts the estimates of the magnitude square of the velocities of the targets.

Figure 3 illustrates the time-evolution of the model estimate in terms of number of particles that followed each model at every time instant after resampling. Note that the changes in the model (time instants 15s, 37.5s, and 75s) were immediately detected by the particle filter.

We also ran experiments in order to obtain statistical averages on the performance of the algorithm. The presented results correspond to a similar scenario as before where at the beginning there were no targets in the field; at time instant $t = 15$s, a first target appeared, and at time instant $t = 37.5$s, a second target appeared and both targets continued within the field until the end of the simulations. We

[3] Recall that this is the number of particles after resampling.

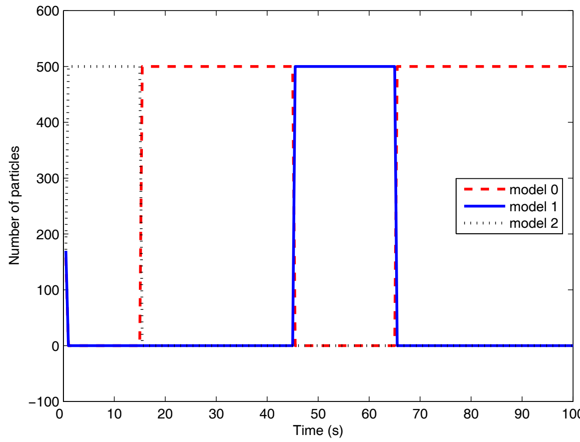

Fig. 3. Evolution of the model estimate with respect to time.

generated one trajectory and we ran 20 realizations of the algorithm using different observation measurements and given a minimum desired signal-to-noise ratio (SNR) when a target was located in the middle of the square formed by 4 adjacent sensors. The probability of incorrect model estimation (P_e) during the simulation time is shown in the following table:

SNR (dB)	P_e
-10	1
-5	0.5
0	0.1
5	0.05
10	0

We also used the data obtained from this experiment to compute the root mean square (RMS) error made in the estimation of the positions. For the k-th target, the RMS is given by

$$RMS_t^j = \sqrt{\frac{1}{20}\sum_{j=1}^{20}\left[(x_{1,k,t}^{est,j} - x_{1,k,t}^{j})^2 + (x_{2,k,t}^{est,j} - x_{2,k,t}^{j})^2\right]}$$

where $[x_{1,k,t}^{j}\ x_{2,k,t}^{j}]^\top$ was the true position of the target at time t in the j-th run, and $[x_{1,t,j}^{est,j}\ x_{2,t,j}^{est,j}]^\top$ was the corresponding estimate obtained by the filter. Figure 4 shows the obtained results for SNR$=$ 0 dB and SNR$=$ 10 dB. Note that these results are in agreement with the table values and that the made errors are acceptable given the dimensions of the sensor field (see Figure 1 (top).)

6. CONCLUSIONS

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We have presented a particle filtering method for detection of the number of targets and estimation of their positions and velocities in a two-dimensional sensor field using acoustic measurements. At every time instant the algorithm tests a set of models which are generated according to a well predefined rule. Simulation results show very good performance of the particle filtering both in detecting the correct number of targets and estimating their positions and velocities in time.

7. REFERENCES

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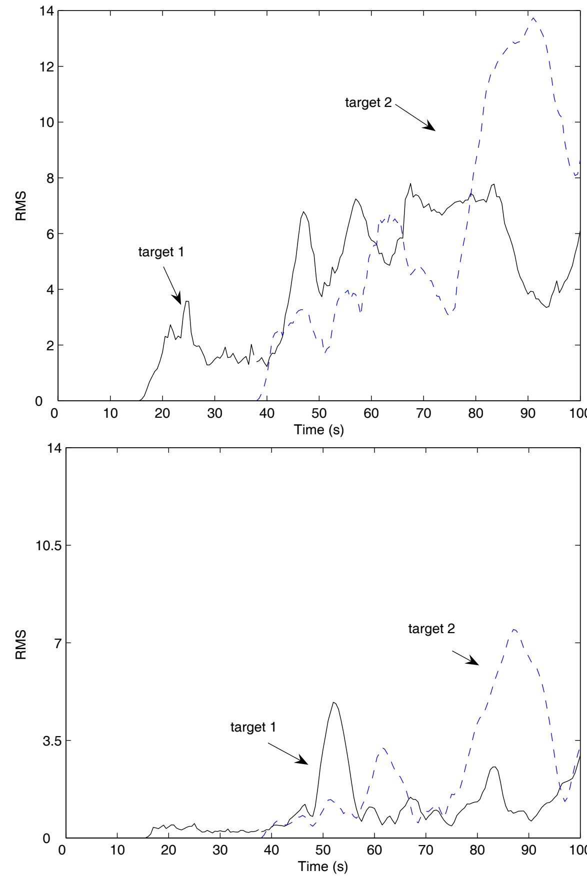

Fig. 4. RMS of the position. Top: SNR$=$ 0 dB. Bottom: SNR$=$ 10 dB.

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